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## Preface

Welcome to *College Physics*, an OpenStax resource. This textbook was written to increase student access to high-quality learning materials, maintaining highest standards of academic rigor at little to no cost.

## About OpenStax

OpenStax is a nonprofit based at Rice University, and it's our mission to improve student access to education. Our first openly licensed college textbook was published in 2012, and our library has since scaled to over 20 books for college and AP courses used by hundreds of thousands of students. Our adaptive learning technology, designed to improve learning outcomes through personalized educational paths, is being piloted in college courses throughout the country. Through our partnerships with philanthropic foundations and our alliance with other educational resource organizations, OpenStax is breaking down the most common barriers to learning and empowering students and instructors to succeed.

## About OpenStax Resources

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## **Errata**

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## **Format**

You can access this textbook for free in web view or PDF through [openstax.org](https://openstax.org), and in low-cost print and iBooks editions.

## **About *College Physics***

*College Physics* meets standard scope and sequence requirements for a two-semester introductory algebra-based physics course. The text is grounded in real-world examples to help students grasp fundamental physics concepts. It requires knowledge of algebra and some trigonometry, but not calculus. *College Physics* includes learning objectives, concept questions, links to labs and simulations, and ample practice opportunities for traditional physics application problems.

## **Coverage and Scope**

College Physics is organized such that topics are introduced conceptually with a steady progression to precise definitions and analytical applications. The analytical aspect (problem solving) is tied back to the conceptual before moving on to another topic. Each introductory chapter, for example, opens with an engaging photograph relevant to the subject of the chapter and interesting applications that are easy for most students to visualize.

Chapter 1: Introduction: The Nature of Science and Physics

Chapter 2: Kinematics

Chapter 3: Two-Dimensional Kinematics

Chapter 4: Dynamics: Force and Newton's Laws of Motion

Chapter 5: Further Applications of Newton's Laws: Friction, Drag, and Elasticity

Chapter 6: Uniform Circular Motion and Gravitation

Chapter 7: Work, Energy, and Energy Resources

Chapter 8: Linear Momentum and Collisions

Chapter 9: Statics and Torque

Chapter 10: Rotational Motion and Angular Momentum

Chapter 11: Fluid Statics

Chapter 12: Fluid Dynamics and Its Biological and Medical Applications

Chapter 13: Temperature, Kinetic Theory, and the Gas Laws

Chapter 14: Heat and Heat Transfer Methods

Chapter 15: Thermodynamics

Chapter 16: Oscillatory Motion and Waves

Chapter 17: Physics of Hearing

Chapter 18: Electric Charge and Electric Field

Chapter 19: Electric Potential and Electric Field

Chapter 20: Electric Current, Resistance, and Ohm's Law

Chapter 21: Circuits and DC Instruments

Chapter 22: Magnetism

Chapter 23: Electromagnetic Induction, AC Circuits, and Electrical Technologies

Chapter 24: Electromagnetic Waves

Chapter 25: Geometric Optics

Chapter 26: Vision and Optical Instruments

Chapter 27: Wave Optics

Chapter 28: Special Relativity

Chapter 29: Introduction to Quantum Physics  
Chapter 30: Atomic Physics  
Chapter 31: Radioactivity and Nuclear Physics  
Chapter 32: Medical Applications of Nuclear Physics  
Chapter 33: Particle Physics  
Chapter 34: Frontiers of Physics  
Appendix A: Atomic Masses  
Appendix B: Selected Radioactive Isotopes  
Appendix C: Useful Information  
Appendix D: Glossary of Key Symbols and Notation

## **Concepts and Calculations**

The ability to calculate does not guarantee conceptual understanding. In order to unify conceptual, analytical, and calculation skills within the learning process, we have integrated Strategies and Discussions throughout the text.

## **Modern Perspective**

The chapters on modern physics are more complete than many other texts on the market, with an entire chapter devoted to medical applications of nuclear physics and another to particle physics. The final chapter of the text, “Frontiers of Physics,” is devoted to the most exciting endeavors in physics. It ends with a module titled “Some Questions We Know to Ask.”

## **Key Features**

### **Modularity**

This textbook is organized as a collection of modules that can be rearranged and modified to suit the needs of a particular professor or class. That being said, modules often contain references to content in other modules, as most topics in physics cannot be discussed in isolation.

### **Learning Objectives**

Every module begins with a set of learning objectives. These objectives are designed to guide the instructor in deciding what content to include or assign, and to guide the student with respect to what he or she can expect to learn. After completing the module and end-of-module exercises, students should be able to demonstrate mastery of the learning objectives.

### **Call-Outs**

Key definitions, concepts, and equations are called out with a special design treatment. Call-outs are designed to catch readers' attention, to make it clear that a specific term, concept, or equation is particularly important, and to provide easy reference for a student reviewing content.

### **Key Terms**

Key terms are in bold and are followed by a definition in context. Definitions of key terms are also listed in the Glossary, which appears at the end of the module.

### **Worked Examples**

Worked examples have four distinct parts to promote both analytical and conceptual skills. Worked examples are introduced in words, always using some application that should be of interest. This is followed by a Strategy section that emphasizes the concepts involved and how solving the problem

relates to those concepts. This is followed by the mathematical Solution and Discussion.

Many worked examples contain multiple-part problems to help the students learn how to approach normal situations, in which problems tend to have multiple parts. Finally, worked examples employ the techniques of the problem-solving strategies so that students can see how those strategies succeed in practice as well as in theory.

### **Problem-Solving Strategies**

Problem-solving strategies are first presented in a special section and subsequently appear at crucial points in the text where students can benefit most from them. Problem-solving strategies have a logical structure that is reinforced in the worked examples and supported in certain places by line drawings that illustrate various steps.

### **Misconception Alerts**

Students come to physics with preconceptions from everyday experiences and from previous courses. Some of these preconceptions are misconceptions, and many are very common among students and the general public. Some are inadvertently picked up through misunderstandings of lectures and texts. The Misconception Alerts feature is designed to point these out and correct them explicitly.

### **Take-Home Investigations**

Take Home Investigations provide the opportunity for students to apply or explore what they have learned with a hands-on activity.

### **Things Great and Small**

In these special topic essays, macroscopic phenomena (such as air pressure) are explained with submicroscopic phenomena (such as atoms bouncing off walls). These essays support the modern perspective by describing aspects of modern physics before they are formally treated in later chapters. Connections are also made between apparently disparate phenomena.

### **Simulations**

Where applicable, students are directed to the interactive PHeT physics simulations developed by the University of Colorado. There they can further explore the physics concepts they have learned about in the module.

### **Summary**

Module summaries are thorough and functional and present all important definitions and equations. Students are able to find the definitions of all terms and symbols as well as their physical relationships. The structure of the summary makes plain the fundamental principles of the module or collection and serves as a useful study guide.

### **Glossary**

At the end of every module or chapter is a Glossary containing definitions of all of the key terms in the module or chapter.

### **End-of-Module Problems**

At the end of every chapter is a set of Conceptual Questions and/or skills-based Problems & Exercises. Conceptual Questions challenge students' ability to explain what they have learned conceptually, independent of the mathematical details. Problems & Exercises challenge students to apply both concepts and skills to solve mathematical physics problems. Online,

every other problem includes an answer that students can reveal immediately by clicking on a “Show Solution” button.

In addition to traditional skills-based problems, there are three special types of end-of-module problems: Integrated Concept Problems, Unreasonable Results Problems, and Construct Your Own Problems. All of these problems are indicated with a subtitle preceding the problem.

### **Integrated Concept Problems**

In Integrated Concept Problems, students are asked to apply what they have learned about two or more concepts to arrive at a solution to a problem. These problems require a higher level of thinking because, before solving a problem, students have to recognize the combination of strategies required to solve it.

### **Unreasonable Results**

In Unreasonable Results Problems, students are challenged to not only apply concepts and skills to solve a problem, but also to analyze the answer with respect to how likely or realistic it really is. These problems contain a premise that produces an unreasonable answer and are designed to further emphasize that properly applied physics must describe nature accurately and is not simply the process of solving equations.

### **Construct Your Own Problem**

These problems require students to construct the details of a problem, justify their starting assumptions, show specific steps in the problem’s solution, and finally discuss the meaning of the result. These types of problems relate well to both conceptual and analytical aspects of physics, emphasizing that physics must describe nature. Often they involve an integration of topics from more than one chapter. Unlike other problems, solutions are not provided since there is no single correct answer.

Instructors should feel free to direct students regarding the level and scope of their considerations. Whether the problem is solved and described correctly will depend on initial assumptions.

## **Additional Resources**

### **Student and Instructor Resources**

We've compiled additional resources for both students and instructors, including Getting Started Guides, an instructor solution manual, and PowerPoint slides. Instructor resources require a verified instructor account, which can be requested on your [openstax.org](https://openstax.org) log-in. Take advantage of these resources to supplement your OpenStax book.

### **Partner Resources**

OpenStax Partners are our allies in the mission to make high-quality learning materials affordable and accessible to students and instructors everywhere. Their tools integrate seamlessly with our OpenStax titles at a low cost. To access the partner resources for your text, visit your book page on [openstax.org](https://openstax.org).

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# Introduction to Science and the Realm of Physics, Physical Quantities, and Units

class="introduction"

Galaxies are  
as immense  
as atoms are  
small. Yet the  
same laws of  
physics  
describe  
both, and all  
the rest of  
nature—an  
indication of  
the  
underlying  
unity in the  
universe. The  
laws of  
physics are  
surprisingly  
few in  
number,  
implying an  
underlying  
simplicity to  
nature's  
apparent  
complexity.  
(credit:  
NASA, JPL-  
Caltech, P.  
Barmby,  
Harvard-  
Smithsonian  
Center for

## Astrophysics )



What is your first reaction when you hear the word “physics”? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

For example, take a look at the image above. This image is of the Andromeda Galaxy, which contains billions of individual stars, huge clouds of gas, and dust. Two smaller galaxies are also visible as bright blue spots in the background. At a staggering 2.5 million light years from the Earth, this galaxy is the nearest one to our own galaxy (which is called the Milky Way). The stars and planets that make up Andromeda might seem to be the furthest thing from most people’s regular, everyday lives. But Andromeda is a great starting point to think about the forces that hold together the universe. The forces that cause Andromeda to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply raise the walls for a new home. The same gravity that causes the stars of Andromeda to rotate and revolve also causes water to flow over hydroelectric dams here on Earth. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater

understanding of the interconnectedness of everything we can see and know in this universe.

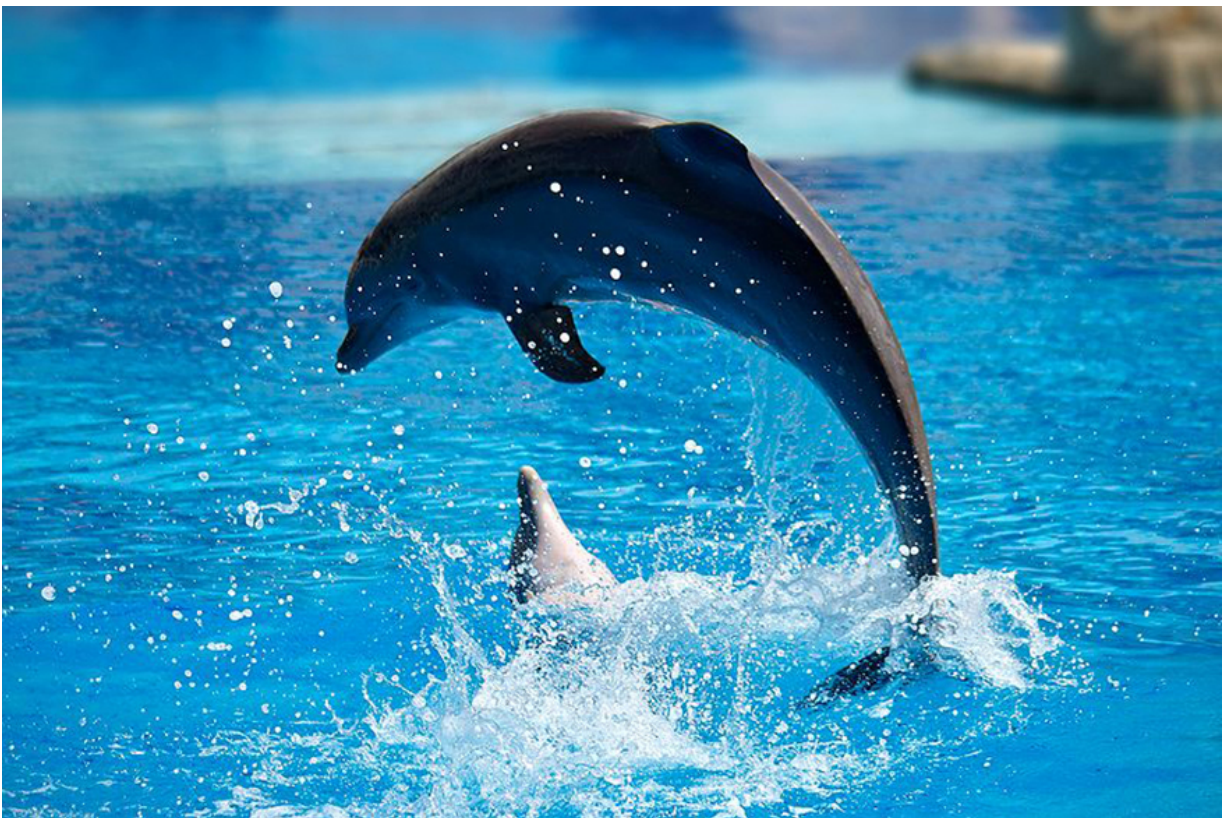
Think now about all of the technological devices that you use on a regular basis. Computers, smart phones, GPS systems, MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, “invisibility cloaks” that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient Greeks, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

## Introduction to Dynamics: Newton's Laws of Motion

class="introduction"

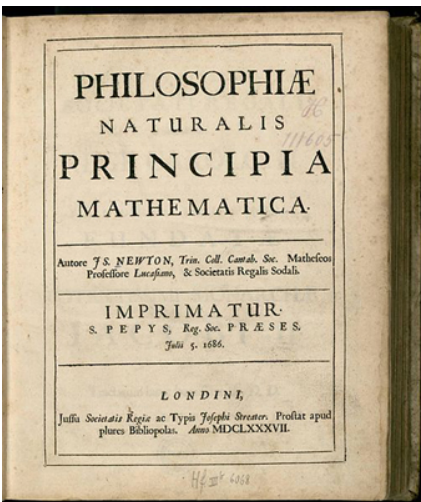
Newton's laws of motion describe the motion of the dolphin's path.  
(credit: Jin Jang)



Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a

dolphin jumping out of the water, or a pole vaulter, or the flight of a bird, or the orbit of a satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

Isaac Newton's (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance into the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. For many centuries natural philosophers had debated the nature of the universe based largely on certain rules of logic with great weight given to the thoughts of earlier classical philosophers such as Aristotle (384–322 BC). Among the many great thinkers who contributed to this change were Newton and Galileo.



Isaac Newton's  
monumental work,  
*Philosophiæ  
Naturalis Principia  
Mathematica*, was  
published in 1687. It  
proposed scientific

laws that are still  
used today to  
describe the motion  
of objects. (credit:  
Service commun de  
la documentation de  
l'Université de  
Strasbourg)

Galileo was instrumental in establishing *observation* as the absolute determinant of truth, rather than “logical” argument. Galileo’s use of the telescope was his most notable achievement in demonstrating the importance of observation. He discovered moons orbiting Jupiter and made other observations that were inconsistent with certain ancient ideas and religious dogma. For this reason, and because of the manner in which he dealt with those in authority, Galileo was tried by the Inquisition and punished. He spent the final years of his life under a form of house arrest. Because others before Galileo had also made discoveries by *observing* the nature of the universe, and because repeated observations verified those of Galileo, his work could not be suppressed or denied. After his death, his work was verified by others, and his ideas were eventually accepted by the church and scientific communities.

Galileo also contributed to the formation of what is now called Newton’s first law of motion. Newton made use of the work of his predecessors, which enabled him to develop laws of motion, discover the law of gravity, invent calculus, and make great contributions to the theories of light and color. It is amazing that many of these developments were made with Newton working alone, without the benefit of the usual interactions that take place among scientists today.

It was not until the advent of modern physics early in the 20th century that it was discovered that Newton’s laws of motion produce a good approximation to motion only when the objects are moving at speeds much, much less than the speed of light and when those objects are larger than the

size of most molecules (about                      in diameter). These constraints define the realm of classical mechanics, as discussed in [Introduction to the Nature of Science and Physics](#). At the beginning of the 20<sup>th</sup> century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, developed quantum theory. This theory does not have the constraints present in classical physics. All of the situations we consider in this chapter, and all those preceding the introduction of relativity in [Special Relativity](#), are in the realm of classical physics.

**Note:**

**Making Connections: Past and Present Philosophy**

*The importance of observation* and the concept of *cause and effect* were not always so entrenched in human thinking. This realization was a part of the evolution of modern physics from natural philosophy. The achievements of Galileo, Newton, Einstein, and others were key milestones in the history of scientific thought. Most of the scientific theories that are described in this book descended from the work of these scientists.

## Newton's First Law of Motion: Inertia

- Define mass and inertia.
- Understand Newton's first law of motion.

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What **Newton's first law of motion** states, however, is the following:

### **Note:**

#### **Newton's First Law of Motion**

A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb “remains.” We can think of this law as preserving the status quo of motion.

Rather than contradicting our experience, **Newton's first law of motion** states that there must be a *cause* (which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)*. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface, we can imagine the object sliding in a straight line indefinitely. Friction is thus the *cause* of the slowing (consistent with Newton's first law). The object would not slow down at all if friction were

completely eliminated. Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object will slow down. Friction is an external force.

Newton's first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important not only here—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law, and Newton, who clarified it, was to ask the fundamental question, “What is the cause?” Thinking in terms of cause and effect is a worldview fundamentally different from the typical ancient Greek approach when questions such as “Why does a tiger have stripes?” would have been answered in Aristotelian fashion, “That is the nature of the beast.” True perhaps, but not a useful insight.

## Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called **inertia**. Newton's first law is often called the **law of inertia**. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its **mass**. Roughly speaking, mass is a measure of the amount of “stuff” (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this

manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

**Exercise:**

**Check Your Understanding**

**Problem:**

Which has more mass: a kilogram of cotton balls or a kilogram of gold?

---

**Solution:**

**Answer**

They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

**Section Summary**

- **Newton's first law of motion** states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the **law of inertia**.
- **Inertia** is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- **Mass** is the quantity of matter in a substance.

**Conceptual Questions**

**Exercise:**

**Problem:** How are inertia and mass related?

**Exercise:**

**Problem:**

What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

## **Glossary**

inertia

the tendency of an object to remain at rest or remain in motion

law of inertia

see Newton's first law of motion

mass

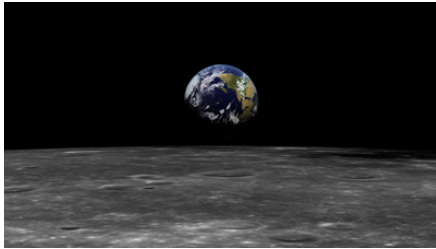
the quantity of matter in a substance; measured in kilograms

Newton's first law of motion

a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

## Physical Quantities and Units

- Perform unit conversions both in the SI and English units.
- Explain the most common prefixes in the SI units and be able to write them in scientific notation.

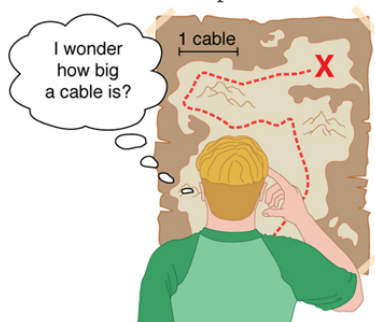


The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit: NASA)

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—all physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See [\[link\]](#).)



Distances given in  
unknown units are  
maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym “SI” is derived from the French *Système International*.

## SI Units: Fundamental and Derived Units

[\[link\]](#) gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Length	Mass	Time	Electric Current
meter (m)	kilogram (kg)	second (s)	ampere (A)

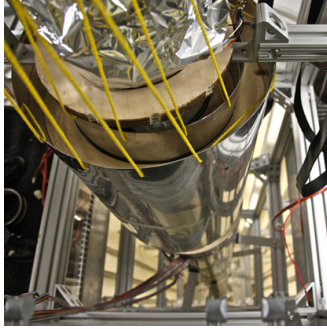
### Fundamental SI Units

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

## Units of Time, Length, and Mass: The Second, Meter, and Kilogram

### The Second

The SI unit for time, the **second**(abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth’s rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. (See [\[link\]](#).) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.



An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall!  
(credit: Steve Jurvetson/Flickr)

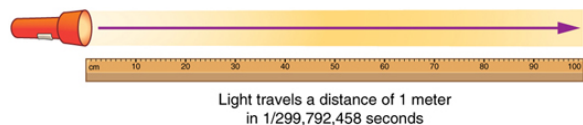
## The Meter

The SI unit for length is the **meter** (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. (See [\[link\]](#).) This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

## The Kilogram

The SI unit for mass is the **kilogram** (abbreviated kg); it is defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram are also kept at the United States' National Institute of Standards

and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses can be ultimately traced to a comparison with the standard mass.



The meter is defined to be the distance light travels in  $1/299,792,458$  of a second in a vacuum. Distance traveled is speed multiplied by time.

Electric current and its accompanying unit, the ampere, will be introduced in [Introduction to Electric Current, Resistance, and Ohm's Law](#) when electricity and magnetism are covered. The initial modules in this textbook are concerned with mechanics, fluids, heat, and waves. In these subjects all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

## Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. [\[link\]](#) gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term **order of magnitude** refers to the scale of a value expressed in the metric system. Each power of 10 in the metric system represents a different order of magnitude. For example,  $10^1$ ,  $10^2$ ,  $10^3$ , and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the *same* order of magnitude. For example, the number 800 can be written as  $8 \times 10^2$ , and the number 450 can be written as  $4.5 \times 10^2$ . Thus, the numbers 800 and 450 are of the same order of magnitude:  $10^2$ . Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of  $10^{-9}$  m, while the diameter of the Sun is on the order of  $10^9$  m.

### Note:

The Quest for Microscopic Standards for Basic Units

The fundamental units described in this chapter are those that produce the greatest accuracy and precision in measurement. There is a sense among physicists that, because there is an underlying microscopic substructure to matter, it would be most satisfying to base our standards of measurement on microscopic objects and fundamental physical phenomena such as the speed of light. A microscopic standard has been accomplished for the standard of time, which is based on the oscillations of the cesium atom.

The standard for length was once based on the wavelength of light (a small-scale length) emitted by a certain type of atom, but it has been supplanted by the more precise measurement of the speed of light. If it becomes possible to measure the mass of atoms or a particular arrangement of atoms such as a silicon sphere to greater precision than the kilogram standard, it may become possible to base mass measurements on the small scale. There are also possibilities that electrical phenomena on the small scale may someday allow us to base a unit of charge on the charge of electrons and protons, but at present current and charge are related to large-scale currents and forces between wires.

Prefix	Symbol	Value <sup>[footnote]</sup> See <a href="#">Appendix A</a> for a discussion of powers of 10.	Example (some are approximate)			
exa	E	$10^{18}$	exameter	Em	$10^{18}$ m	distance light travels in a century
peta	P	$10^{15}$	petasecond	Ps	$10^{15}$ s	30 million years
tera	T	$10^{12}$	terawatt	TW	$10^{12}$ W	powerful laser output
giga	G	$10^9$	gigahertz	GHz	$10^9$ Hz	a microwave frequency
mega	M	$10^6$	megacurie	MCi	$10^6$ Ci	high radioactivity
kilo	k	$10^3$	kilometer	km	$10^3$ m	about 6/10 mile
hecto	h	$10^2$	hectoliter	hL	$10^2$ L	26 gallons

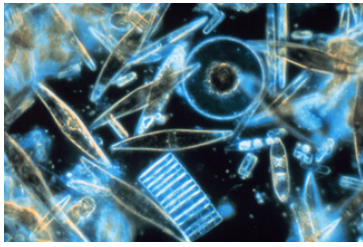
Prefix	Symbol	Value <sup><a href="#">[footnote]</a></sup> See <a href="#">Appendix A</a> for a discussion of powers of 10.	Example (some are approximate)			
deka	da	$10^1$	dekagram	dag	$10^1$ g	teaspoon of butter
—	—	$10^0$ (=1)				
deci	d	$10^{-1}$	deciliter	dL	$10^{-1}$ L	less than half a soda
centi	c	$10^{-2}$	centimeter	cm	$10^{-2}$ m	fingertip thickness
milli	m	$10^{-3}$	millimeter	mm	$10^{-3}$ m	flea at its shoulders
micro	$\mu$	$10^{-6}$	micrometer	$\mu\text{m}$	$10^{-6}$ m	detail in microscope
nano	n	$10^{-9}$	nanogram	ng	$10^{-9}$ g	small speck of dust
pico	p	$10^{-12}$	picofarad	pF	$10^{-12}$ F	small capacitor in radio
femto	f	$10^{-15}$	femtometer	fm	$10^{-15}$ m	size of a proton
atto	a	$10^{-18}$	attosecond	as	$10^{-18}$ s	time light crosses an atom

Metric Prefixes for Powers of 10 and their Symbols

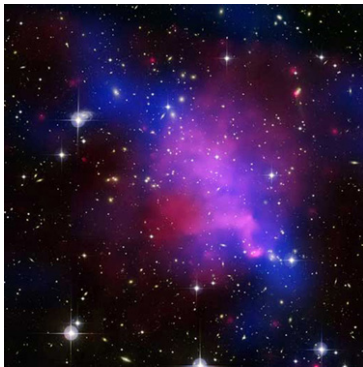
### Known Ranges of Length, Mass, and Time

The vastness of the universe and the breadth over which physics applies are illustrated by the wide range of examples of known lengths, masses, and times in [\[link\]](#). Examination of this table will give you some

feeling for the range of possible topics and numerical values. (See [\[link\]](#) and [\[link\]](#).)



Tiny phytoplankton  
swims among crystals of  
ice in the Antarctic Sea.  
They range from a few  
micrometers to as much  
as 2 millimeters in length.  
(credit: Prof. Gordon T.  
Taylor, Stony Brook  
University; NOAA Corps  
Collections)



Galaxies collide 2.4  
billion light years away  
from Earth. The  
tremendous range of  
observable phenomena in  
nature challenges the  
imagination. (credit:  
NASA/CXC/UVic./A.  
Mahdavi et al.  
Optical/lensing:  
CFHT/UVic./H. Hoekstra  
et al.)

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:

Equation:

80 m × (1 km / 1000 m) = 0.080 km.

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

Click [\[link\]](#) for a more complete list of conversion factors.

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
10 <sup>-18</sup>	Present experimental limit to smallest observable detail	10 <sup>-30</sup>	Mass of an electron (9.11 × 10 <sup>-31</sup> kg)	10 <sup>-23</sup>	Time for light to cross a proton
10 <sup>-15</sup>	Diameter of a proton	10 <sup>-27</sup>	Mass of a hydrogen atom (1.67 × 10 <sup>-27</sup> kg)	10 <sup>-22</sup>	Mean life of an extremely unstable nucleus

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
$10^{-14}$	Diameter of a uranium nucleus	$10^{-15}$	Mass of a bacterium	$10^{-15}$	Time for one oscillation of visible light
$10^{-10}$	Diameter of a hydrogen atom	$10^{-5}$	Mass of a mosquito	$10^{-13}$	Time for one vibration of an atom in a solid
$10^{-8}$	Thickness of membranes in cells of living organisms	$10^{-2}$	Mass of a hummingbird	$10^{-8}$	Time for one oscillation of an FM radio wave
$10^{-6}$	Wavelength of visible light	1	Mass of a liter of water (about a quart)	$10^{-3}$	Duration of a nerve impulse
$10^{-3}$	Size of a grain of sand	$10^2$	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	$10^3$	Mass of a car	$10^5$	One day ( $8.64 \times 10^4$ s)
$10^2$	Length of a football field	$10^8$	Mass of a large ship	$10^7$	One year (y) ( $3.16 \times 10^7$ s)
$10^4$	Greatest ocean depth	$10^{12}$	Mass of a large iceberg	$10^9$	About half the life expectancy of a human
$10^7$	Diameter of the Earth	$10^{15}$	Mass of the nucleus of a comet	$10^{11}$	Recorded history
$10^{11}$	Distance from the Earth to the Sun	$10^{23}$	Mass of the Moon ( $7.35 \times 10^{22}$ kg)	$10^{17}$	Age of the Earth
$10^{16}$	Distance traveled by light in 1 year (a light year)	$10^{25}$	Mass of the Earth ( $5.97 \times 10^{24}$ kg)	$10^{18}$	Age of the universe
$10^{21}$	Diameter of the Milky Way galaxy	$10^{30}$	Mass of the Sun ( $1.99 \times 10^{30}$ kg)		

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
$10^{22}$	Distance from the Earth to the nearest large galaxy (Andromeda)	$10^{42}$	Mass of the Milky Way galaxy (current upper limit)		
$10^{26}$	Distance from the Earth to the edges of the known universe	$10^{53}$	Mass of the known universe (current upper limit)		

### Approximate Values of Length, Mass, and Time

#### Example:

##### Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

##### Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

##### Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

##### Equation:

$$\text{average speed} = \frac{\text{distance}}{\text{time}}.$$

(2) Substitute the given values for distance and time.

##### Equation:

$$\text{average speed} = \frac{10.0 \text{ km}}{20.0 \text{ min}} = 0.500 \frac{\text{km}}{\text{min}}.$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/hr. Thus,

##### Equation:

$$\text{average speed} = 0.500 \frac{\text{km}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ h}} = 30.0 \frac{\text{km}}{\text{h}}.$$

##### Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

**Equation:**

$$\frac{\text{km}}{\text{min}} \times \frac{1 \text{ hr}}{60 \text{ min}} = \frac{1}{60} \frac{\text{km} \cdot \text{hr}}{\text{min}^2},$$

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/hr does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

**Solution for (b)**

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

**Equation:**

$$\text{Average speed} = 30.0 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ h}}{3,600 \text{ s}} \times \frac{1,000 \text{ m}}{1 \text{ km}},$$

**Equation:**

$$\text{Average speed} = 8.33 \frac{\text{m}}{\text{s}}.$$

**Discussion for (b)**

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits.

Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces? The module [Accuracy, Precision, and Significant Figures](#) will help you answer these questions.

**Note:**

**Nonstandard Units**

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different “weights and measures.” Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

**Exercise:**  
**Check Your Understanding**

**Problem:**

Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

---

**Solution:**

The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or  $10^{-3}$  seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

**Exercise:**  
**Check Your Understanding**

**Problem:**

One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

---

**Solution:**

The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

**Summary**

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

**Conceptual Questions**

**Exercise:**

**Problem:** Identify some advantages of metric units.

**Problems & Exercises**

**Exercise:****Problem:**

The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?

---

**Solution:**

- a. 27.8 m/s
- b. 62.1 mph

**Exercise:****Problem:**

A car is traveling at a speed of 33 m/s. (a) What is its speed in kilometers per hour? (b) Is it exceeding the 90 km/h speed limit?

**Exercise:****Problem:**

Show that  $1.0 \text{ m/s} = 3.6 \text{ km/h}$ . Hint: Show the explicit steps involved in converting  $1.0 \text{ m/s} = 3.6 \text{ km/h}$ .

---

**Solution:**

$$\begin{aligned}\frac{1.0 \text{ m}}{\text{s}} &= \frac{1.0 \text{ m}}{\text{s}} \times \frac{3600 \text{ s}}{1 \text{ hr}} \times \frac{1 \text{ km}}{1000 \text{ m}} \\ &= 3.6 \text{ km/h.}\end{aligned}$$

**Exercise:****Problem:**

American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)

**Exercise:****Problem:**

Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)

---

**Solution:**

length: 377 ft;  $4.53 \times 10^3$  in. width: 280 ft;  $3.3 \times 10^3$  in.

**Exercise:****Problem:**

What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)

**Exercise:**

**Problem:**

Mount Everest, at 29,028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3,281 feet.)

---

**Solution:**

8.847 km

**Exercise:**

**Problem:** The speed of sound is measured to be 342 m/s on a certain day. What is this in km/h?

**Exercise:****Problem:**

Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?

---

**Solution:**

(a)  $1.3 \times 10^{-9}$  m

(b) 40 km/My

**Exercise:****Problem:**

(a) Refer to [\[link\]](#) to determine the average distance between the Earth and the Sun. Then calculate the average speed of the Earth in its orbit in kilometers per second. (b) What is this in meters per second?

**Glossary**

physical quantity

a characteristic or property of an object that can be measured or calculated from other measurements

units

a standard used for expressing and comparing measurements

SI units

the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

English units

system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

fundamental units

units that can only be expressed relative to the procedure used to measure them

derived units

units that can be calculated using algebraic combinations of the fundamental units

second

the SI unit for time, abbreviated (s)

meter

the SI unit for length, abbreviated (m)

kilogram

the SI unit for mass, abbreviated (kg)

metric system

a system in which values can be calculated in factors of 10

order of magnitude

refers to the size of a quantity as it relates to a power of 10

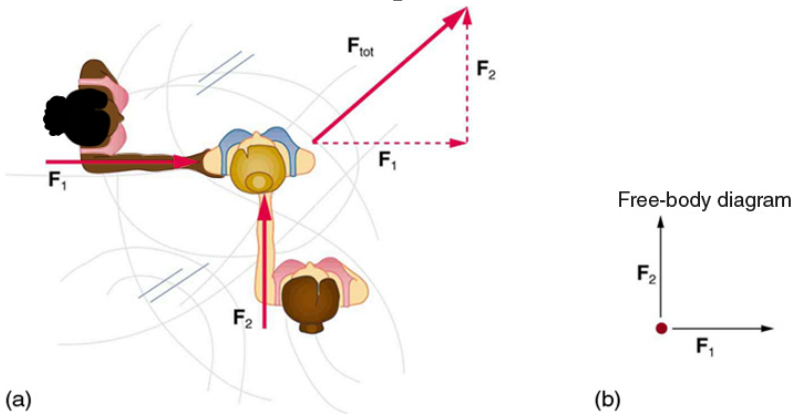
conversion factor

a ratio expressing how many of one unit are equal to another unit

## Development of Force Concept

- Understand the definition of force.

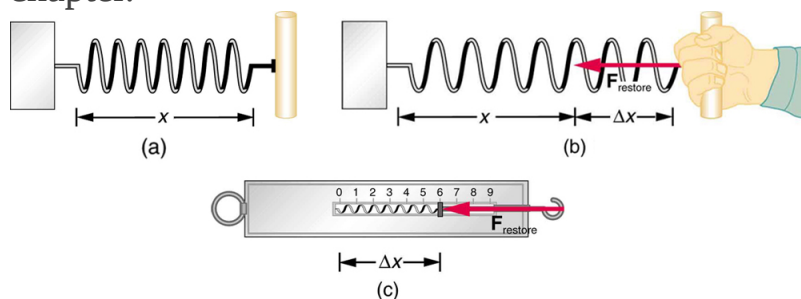
**Dynamics** is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of **force**—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, as illustrated in [\[link\]](#), we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors, as illustrated in [\[link\]\(a\)](#) for two ice skaters. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or by trigonometric methods. These ideas were developed in [Two-Dimensional Kinematics](#).



Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater.

[\[link\]](#)(b) is our first example of a **free-body diagram**, which is a technique used to illustrate all the **external forces** acting on a body. The body is represented by a single isolated point (or free body), and only those forces acting *on* the body from the outside (external forces) are shown. (These forces are the only ones shown, because only external forces acting on the body affect its motion. We can ignore any internal forces within the body.) Free-body diagrams are very useful in analyzing forces acting on a system and are employed extensively in the study and application of Newton's laws of motion.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in [\[link\]](#), and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force. Many other possibilities exist for standard forces. (One that we will encounter in [Magnetism](#) is the magnetic force between two wires carrying electric current.) Some alternative definitions of force will be given later in this chapter.



The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length  $x$  when undistorted. (b) When stretched a distance  $\Delta x$ , the spring exerts a restoring force,  $\mathbf{F}_{\text{restore}}$ , which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force  $\mathbf{F}_{\text{restore}}$  is exerted on whatever is attached to the hook. Here  $\mathbf{F}_{\text{restore}}$  has a

magnitude of 6 units in the force standard being employed.

**Note:**

**Take-Home Experiment: Force Standards**

To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

## Section Summary

- **Dynamics** is the study of how forces affect the motion of objects.
- **Force** is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.
- **External forces** are any outside forces that act on a body. A **free-body diagram** is a drawing of all external forces acting on a body.

## Conceptual Questions

**Exercise:**

**Problem:**

Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.

**Exercise:****Problem:**

What properties do forces have that allow us to classify them as vectors?

**Glossary**

dynamics

the study of how forces affect the motion of objects and systems

external force

a force acting on an object or system that originates outside of the object or system

free-body diagram

a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

force

a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

## Vectors, Scalars, and Coordinate Systems

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.



The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the  $x$ -coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

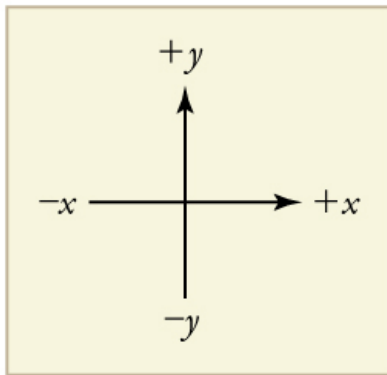
The direction of a vector in one-dimensional motion is given simply by a plus (+) or minus (−) sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a 20°C temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a −20°C temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

## Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in [\[link\]](#), it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are

running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.



It is usually convenient to consider motion upward or to the right as positive (+) and motion downward or to the left as negative (−).

### **Exercise:**

#### **Check Your Understanding**

##### **Problem:**

A person's speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

---

##### **Solution:**

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

## Section Summary

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

## Conceptual Questions

### Exercise:

#### Problem:

A student writes, “A bird that is diving for prey has a speed of  $-10\text{ m/s}$ .” What is wrong with the student’s statement? What has the student actually described? Explain.

### Exercise:

**Problem:** What is the speed of the bird in [\[link\]](#)?

### Exercise:

#### Problem:

Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

### Exercise:

**Problem:**

A weather forecast states that the temperature is predicted to be  $-5^{\circ}\text{C}$  the following day. Is this temperature a vector or a scalar quantity? Explain.

**Glossary**

scalar

a quantity that is described by magnitude, but not direction

vector

a quantity that is described by both magnitude and direction

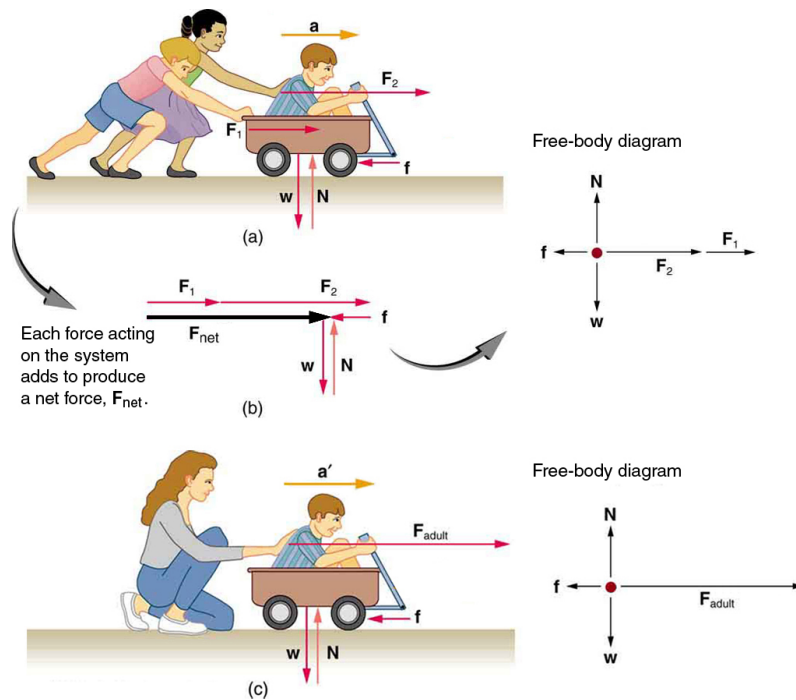
## Newton's Second Law of Motion: Concept of a System

- Define net force, external force, and system.
- Understand Newton's second law of motion.
- Apply Newton's second law to determine the weight of an object.

**Newton's second law of motion** is closely related to Newton's first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton's second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton's second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an **acceleration**. Newton's first law says that a net external force causes a change in motion; thus, we see that a *net external force causes acceleration*.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct—an **external force** acts from outside the **system** of interest. For example, in [\[link\]](#)(a) the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again looking at [\[link\]](#)(a), the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton's first law. (The internal forces actually cancel, as we shall see in the next section.) *You must define the boundaries of the system before you can determine which forces are external.* Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton's laws. This concept will be revisited many times on our journey through physics.



Different forces exerted on the same mass produce different accelerations. (a) Two children push a wagon with a child in it. Arrows representing all external forces are shown. The system of interest is the wagon and its rider. The weight  $w$  of the system and the support of the ground  $N$  are also shown for completeness and are assumed to cancel. The vector  $f$  represents the friction acting on the wagon, and it acts to the left, opposing the motion of the wagon. (b) All of the external forces acting on the system add together to produce a net force,  $F_{net}$ . The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, we draw the vectors collinearly. (c) A larger net external force produces a larger

acceleration ( $\mathbf{a}' > \mathbf{a}$ ) when an adult pushes the child.

Now, it seems reasonable that acceleration should be directly proportional to and in the same direction as the net (total) external force acting on a system. This assumption has been verified experimentally and is illustrated in [\[link\]](#). In part (a), a smaller force causes a smaller acceleration than the larger force illustrated in part (c). For completeness, the vertical forces are also shown; they are assumed to cancel since there is no acceleration in the vertical direction. The vertical forces are the weight  $\mathbf{w}$  and the support of the ground  $\mathbf{N}$ , and the horizontal force  $\mathbf{f}$  represents the force of friction. These will be discussed in more detail in later sections. For now, we will define **friction** as a force that opposes the motion past each other of objects that are touching. [\[link\]](#)(b) shows how vectors representing the external forces add together to produce a net force,  $\mathbf{F}_{\text{net}}$ .

To obtain an equation for Newton's second law, we first write the relationship of acceleration and net external force as the proportionality **Equation:**

$$\mathbf{a} \propto \mathbf{F}_{\text{net}},$$

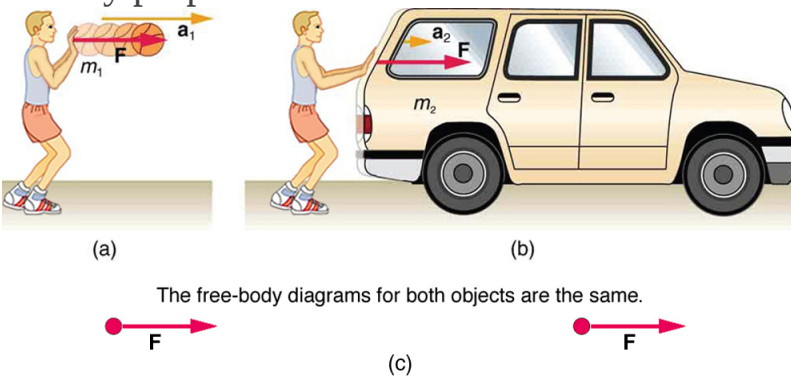
where the symbol  $\propto$  means “proportional to,” and  $\mathbf{F}_{\text{net}}$  is the **net external force**. (The net external force is the vector sum of all external forces and can be determined graphically, using the head-to-tail method, or analytically, using components. The techniques are the same as for the addition of other vectors, and are covered in [Two-Dimensional Kinematics](#).) This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*. Once the system of interest is chosen, it is important to identify the external forces and ignore the internal ones. It is a tremendous simplification not to have to consider the numerous internal forces acting between objects within the system, such as muscular forces within the child's body, let alone the myriad of forces between atoms in the objects, but by doing so, we can easily solve some very complex problems with only minimal error due to our simplification

Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. And indeed, as illustrated in [\[link\]](#), the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

**Equation:**

$$\mathbf{a} \propto \frac{1}{m}$$

where  $m$  is the mass of the system. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is exactly linearly proportional to the net external force.



The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems.

It has been found that the acceleration of an object depends *only* on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

**Note:**

**Newton's Second Law of Motion**

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

In equation form, Newton's second law of motion is

**Equation:**

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}.$$

This is often written in the more familiar form

**Equation:**

$$\mathbf{F}_{\text{net}} = m\mathbf{a}.$$

When only the magnitude of force and acceleration are considered, this equation is simply

**Equation:**

$$F_{\text{net}} = ma.$$

Although these last two equations are really the same, the first gives more insight into what Newton's second law means. The law is a *cause and effect relationship* among three quantities that is not simply based on their definitions. The validity of the second law is completely based on experimental verification.

## Units of Force

$\mathbf{F}_{\text{net}} = m\mathbf{a}$  is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the **newton** (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of  $1\text{m/s}^2$ . That is, since  $\mathbf{F}_{\text{net}} = m\mathbf{a}$ ,

**Equation:**

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2.$$

While almost the entire world uses the newton for the unit of force, in the United States the most familiar unit of force is the pound (lb), where  $1 \text{ N} = 0.225 \text{ lb}$ .

## Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its **weight  $\mathbf{w}$** . Weight can be denoted as a vector  $\mathbf{w}$  because it has a direction; *down* is, by definition, the direction of gravity, and hence weight is a downward force. The magnitude of weight is denoted as  $w$ . Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration  $g$ . Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass  $m$  falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude  $w$ . Newton's second law states that the magnitude of the net external force on an object is  $F_{\text{net}} = ma$ .

Since the object experiences only the downward force of gravity,  $F_{\text{net}} = w$ . We know that the acceleration of an object due to gravity is  $g$ , or  $a = g$ . Substituting these into Newton's second law gives

**Note:****Weight**

This is the equation for *weight*—the gravitational force on a mass  $m$ :

**Equation:**

$$w = mg.$$

Since  $g = 9.80 \text{ m/s}^2$  on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:

**Equation:**

$$w = mg = (1.0 \text{ kg})(9.80 \text{ m/s}^2) = 9.8 \text{ N}.$$

Recall that  $g$  can take a positive or negative value, depending on the positive direction in the coordinate system. Be sure to take this into consideration when solving problems with weight.

When the net external force on an object is its weight, we say that it is in **free-fall**. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity  $g$  varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth's surface. On the Moon, for example, the acceleration due to gravity is only  $1.67 \text{ m/s}^2$ . A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that *the weight of an object is the gravitational force on it from the nearest large body*, such as Earth, the Moon, the Sun, and so on. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of “weightlessness” and

“microgravity,” they are really referring to the phenomenon we call “free-fall” in physics. We shall use the above definition of weight, and we will make careful distinctions between free-fall and actual weightlessness.

It is important to be aware that weight and mass are very different physical quantities, although they are closely related. Mass is the quantity of matter (how much “stuff”) and does not vary in classical physics, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our “weight” in kilograms, but never in the correct units of newtons.

**Note:**

**Common Misconceptions: Mass vs. Weight**

Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram (or the “slug” in English units). Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object ( $m$ ) multiplied by the acceleration due to gravity ( $g$ ). Like any other force, weight is measured in terms of newtons (or pounds in English units).

Assuming the mass of an object is kept intact, it will remain the same, regardless of its location. However, because weight depends on the acceleration due to gravity, the weight of an object *can change* when the object enters into a region with stronger or weaker gravity. For example, the acceleration due to gravity on the Moon is  $1.67 \text{ m/s}^2$  (which is much less than the acceleration due to gravity on Earth,  $9.80 \text{ m/s}^2$ ). If you measured your weight on Earth and then measured your weight on the Moon, you would find that you “weigh” much less, even though you do not look any skinnier. This is because the force of gravity is weaker on the Moon. In fact, when people say that they are “losing weight,” they really

mean that they are losing “mass” (which in turn causes them to weigh less).

**Note:**

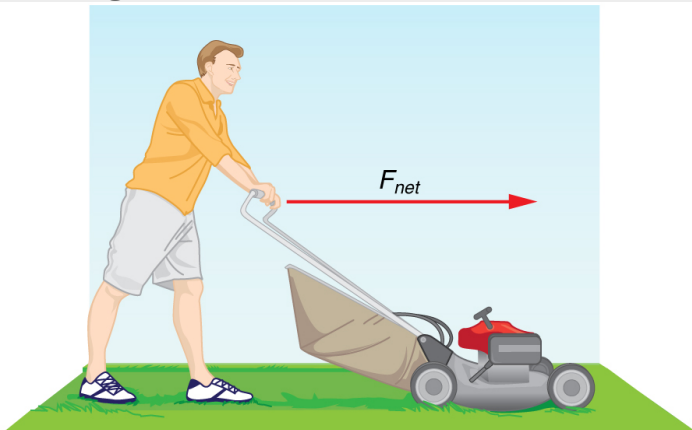
**Take-Home Experiment: Mass and Weight**

What do bathroom scales measure? When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same “mass” on Earth as on the Moon?

**Example:**

**What Acceleration Can a Person Produce when Pushing a Lawn Mower?**

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb) parallel to the ground. The mass of the mower is 24 kg. What is its acceleration?



The net force on a lawn mower is 51

N to the right. At what rate does the lawn mower accelerate to the right?

### Strategy

Since  $\mathbf{F}_{\text{net}}$  and  $m$  are given, the acceleration can be calculated directly from Newton's second law as stated in  $\mathbf{F}_{\text{net}} = m\mathbf{a}$ .

### Solution

The magnitude of the acceleration  $a$  is  $a = \frac{F_{\text{net}}}{m}$ . Entering known values gives

**Equation:**

$$a = \frac{51 \text{ N}}{24 \text{ kg}}$$

Substituting the units  $\text{kg} \cdot \text{m}/\text{s}^2$  for N yields

**Equation:**

$$a = \frac{51 \text{ kg} \cdot \text{m}/\text{s}^2}{24 \text{ kg}} = 2.1 \text{ m}/\text{s}^2.$$

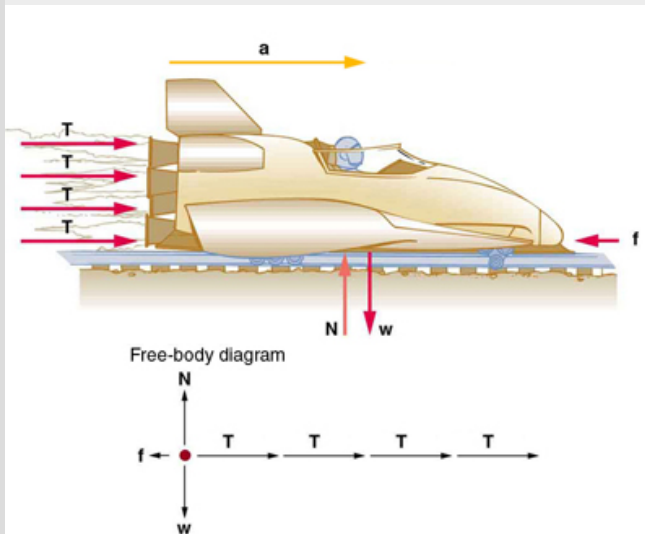
### Discussion

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moves forward), and the vertical forces must cancel if there is to be no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long because the person's top speed would soon be reached.

### Example:

### What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust  $\mathbf{T}$ , for the four-rocket propulsion system shown in [\[link\]](#). The sled's initial acceleration is  $49 \text{ m/s}^2$ , the mass of the system is  $2100 \text{ kg}$ , and the force of friction opposing the motion is known to be  $650 \text{ N}$ .



A sled experiences a rocket thrust that accelerates it to the right. Each rocket creates an identical thrust  $\mathbf{T}$ . As in other situations where there is only horizontal acceleration, the vertical forces cancel. The ground exerts an upward force  $\mathbf{N}$  on the system that is equal in magnitude and opposite in direction to its weight,  $\mathbf{w}$ . The system here is the sled, its rockets, and rider, so none of the forces *between* these objects are considered. The arrow representing friction ( $\mathbf{f}$ ) is drawn larger than scale.

**Strategy**

Although there are forces acting vertically and horizontally, we assume the vertical forces cancel since there is no vertical acceleration. This leaves us with only horizontal forces and a simpler one-dimensional problem.

Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in the figure.

**Solution**

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines.

Since we have defined the direction of the force and acceleration as acting "to the right," we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

**Equation:**

$$F_{\text{net}} = ma,$$

where  $F_{\text{net}}$  is the net force along the horizontal direction. We can see from [\[link\]](#) that the engine thrusts add, while friction opposes the thrust. In equation form, the net external force is

**Equation:**

$$F_{\text{net}} = 4T - f.$$

Substituting this into Newton's second law gives

**Equation:**

$$F_{\text{net}} = ma = 4T - f.$$

Using a little algebra, we solve for the total thrust  $4T$ :

**Equation:**

$$4T = ma + f.$$

Substituting known values yields

**Equation:**

$$4T = ma + f = (2100 \text{ kg})(49 \text{ m/s}^2) + 650 \text{ N}.$$

So the total thrust is

**Equation:**

$$4T = 1.0 \times 10^5 \text{ N},$$

and the individual thrusts are

**Equation:**

$$T = \frac{1.0 \times 10^5 \text{ N}}{4} = 2.6 \times 10^4 \text{ N}.$$

### Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and the setup designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 *g*'s. (Recall that *g*, the acceleration due to gravity, is 9.80 m/s<sup>2</sup>. When we say that an acceleration is 45 *g*'s, it is 45 × 9.80 m/s<sup>2</sup>, which is approximately 440 m/s<sup>2</sup>.) While living subjects are not used any more, land speeds of 10,000 km/h have been obtained with rocket sleds. In this example, as in the preceding one, the system of interest is obvious. We will see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.

Newton's second law of motion is more than a definition; it is a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature. The next section introduces the third and final law of motion.

### Section Summary

- Acceleration, **a**, is defined as a change in velocity, meaning a change in its magnitude or direction, or both.
- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the

system.

- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is  $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$ .
- This is often written in the more familiar form:  $\mathbf{F}_{\text{net}} = m\mathbf{a}$ .
- The weight  $\mathbf{w}$  of an object is defined as the force of gravity acting on an object of mass  $m$ . The object experiences an acceleration due to gravity  $\mathbf{g}$ :

**Equation:**

$$\mathbf{w} = m\mathbf{g}.$$

- If the only force acting on an object is due to gravity, the object is in free fall.
- Friction is a force that opposes the motion past each other of objects that are touching.

## Conceptual Questions

**Exercise:**

**Problem:**

Which statement is correct? (a) Net force causes motion. (b) Net force causes change in motion. Explain your answer and give an example.

**Exercise:**

**Problem:**

Why can we neglect forces such as those holding a body together when we apply Newton's second law of motion?

**Exercise:**

**Problem:**

Explain how the choice of the “system of interest” affects which forces must be considered when applying Newton’s second law of motion.

**Exercise:**

**Problem:**

Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.

**Exercise:**

**Problem:**

A system can have a nonzero velocity while the net external force on it is zero. Describe such a situation.

**Exercise:**

**Problem:**

A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory?

**Exercise:**

**Problem:**

(a) Give an example of different net external forces acting on the same system to produce different accelerations. (b) Give an example of the same net external force acting on systems of different masses, producing different accelerations. (c) What law accurately describes both effects? State it in words and as an equation.

**Exercise:**

**Problem:**

If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.

**Exercise:**

**Problem:**

If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?

**Exercise:****Problem:**

The gravitational force on the basketball in [\[link\]](#) is ignored. When gravity *is* taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?

**Problem Exercises**

**You may assume data taken from illustrations is accurate to three digits.**

**Exercise:****Problem:**

A 63.0-kg sprinter starts a race with an acceleration of  $4.20 \text{ m/s}^2$ . What is the net external force on him?

---

**Solution:**

265 N

**Exercise:****Problem:**

If the sprinter from the previous problem accelerates at that rate for 20 m, and then maintains that velocity for the remainder of the 100-m dash, what will be his time for the race?

**Exercise:**

**Problem:**

A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of its acceleration.

---

**Solution:**

$$13.3 \text{ m/s}^2$$

**Exercise:****Problem:**

Since astronauts in orbit are apparently weightless, a clever method of measuring their masses is needed to monitor their mass gains or losses to adjust diets. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted and the astronaut's acceleration is measured to be  $0.893 \text{ m/s}^2$ . (a) Calculate her mass. (b) By exerting a force on the astronaut, the vehicle in which they orbit experiences an equal and opposite force. Discuss how this would affect the measurement of the astronaut's acceleration. Propose a method in which recoil of the vehicle is avoided.

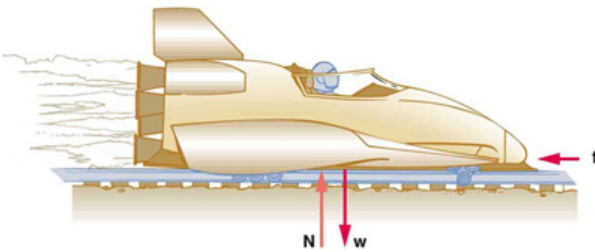
**Exercise:****Problem:**

In [\[link\]](#), the net external force on the 24-kg mower is stated to be 51 N. If the force of friction opposing the motion is 24 N, what force  $F$  (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force  $F$  is removed. How far will the mower go before stopping?

**Exercise:**

**Problem:**

The same rocket sled drawn in [\[link\]](#) is decelerated at a rate of  $196 \text{ m/s}^2$ . What force is necessary to produce this deceleration? Assume that the rockets are off. The mass of the system is 2100 kg.

**Exercise:****Problem:**

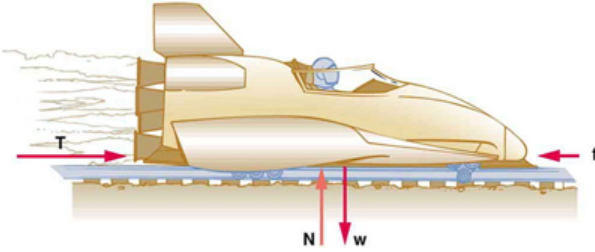
(a) If the rocket sled shown in [\[link\]](#) starts with only one rocket burning, what is the magnitude of its acceleration? Assume that the mass of the system is 2100 kg, the thrust  $T$  is  $2.4 \times 10^4 \text{ N}$ , and the force of friction opposing the motion is known to be 650 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?

---

**Solution:**

(a)  $12 \text{ m/s}^2$ .

(b) The acceleration is not one-fourth of what it was with all rockets burning because the frictional force is still as large as it was with all rockets burning.



### Exercise:

#### Problem:

What is the deceleration of the rocket sled if it comes to rest in 1.1 s from a speed of 1000 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)

### Exercise:

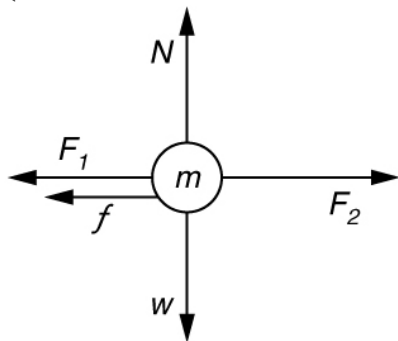
#### Problem:

Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg. (a) What is the system of interest if the acceleration of the child in the wagon is to be calculated? (b) Draw a free-body diagram, including all forces acting on the system. (c) Calculate the acceleration. (d) What would the acceleration be if friction were 15.0 N?

#### Solution:

(a) The system is the child in the wagon plus the wagon.

(b)



(c)  $a = 0.130 \text{ m/s}^2$  in the direction of the second child's push.

(d)  $a = 0.00 \text{ m/s}^2$

### Exercise:

#### Problem:

A powerful motorcycle can produce an acceleration of  $3.50 \text{ m/s}^2$  while traveling at  $90.0 \text{ km/h}$ . At that speed the forces resisting motion, including friction and air resistance, total  $400 \text{ N}$ . (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force the motorcycle exerts backward on the ground to produce its acceleration if the mass of the motorcycle with rider is  $245 \text{ kg}$ ?

### Exercise:

#### Problem:

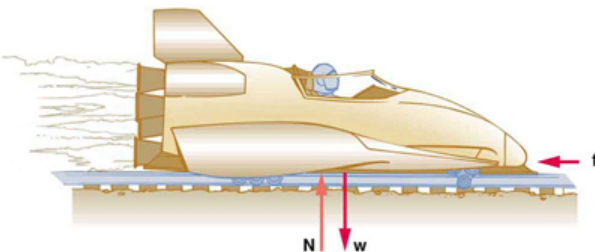
The rocket sled shown in [\[link\]](#) accelerates at a rate of  $49.0 \text{ m/s}^2$ . Its passenger has a mass of  $75.0 \text{ kg}$ . (a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight by using a ratio. (b) Calculate the direction and magnitude of the total force the seat exerts against his body.

---

#### Solution:

(a)  $3.68 \times 10^3 \text{ N}$ . This force is 5.00 times greater than his weight.

(b)  $3750 \text{ N}$ ;  $11.3^\circ$  above horizontal



**Exercise:****Problem:**

Repeat the previous problem for the situation in which the rocket sled decelerates at a rate of  $201 \text{ m/s}^2$ . In this problem, the forces are exerted by the seat and restraining belts.

**Exercise:****Problem:**

The weight of an astronaut plus his space suit on the Moon is only 250 N. How much do they weigh on Earth? What is the mass on the Moon? On Earth?

---

**Solution:**

$1.5 \times 10^3 \text{ N}$ , 150 kg, 150 kg

**Exercise:****Problem:**

Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 10,000 kg. The thrust of its engines is 30,000 N. (a) Calculate its the magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.

**Glossary**

acceleration

the rate at which an object's velocity changes over a period of time

free-fall

a situation in which the only force acting on an object is the force due to gravity

friction

a force past each other of objects that are touching; examples include rough surfaces and air resistance

net external force

the vector sum of all external forces acting on an object or system; causes a mass to accelerate

Newton's second law of motion

the net external force  $\mathbf{F}_{\text{net}}$  on an object with mass  $m$  is proportional to and in the same direction as the acceleration of the object,  $\mathbf{a}$ , and inversely proportional to the mass; defined mathematically as

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$$

system

defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces

weight

the force  $\mathbf{w}$  due to gravity acting on an object of mass  $m$ ; defined mathematically as:  $\mathbf{w} = m\mathbf{g}$ , where  $\mathbf{g}$  is the magnitude and direction of the acceleration due to gravity

## Physics: An Introduction

- Explain the difference between a principle and a law.
- Explain the difference between a model and a theory.



The flight formations of migratory birds such as Canada geese are governed by the laws of physics.  
(credit: David Merrett)

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be

converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

## Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone ([\[link\]](#)). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and

circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another.



The Apple  
“iPhone” is a  
common  
smart phone  
with a GPS  
function.

Physics  
describes the  
way that  
electricity  
flows through  
the circuits of  
this device.  
Engineers use  
their  
knowledge of  
physics to  
construct an

iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: @gletham GIS, Social, Mobile Tech Images)

## **Applications of Physics**

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See [\[link\]](#) and [\[link\]](#).) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car's ignition system as well as the transmission of electrical signals through our body's nervous system are

much easier to understand when you think about them in terms of basic physics.

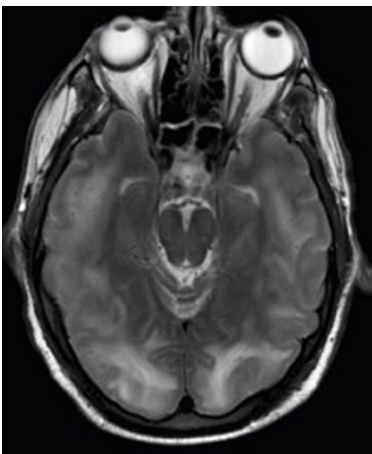
Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes ([\[link\]](#) and [\[link\]](#)). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

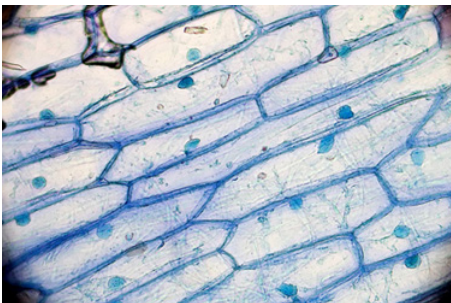
It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.



The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz)

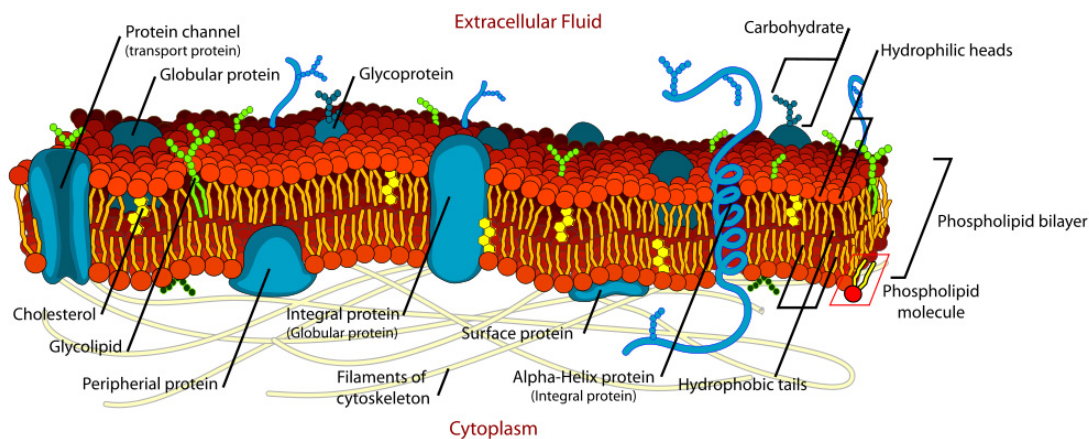


These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined.  
(credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik)



Physics, chemistry,

and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin)

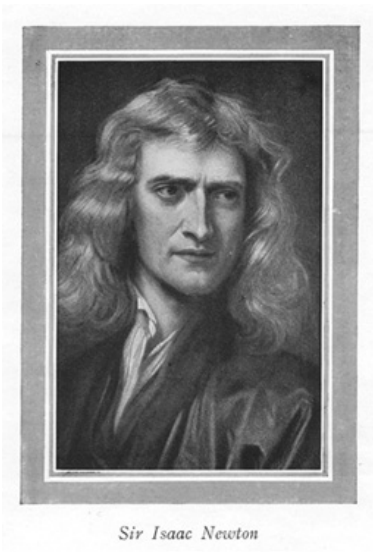


An artist's rendition of the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz)

## Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not

create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See [\[link\]](#) and [\[link\]](#).) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.



**Isaac Newton**  
(1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post

seriously,  
inventing reeding  
(or creating  
ridges) on the  
edge of coins to  
prevent  
unscrupulous  
people from  
trimming the  
silver off of them  
before using them  
as currency.  
(credit: Arthur  
Shuster and  
Arthur E. Shipley:  
*Britain's Heritage  
of Science*.  
London, 1917.)



**Marie Curie**  
(1867–1934)  
sacrificed

monetary assets  
to help finance  
her early  
research and  
damaged her  
physical well-  
being with  
radiation  
exposure. She is  
the only person  
to win Nobel  
prizes in both  
physics and  
chemistry. One  
of her daughters  
also won a  
Nobel Prize.  
(credit:  
Wikimedia  
Commons)

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

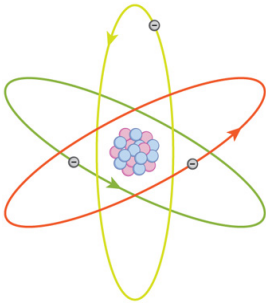
A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the

nucleus, analogous to the way planets orbit the Sun. (See [\[link\]](#).) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A **law** uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation  $\mathbf{F} = m\mathbf{a}$ . A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal's principle, which is applicable only in fluids), but the distinction

between laws and principles often is not carefully made.



What is a  
model?

This  
planetary  
model of  
the atom  
shows  
electrons  
orbiting the  
nucleus. It  
is a  
drawing  
that we use  
to form a  
mental  
image of  
the atom  
that we  
cannot see  
directly  
with our  
eyes  
because it  
is too  
small.

**Note:****Models, Theories, and Laws**

Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unobserved*. These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if *experiment* does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

**Note:****The Scientific Method**

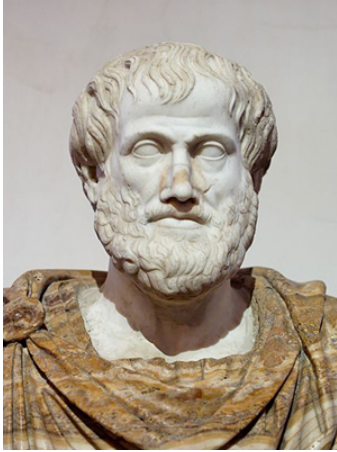
As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist

typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

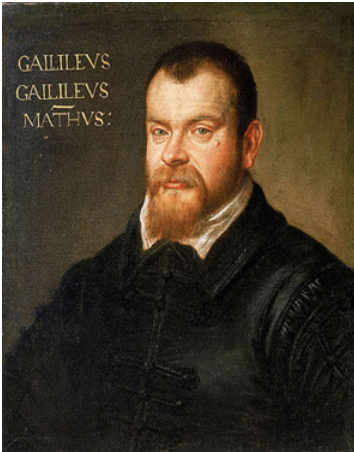
## The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called “natural philosophy.” From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See [\[link\]](#), [\[link\]](#), and [\[link\]](#).) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.



Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher **Aristotle** (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry.  
(credit: Jastrow

(2006)/Ludovisi  
Collection)



**Galileo Galilei**  
(1564–1642) laid  
the foundation of  
modern  
experimentation  
and made  
contributions in  
mathematics,  
physics, and  
astronomy.  
(credit:  
Domenico  
Tintoretto)



**Niels Bohr**  
(1885–1962)  
made  
fundamental  
contributions to  
the development  
of quantum  
mechanics, one  
part of modern  
physics. (credit:  
United States  
Library of  
Congress Prints  
and Photographs  
Division)

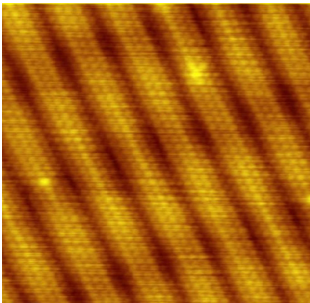
Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us

conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom's properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually “picture” the atom.

**Note:**

**Limits on the Laws of Classical Physics**

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.



Using a  
scanning  
tunneling  
microscope  
(STM),  
scientists can  
see the  
individual  
atoms that

compose this  
sheet of gold.  
(credit:  
Erwinrossen)

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

**Modern physics** itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics*, and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results. We will find, however, that we can do a great deal of modern physics with the algebra and trigonometry used in this text.

**Exercise:**

**Check Your Understanding**

**Problem:**

A friend tells you he has learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you he had learned about a scientific theory rather than a law?

---

**Solution:**

Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

**Note:**

PhET Explorations: Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g.  $y = bx$ ) to see how they add to generate the polynomial curve.

[https://phet.colorado.edu/sims/equation-grapher/equation-grapher\\_en.html](https://phet.colorado.edu/sims/equation-grapher/equation-grapher_en.html)

**Summary**

- Science seeks to discover and describe the underlying order and simplicity in nature.
- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

## Conceptual Questions

### Exercise:

#### Problem:

Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?

### Exercise:

**Problem:** How does a model differ from a theory?

### Exercise:

#### Problem:

If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?

### Exercise:

**Problem:** What determines the validity of a theory?

### Exercise:

#### Problem:

Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?

### Exercise:

#### Problem:

Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

### Exercise:

**Problem:**

Classical physics is a good approximation to modern physics under certain circumstances. What are they?

**Exercise:**

**Problem:** When is it *necessary* to use relativistic quantum mechanics?

**Exercise:****Problem:**

Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

**Glossary**

classical physics

physics that was developed from the Renaissance to the end of the 19th century

physics

the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

model

representation of something that is often too difficult (or impossible) to display directly

theory

an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

law

a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence

and repeated experiments

scientific method

a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

modern physics

the study of relativity, quantum mechanics, or both

relativity

the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

quantum mechanics

the study of objects smaller than can be seen with a microscope

## Approximation

- Make reasonable approximations based on given data.

On many occasions, physicists, other scientists, and engineers need to make **approximations** or “guesstimates” for a particular quantity. What is the distance to a certain destination? What is the approximate density of a given item? About how large a current will there be in a circuit? Many approximate numbers are based on formulae in which the input quantities are known only to a limited accuracy. As you develop problem-solving skills (that can be applied to a variety of fields through a study of physics), you will also develop skills at approximating. You will develop these skills through thinking more quantitatively, and by being willing to take risks. As with any endeavor, experience helps, as well as familiarity with units. These approximations allow us to rule out certain scenarios or unrealistic numbers. Approximations also allow us to challenge others and guide us in our approaches to our scientific world. Let us do two examples to illustrate this concept.

### **Example:**

#### **Approximate the Height of a Building**

Can you approximate the height of one of the buildings on your campus, or in your neighborhood? Let us make an approximation based upon the height of a person. In this example, we will calculate the height of a 39-story building.

#### **Strategy**

Think about the average height of an adult male. We can approximate the height of the building by scaling up from the height of a person.

#### **Solution**

Based on information in the example, we know there are 39 stories in the building. If we use the fact that the height of one story is approximately equal to about the length of two adult humans (each human is about 2-m tall), then we can estimate the total height of the building to be

#### **Equation:**

$$\frac{2 \text{ m}}{1 \text{ person}} \times \frac{2 \text{ person}}{1 \text{ story}} \times 39 \text{ stories} = 156 \text{ m.}$$

### Discussion

You can use known quantities to determine an approximate measurement of unknown quantities. If your hand measures 10 cm across, how many hand lengths equal the width of your desk? What other measurements can you approximate besides length?

### Example:

#### Approximating Vast Numbers: a Trillion Dollars



A bank stack contains one-hundred \$100 bills, and is worth \$10,000. How many bank stacks make up a trillion dollars? (credit: Andrew Magill)

The U.S. federal deficit in the 2008 fiscal year was a little greater than \$10 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in \$100 bills. If you made 100-bill stacks and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here)

because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

### **Strategy**

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped \$100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

### **Solution**

(1) Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is:

#### **Equation:**

$$\begin{aligned}\text{volume of stack} &= \text{length} \times \text{width} \times \text{height}, \\ \text{volume of stack} &= 6 \text{ in.} \times 3 \text{ in.} \times 0.5 \text{ in.}, \\ \text{volume of stack} &= 9 \text{ in.}^3.\end{aligned}$$

(2) Calculate the number of stacks. Note that a trillion dollars is equal to  $\$1 \times 10^{12}$ , and a stack of one-hundred \$100 bills is equal to \$10,000, or  $\$1 \times 10^4$ . The number of stacks you will have is:

#### **Equation:**

$$\$1 \times 10^{12} (\text{a trillion dollars}) / \$1 \times 10^4 \text{ per stack} = 1 \times 10^8 \text{ stacks.}$$

(3) Calculate the area of a football field in square inches. The area of a football field is 100 yd  $\times$  50 yd, which gives 5,000 yd<sup>2</sup>. Because we are working in inches, we need to convert square yards to square inches:

#### **Equation:**

$$\begin{aligned}\text{Area} &= 5,000 \text{ yd}^2 \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{12 \text{ in.}}{1 \text{ ft}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 6,480,000 \text{ in.}^2, \\ \text{Area} &\approx 6 \times 10^6 \text{ in.}^2.\end{aligned}$$

This conversion gives us  $6 \times 10^6 \text{ in.}^2$  for the area of the field. (Note that we are using only one significant figure in these calculations.)

(4) Calculate the total volume of the bills. The volume of all the \$100-bill stacks is  $9 \text{ in.}^3/\text{stack} \times 10^8 \text{ stacks} = 9 \times 10^8 \text{ in.}^3$ .

(5) Calculate the height. To determine the height of the bills, use the equation:

**Equation:**

$$\text{volume of bills} = \text{area of field} \times \text{height of money:}$$

$$\text{Height of money} = \frac{\text{volume of bills}}{\text{area of field}},$$

$$\text{Height of money} = \frac{9 \times 10^8 \text{ in.}^3}{6 \times 10^6 \text{ in.}^2} = 1.33 \times 10^2 \text{ in.},$$

$$\text{Height of money} \approx 1 \times 10^2 \text{ in.} = 100 \text{ in.}$$

The height of the money will be about 100 in. high. Converting this value to feet gives

**Equation:**

$$100 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = 8.33 \text{ ft} \approx 8 \text{ ft.}$$

### Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough “guesstimates” versus carefully calculated approximations?

### Exercise:

#### Check Your Understanding

##### Problem:

Using mental math and your understanding of fundamental units, approximate the area of a regulation basketball court. Describe the process you used to arrive at your final approximation.

---

##### Solution:

An average male is about two meters tall. It would take approximately 15 men laid out end to end to cover the length, and about 7 to cover the width. That gives an approximate area of  $420 \text{ m}^2$ .

## Summary

Scientists often approximate the values of quantities to perform calculations and analyze systems.

## Problems & Exercises

### Exercise:

**Problem:** How many heartbeats are there in a lifetime?

---

#### Solution:

Sample answer:  $2 \times 10^9$  heartbeats

### Exercise:

#### Problem:

A generation is about one-third of a lifetime. Approximately how many generations have passed since the year 0 AD?

### Exercise:

#### Problem:

How many times longer than the mean life of an extremely unstable atomic nucleus is the lifetime of a human? (Hint: The lifetime of an unstable atomic nucleus is on the order of  $10^{-22} \text{ s}$ .)

---

#### Solution:

Sample answer:  $2 \times 10^{31}$  if an average human lifetime is taken to be about 70 years.

**Exercise:****Problem:**

Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint: The mass of a hydrogen atom is on the order of  $10^{-27}$  kg and the mass of a bacterium is on the order of  $10^{-15}$  kg.)



This color-enhanced photo shows *Salmonella typhimurium* (red) attacking human cells. These bacteria are commonly known for causing foodborne illness. Can you estimate the number of atoms in each bacterium? (credit: Rocky Mountain Laboratories, NIAID, NIH)

**Exercise:**

**Problem:**

Approximately how many atoms thick is a cell membrane, assuming all atoms there average about twice the size of a hydrogen atom?

---

**Solution:**

Sample answer: 50 atoms

**Exercise:****Problem:**

(a) What fraction of Earth's diameter is the greatest ocean depth? (b) The greatest mountain height?

**Exercise:****Problem:**

(a) Calculate the number of cells in a hummingbird assuming the mass of an average cell is ten times the mass of a bacterium. (b) Making the same assumption, how many cells are there in a human?

---

**Solution:**

Sample answers:

(a)  $10^{12}$  cells/hummingbird

(b)  $10^{16}$  cells/human

**Exercise:****Problem:**

Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?

**Glossary**

approximation

an estimated value based on prior experience and reasoning

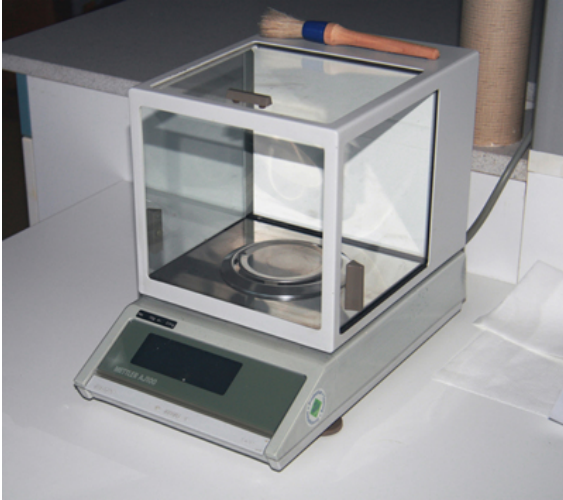
## Accuracy, Precision, and Significant Figures

- Determine the appropriate number of significant figures in both addition and subtraction, as well as multiplication and division calculations.
- Calculate the percent uncertainty of a measurement.



A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The “known masses” are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams.

(credit: Serge Melki)



Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit: Karel Jakubec)

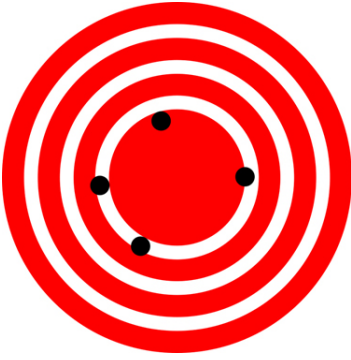
## Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in.

These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The **precision** of a measurement system refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In [\[link\]](#), you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in [\[link\]](#), the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.



A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy.  
(credit: Dark Evil)



In this figure,  
the dots are  
concentrated  
rather closely to  
one another,  
indicating high  
precision, but  
they are rather  
far away from  
the actual  
location of the  
restaurant,  
indicating low  
accuracy.  
(credit: Dark  
Evil)

## Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the

uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement,  $A$ , is often denoted as  $\delta A$  (“delta  $A$ ”), so the measurement result would be recorded as  $A \pm \delta A$ . In our paper example, the length of the paper could be expressed as  $11 \text{ in.} \pm 0.2$ .

The factors contributing to uncertainty in a measurement include:

1. Limitations of the measuring device,
2. The skill of the person making the measurement,
3. Irregularities in the object being measured,
4. Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

**Note:**

**Making Connections: Real-World Connections – Fevers or Chills?**

Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check his or her temperature with a thermometer. What if the uncertainty of the thermometer were  $3.0^{\circ}\text{C}$ ? If the child’s temperature reading was  $37.0^{\circ}\text{C}$  (which is normal body temperature), the “true” temperature could be anywhere from a

hypothermic 34.0°C to a dangerously high 40.0°C. A thermometer with an uncertainty of 3.0°C would be useless.

## Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement  $A$  is expressed with uncertainty,  $\delta A$ , the **percent uncertainty** (%unc) is defined to be

**Equation:**

$$\% \text{ unc} = \frac{\delta A}{A} \times 100\%.$$

### Example:

#### Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

Week 1 weight: 4.8 lb

Week 2 weight: 5.3 lb

Week 3 weight: 4.9 lb

Week 4 weight: 5.4 lb

You determine that the weight of the 5-lb bag has an uncertainty of  $\pm 0.4$  lb. What is the percent uncertainty of the bag's weight?

#### Strategy

First, observe that the expected value of the bag's weight,  $A$ , is 5 lb. The uncertainty in this value,  $\delta A$ , is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight:

**Equation:**

$$\% \text{ unc} = \frac{\delta A}{A} \times 100\%.$$

**Solution**

Plug the known values into the equation:

**Equation:**

$$\% \text{ unc} = \frac{0.4 \text{ lb}}{5 \text{ lb}} \times 100\% = 8\%.$$

**Discussion**

We can conclude that the weight of the apple bag is  $5 \text{ lb} \pm 8\%$ . Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

## Uncertainties in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements going into the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used for multiplication or division. This method says that *the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation*. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2% and 1%, respectively, then the area of the floor is  $12.0 \text{ m}^2$  and has an uncertainty of 3%. (Expressed as an area this is  $0.36 \text{ m}^2$ , which we round to  $0.4 \text{ m}^2$  since the area of the floor is given to a tenth of a square meter.)

**Exercise:****Check Your Understanding**

**Problem:**

A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of  $\pm 0.05$  s. Runners on the track coach's team regularly clock 100-m sprints of 11.49 s to 15.01 s. At the school's last track meet, the first-place sprinter came in at 12.04 s and the second-place sprinter came in at 12.07 s. Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?

---

**Solution:**

No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

**Precision of Measuring Tools and Significant Figures**

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest 0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be 36.7 cm. You could not express this value as 36.71 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36.6 cm and 36.7 cm, and he or she must estimate the value of the last digit. Using the

method of **significant figures**, the rule is that *the last digit written down in a measurement is the first digit with some uncertainty*. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value 36.7 cm has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

## Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) *Zeros are significant except when they serve only as placekeepers.*

### Exercise:

#### Check Your Understanding

##### Problem:

Determine the number of significant figures in the following measurements:

- a. 0.0009
- b. 15,450.0
- c.  $6 \times 10^3$
- d. 87.990
- e. 30.42

---

##### Solution:

- (a) 1; the zeros in this number are placekeepers that indicate the decimal point
- (b) 6; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant
- (c) 1; the value  $10^3$  signifies the decimal place, not the number of measured values
- (d) 5; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant
- (e) 4; any zeros located in between significant figures in a number are also significant

## Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, *the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value*. There are two different rules, one for multiplication and division and the other for addition and subtraction, as discussed below.

**1. For multiplication and division:** *The result should have the same number of significant figures as the quantity having the least significant figures entering into the calculation.* For example, the area of a circle can be calculated from its radius using  $A = \pi r^2$ . Let us see how many significant figures the area has if the radius has only two—say,  $r = 1.2$  m. Then,

**Equation:**

$$A = \pi r^2 = (3.1415927...) \times (1.2 \text{ m})^2 = 4.5238934 \text{ m}^2$$

is what you would get using a calculator that has an eight-digit output. But because the radius has only two significant figures, it limits the calculated

quantity to two significant figures or

**Equation:**

$$A=4.5 \text{ m}^2,$$

even though  $\pi$  is good to at least eight digits.

**2. For addition and subtraction:** *The answer can contain no more decimal places than the least precise measurement.* Suppose that you buy 7.56-kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg. Then you drop off 6.052-kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

**Equation:**

$$\begin{array}{r} 7.56 \text{ kg} \\ - 6.052 \text{ kg} \\ \hline +13.7 \text{ kg} \\ 15.208 \text{ kg} \end{array} = 15.2 \text{ kg}.$$

Next, we identify the least precise measurement: 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

### Significant Figures in this Text

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will also have fewer significant

figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and more than three significant figures will be used. Finally, if a number is *exact*, such as the two in the formula for the circumference of a circle,  $c = 2\pi r$ , it does not affect the number of significant figures in a calculation.

**Exercise:**

**Check Your Understanding**

**Problem:**

Perform the following calculations and express your answer using the correct number of significant digits.

- (a) A woman has two bags weighing 13.5 pounds and one bag with a weight of 10.2 pounds. What is the total weight of the bags?
- (b) The force  $F$  on an object is equal to its mass  $m$  multiplied by its acceleration  $a$ . If a wagon with mass 55 kg accelerates at a rate of  $0.0255 \text{ m/s}^2$ , what is the force on the wagon? (The unit of force is called the newton, and it is expressed with the symbol N.)

---

**Solution:**

- (a) 37.2 pounds; Because the number of bags is an exact value, it is not considered in the significant figures.
- (b) 1.4 N; Because the value 55 kg has only two significant figures, the final value must also contain two significant figures.

**Note:**

**PhET Explorations: Estimation**

Explore size estimation in one, two, and three dimensions! Multiple levels of difficulty allow for progressive skill improvement.

[https://phet.colorado.edu/sims/estimation/estimation\\_en.html](https://phet.colorado.edu/sims/estimation/estimation_en.html)

## Summary

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

## Conceptual Questions

### Exercise:

#### Problem:

What is the relationship between the accuracy and uncertainty of a measurement?

### Exercise:

#### Problem:

Prescriptions for vision correction are given in units called *diopters* (D). Determine the meaning of that unit. Obtain information (perhaps by calling an optometrist or performing an internet search) on the minimum uncertainty with which corrections in diopters are determined and the accuracy with which corrective lenses can be produced. Discuss the sources of uncertainties in both the prescription and accuracy in the manufacture of lenses.

## Problems & Exercises

Express your answers to problems in this section to the correct number of significant figures and proper units.

**Exercise:**

**Problem:**

Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?

---

**Solution:**

2 kg

**Exercise:**

**Problem:**

A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?

**Exercise:**

**Problem:**

(a) A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads 90 km/h? (b) Convert this range to miles per hour. (1 km = 0.6214 mi)

---

**Solution:**

a. 85.5 to 94.5 km/h

b. 53.1 to 58.7 mi/h

**Exercise:**

**Problem:**

An infant's pulse rate is measured to be  $130 \pm 5$  beats/min. What is the percent uncertainty in this measurement?

**Exercise:****Problem:**

(a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats does he or she have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?

---

**Solution:**

(a)  $7.6 \times 10^7$  beats

(b)  $7.57 \times 10^7$  beats

(c)  $7.57 \times 10^7$  beats

**Exercise:****Problem:**

A can contains 375 mL of soda. How much is left after 308 mL is removed?

**Exercise:****Problem:**

State how many significant figures are proper in the results of the following calculations: (a)  $(106.7)(98.2)/(46.210)(1.01)$  (b)  $(18.7)^2$  (c)  $(1.60 \times 10^{-19})(3712)$ .

---

**Solution:**

a. 3

b. 3

c. 3

**Exercise:**

**Problem:**

(a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?

**Exercise:****Problem:**

(a) If your speedometer has an uncertainty of 2.0 km/h at a speed of 90 km/h, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads 60 km/h, what is the range of speeds you could be going?

---

**Solution:**

a) 2.2%

(b) 59 to 61 km/h

**Exercise:****Problem:**

(a) A person's blood pressure is measured to be  $120 \pm 2$  mm Hg. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of 80 mm Hg?

**Exercise:****Problem:**

A person measures his or her heart rate by counting the number of beats in 30 s. If  $40 \pm 1$  beats are counted in  $30.0 \pm 0.5$  s, what is the heart rate and its uncertainty in beats per minute?

---

**Solution:**

$80 \pm 3$  beats/min

**Exercise:**

**Problem:** What is the area of a circle 3.102 cm in diameter?

**Exercise:**

**Problem:**

If a marathon runner averages 9.5 mi/h, how long does it take him or her to run a 26.22-mi marathon?

---

**Solution:**

2.8 h

**Exercise:**

**Problem:**

A marathon runner completes a 42.188-km course in 2 h, 30 min, and 12 s. There is an uncertainty of 25 m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?

**Exercise:**

**Problem:**

The sides of a small rectangular box are measured to be  $1.80 \pm 0.01$  cm,  $2.05 \pm 0.02$  cm, and  $3.1 \pm 0.1$  cm long. Calculate its volume and uncertainty in cubic centimeters.

---

**Solution:**

$11 \pm 1$  cm<sup>3</sup>

**Exercise:**

**Problem:**

When non-metric units were used in the United Kingdom, a unit of mass called the *pound-mass* (lbm) was employed, where  $1 \text{ lbm} = 0.4539 \text{ kg}$ . (a) If there is an uncertainty of 0.0001 kg in the pound-mass unit, what is its percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?

**Exercise:****Problem:**

The length and width of a rectangular room are measured to be  $3.955 \pm 0.005 \text{ m}$  and  $3.050 \pm 0.005 \text{ m}$ . Calculate the area of the room and its uncertainty in square meters.

---

**Solution:**

$$12.06 \pm 0.04 \text{ m}^2$$

**Exercise:****Problem:**

A car engine moves a piston with a circular cross section of  $7.500 \pm 0.002 \text{ cm}$  diameter a distance of  $3.250 \pm 0.001 \text{ cm}$  to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.

**Glossary**

accuracy

the degree to which a measured value agrees with correct value for that measurement

method of adding percents

the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

percent uncertainty

the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage

precision

the degree to which repeated measurements agree with each other

significant figures

express the precision of a measuring tool used to measure a value

uncertainty

a quantitative measure of how much your measured values deviate from a standard or expected value

## Friction

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitude of static and kinetic friction.

**Friction** is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

### **Note:**

#### Friction

Friction is a force that opposes relative motion between systems in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the objects.

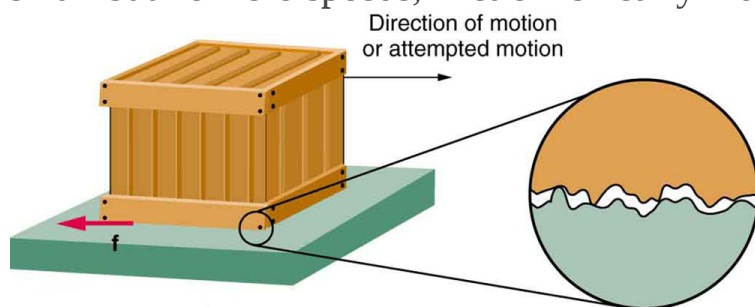
### **Note:**

#### Kinetic Friction

If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

[\[link\]](#) is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.



Frictional forces, such as  $f$ , always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of

the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction**  $f_s$  is

**Equation:**

$$f_s \leq \mu_s N,$$

where  $\mu_s$  is the coefficient of static friction and  $N$  is the magnitude of the normal force (the force perpendicular to the surface).

**Note:**

Magnitude of Static Friction

Magnitude of static friction  $f_s$  is

**Equation:**

$$f_s \leq \mu_s N,$$

where  $\mu_s$  is the coefficient of static friction and  $N$  is the magnitude of the normal force.

The symbol  $\leq$  means *less than or equal to*, implying that static friction can have a minimum and a maximum value of  $\mu_s N$ . Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds  $f_{s(\max)}$ , the object will move. Thus

**Equation:**

$$f_{s(\max)} = \mu_s N.$$

Once an object is moving, the **magnitude of kinetic friction**  $f_k$  is given by **Equation:**

$$f_k = \mu_k N,$$

where  $\mu_k$  is the coefficient of kinetic friction. A system in which  $f_k = \mu_k N$  is described as a system in which *friction behaves simply*.

**Note:****Magnitude of Kinetic Friction**

The magnitude of kinetic friction  $f_k$  is given by

**Equation:**

$$f_k = \mu_k N,$$

where  $\mu_k$  is the coefficient of kinetic friction.

As seen in [\[link\]](#), the coefficients of kinetic friction are less than their static counterparts. That values of  $\mu$  in [\[link\]](#) are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

<b>System</b>	<b>Static friction</b> $\mu_s$	<b>Kinetic friction</b> $\mu_k$
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7

System	Static friction $\mu_s$	Kinetic friction $\mu_k$
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.04	0.02

### Coefficients of Static and Kinetic Friction

The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight,  $W = mg = (100 \text{ kg})(9.80 \text{ m/s}^2) = 980 \text{ N}$ , perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than  $f_{s(\text{max})} = \mu_s N = (0.45)(980 \text{ N}) = 440 \text{ N}$  to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ( $f_k = \mu_k N = (0.30)(980 \text{ N}) = 290 \text{ N}$ ) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

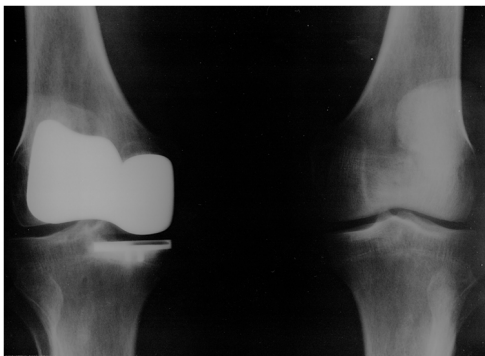
#### **Note:**

##### Take-Home Experiment

Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table,

simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint ([link](#)). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.



Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op x rays of the right knee joint replacement. (credit: Mike Baird, Flickr)

Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor's clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to move freely over the skin.

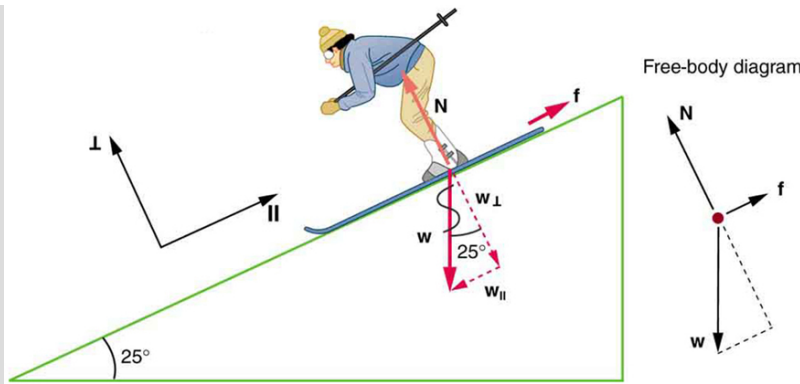
### **Example:**

#### **Skiing Exercise**

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

#### **Strategy**

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force  $N$  as  $f_k = \mu_k N$ ; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See the skier and free-body diagram in [\[link\]](#).)



The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier).  $N$  (the normal force) is perpendicular to the slope, and  $f$  (the friction) is parallel to the slope, but  $w$  (the skier's weight) has components along both axes, namely  $w_{\perp}$  and  $w_{\parallel}$ .  $N$  is equal in magnitude to  $w_{\perp}$ , so there is no motion perpendicular to the slope. However,  $f$  is less than  $w_{\parallel}$  in magnitude, so there is acceleration down the slope (along the x-axis).

That is,

**Equation:**

$$N = w_{\perp} = w \cos 25^{\circ} = mg \cos 25^{\circ}.$$

Substituting this into our expression for kinetic friction, we get

**Equation:**

$$f_k = \mu_k mg \cos 25^{\circ},$$

which can now be solved for the coefficient of kinetic friction  $\mu_k$ .

**Solution**

Solving for  $\mu_k$  gives

**Equation:**

$$\mu_k = \frac{f_k}{N} = \frac{f_k}{w \cos 25^\circ} = \frac{f_k}{mg \cos 25^\circ}.$$

Substituting known values on the right-hand side of the equation,

**Equation:**

$$\mu_k = \frac{45.0 \text{ N}}{(62 \text{ kg})(9.80 \text{ m/s}^2)(0.906)} = 0.082.$$

**Discussion**

This result is a little smaller than the coefficient listed in [\[link\]](#) for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass  $m$  slides down a slope that makes an angle  $\theta$  with the horizontal, friction is given by  $f_k = \mu_k mg \cos \theta$ . All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter's Problems and Exercises.

**Note:****Take-Home Experiment**

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in [\[link\]](#), the kinetic friction on a slope  $f_k = \mu_k mg \cos \theta$ . The component of the weight down the slope is equal to  $mg \sin \theta$  (see the free-body diagram in [\[link\]](#)). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out:

**Equation:**

$$f_k = F_{g_x}$$

**Equation:**

$$\mu_k mg \cos \theta = mg \sin \theta.$$

Solving for  $\mu_k$ , we find that

**Equation:**

$$\mu_k = \frac{mg \sin \theta}{mg \cos \theta} = \tan \theta.$$

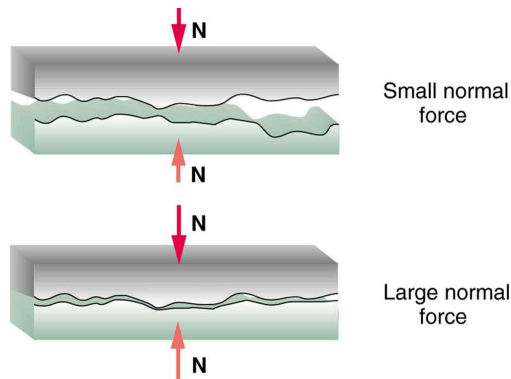
Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find  $\mu_k$ . Note that the coin will not start to slide at all until an angle greater than  $\theta$  is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Discuss how this may affect the value for  $\mu_k$  and its uncertainty.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

**Note:****Making Connections: Submicroscopic Explanations of Friction**

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

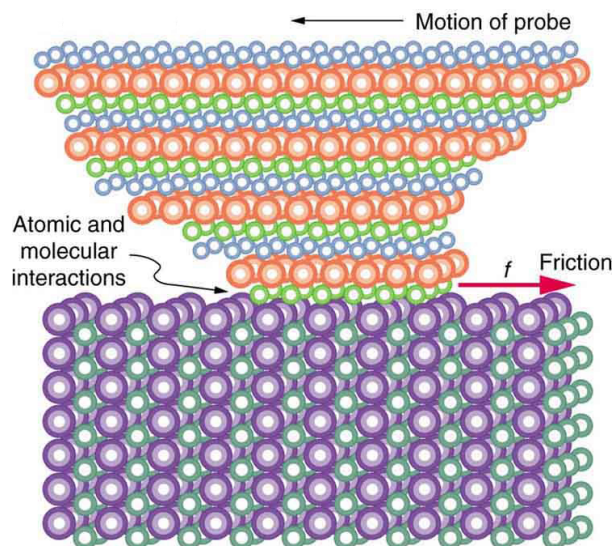
[\[link\]](#) illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.



Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur

between atoms and molecules on the surfaces. [\[link\]](#) shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of  $10^{12}$ ) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.



The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

**Note:**

PhET Explorations: Forces and Motion

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

[Forces](#)  
[and](#)  
[Motion](#)  
[Simulation](#)

## Section Summary

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force  $N$  pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction  $f_s$  between systems stationary relative to one another is given by

**Equation:**

$$f_s \leq \mu_s N,$$

where  $\mu_s$  is the coefficient of static friction, which depends on both of the materials.

- The kinetic friction force  $f_k$  between systems moving relative to one another is given by

**Equation:**

$$f_k = \mu_k N,$$

where  $\mu_k$  is the coefficient of kinetic friction, which also depends on both materials.

## Conceptual Questions

### Exercise:

#### Problem:

Define normal force. What is its relationship to friction when friction behaves simply?

### Exercise:

#### Problem:

The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.

### Exercise:

#### Problem:

When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.

### Exercise:

#### Problem:

When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

## Problems & Exercises

### Exercise:

#### Problem:

A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, he quickly calculates the normal force. What is it?

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#### Solution:

5.00 N

### Exercise:

#### Problem:

(a) When rebuilding her car's engine, a physics major must exert 300 N of force to insert a dry steel piston into a steel cylinder. What is the magnitude of the normal force between the piston and cylinder? (b) What is the magnitude of the force would she have to exert if the steel parts were oiled?

### Exercise:

#### Problem:

(a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

### Exercise:

**Problem:**

Suppose you have a 120-kg wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be?

---

**Solution:**

(a) 588 N

(b)  $1.96 \text{ m/s}^2$

**Exercise:****Problem:**

(a) If half of the weight of a small  $1.00 \times 10^3 \text{ kg}$  utility truck is supported by its two drive wheels, what is the magnitude of the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.

**Exercise:****Problem:**

A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.

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**Solution:**

(a)  $3.29 \text{ m/s}^2$

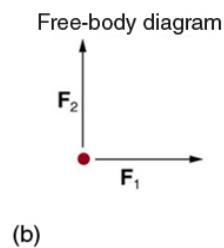
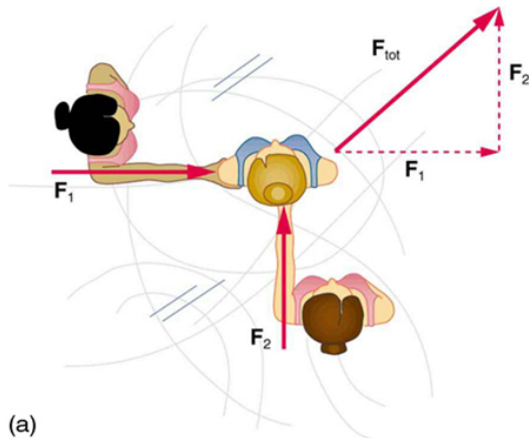
(b)  $3.52 \text{ m/s}^2$

(c) 980 N; 945 N

**Exercise:**

**Problem:**

Consider the 65.0-kg ice skater being pushed by two others shown in [\[link\]](#). (a) Find the direction and magnitude of  $\mathbf{F}_{\text{tot}}$ , the total force exerted on her by the others, given that the magnitudes  $F_1$  and  $F_2$  are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of  $\mathbf{F}_{\text{tot}}$ ? (c) What is her acceleration assuming she is already moving in the direction of  $\mathbf{F}_{\text{tot}}$ ? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)



**Exercise:**

**Problem:**

Show that the acceleration of any object down a frictionless incline that makes an angle  $\theta$  with the horizontal is  $a = g \sin \theta$ . (Note that this acceleration is independent of mass.)

**Exercise:**

**Problem:**

Show that the acceleration of any object down an incline where friction behaves simply (that is, where  $f_k = \mu_k N$ ) is  $a = g(\sin \theta - \mu_k \cos \theta)$ . Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small ( $\mu_k = 0$ ).

**Exercise:****Problem:**

Calculate the deceleration of a snow boarder going up a  $5.0^\circ$  slope assuming the coefficient of friction for waxed wood on wet snow. The result of [\[link\]](#) may be useful, but be careful to consider the fact that the snow boarder is going uphill. Explicitly show how you follow the steps in [Problem-Solving Strategies](#).

---

**Solution:**

$$1.83 \text{ m/s}^2$$

**Exercise:****Problem:**

(a) Calculate the acceleration of a skier heading down a  $10.0^\circ$  slope, assuming the coefficient of friction for waxed wood on wet snow. (b) Find the angle of the slope down which this skier could coast at a constant velocity. You can neglect air resistance in both parts, and you will find the result of [\[link\]](#) to be useful. Explicitly show how you follow the steps in the [Problem-Solving Strategies](#).

**Exercise:**

**Problem:**

If an object is to rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes. Show that the maximum angle of an incline above the horizontal for which an object will not slide down is  $\theta = \tan^{-1} \mu_s$ . You may use the result of the previous problem. Assume that  $a = 0$  and that static friction has reached its maximum value.

**Exercise:****Problem:**

Calculate the maximum deceleration of a car that is heading down a  $6^\circ$  slope (one that makes an angle of  $6^\circ$  with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that  $\mu_s = 0.100$ , the same as for shoes on ice.

**Exercise:****Problem:**

Calculate the maximum acceleration of a car that is heading up a  $4^\circ$  slope (one that makes an angle of  $4^\circ$  with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that  $\mu_s = 0.100$ , the same as for shoes on ice.

---

**Solution:**

(a)  $4.20 \text{ m/s}^2$

(b)  $2.74 \text{ m/s}^2$

(c)  $-0.195 \text{ m/s}^2$

**Exercise:**

**Problem:** Repeat [\[link\]](#) for a car with four-wheel drive.

**Exercise:**

**Problem:**

A freight train consists of two  $8.00 \times 10^5$ -kg engines and 45 cars with average masses of  $5.50 \times 10^5$  kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of  $5.00 \times 10^{-2} \text{ m/s}^2$  if the force of friction is  $7.50 \times 10^5 \text{ N}$ , assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the magnitude of the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

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**Solution:**

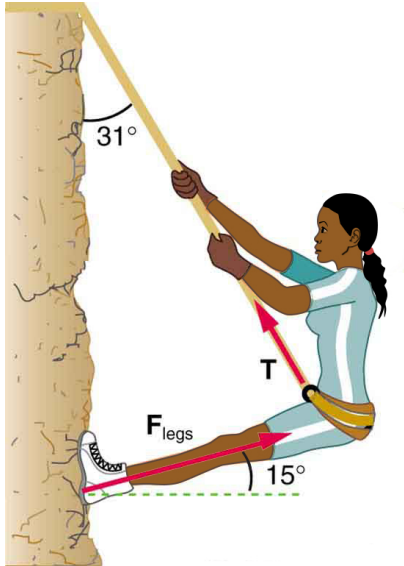
(a)  $1.03 \times 10^6 \text{ N}$

(b)  $3.48 \times 10^5 \text{ N}$

**Exercise:**

**Problem:**

Consider the 52.0-kg mountain climber in [\[link\]](#). (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?



Part of the climber's weight is supported by her rope and part by friction between her feet and the rock face.

### Exercise:

#### Problem:

A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown in [\[link\]](#)(a). (a) Calculate the minimum force  $F$  he must exert to get the block moving. (b) What is the magnitude of its acceleration once it starts to move, if that force is maintained?

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#### Solution:

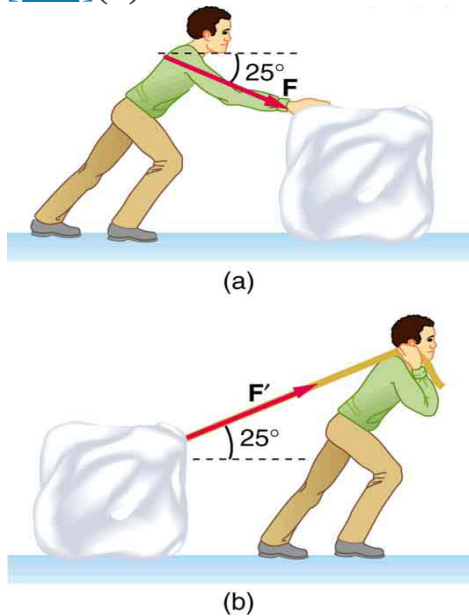
(a) 51.0 N

(b)  $0.720 \text{ m/s}^2$

## Exercise:

### Problem:

Repeat [\[link\]](#) with the contestant pulling the block of ice with a rope over his shoulder at the same angle above the horizontal as shown in [\[link\]](#)(b).



Which method of sliding a block of ice requires less force—(a) pushing or (b) pulling at the same angle above the horizontal?

## Glossary

### friction

a force that opposes relative motion or attempts at motion between systems in contact

### kinetic friction

a force that opposes the motion of two systems that are in contact and moving relative to one another

static friction

a force that opposes the motion of two systems that are in contact and are not moving relative to one another

magnitude of static friction

$f_s \leq \mu_s N$ , where  $\mu_s$  is the coefficient of static friction and  $N$  is the magnitude of the normal force

magnitude of kinetic friction

$f_k = \mu_k N$ , where  $\mu_k$  is the coefficient of kinetic friction

## Drag Forces

- Express mathematically the drag force.
- Discuss the applications of drag force.
- Define terminal velocity.
- Determine the terminal velocity given mass.

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion. Like friction, the **drag force** always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force  $F_D$  is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as  $F_D \propto v^2$ . When taking into account other factors, this relationship becomes

**Equation:**

$$F_D = \frac{1}{2} C \rho A v^2,$$

where  $C$  is the drag coefficient,  $A$  is the area of the object facing the fluid, and  $\rho$  is the density of the fluid. (Recall that density is mass per unit volume.) This equation can also be written in a more generalized fashion as  $F_D = b v^2$ , where  $b$  is a constant equivalent to  $0.5 C \rho A$ . We have set the exponent for these equations as 2 because, when an object is moving at high velocity through air, the magnitude of the drag force is proportional to the square of the speed. As we shall see in a few pages on fluid dynamics, for small particles moving at low speeds in a fluid, the exponent is equal to 1.

**Note:****Drag Force**

Drag force  $F_D$  is found to be proportional to the square of the speed of the object. Mathematically

**Equation:**

$$F_D \propto v^2$$

**Equation:**

$$F_D = \frac{1}{2} C \rho A v^2,$$

where  $C$  is the drag coefficient,  $A$  is the area of the object facing the fluid, and  $\rho$  is the density of the fluid.

Athletes as well as car designers seek to reduce the drag force to lower their race times. (See [\[link\]](#)). “Aerodynamic” shaping of an automobile can reduce the drag force and so increase a car’s gas mileage.



From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed. They are shaped like a bullet with tapered fins. (credit:

U.S. Army, via  
Wikimedia Commons)

The value of the drag coefficient,  $C$ , is determined empirically, usually with the use of a wind tunnel. (See [\[link\]](#)).



NASA researchers  
test a model plane  
in a wind tunnel.  
(credit:  
NASA/Ames)

The drag coefficient can depend upon velocity, but we will assume that it is a constant here. [\[link\]](#) lists some typical drag coefficients for a variety of objects. Notice that the drag coefficient is a dimensionless quantity. At highway speeds, over 50% of the power of a car is used to overcome air drag. The most fuel-efficient cruising speed is about 70–80 km/h (about 45–50 mi/h). For this reason, during the 1970s oil crisis in the United States, maximum speeds on highways were set at about 90 km/h (55 mi/h).

<b>Object</b>	<b><math>C</math></b>
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.0
Circular flat plate	1.12

**Drag Coefficient Values** Typical values of drag coefficient  $C$ .

Substantial research is under way in the sporting world to minimize drag. The dimples on golf balls are being redesigned as are the clothes that athletes wear. Bicycle racers and some swimmers and runners wear full bodysuits. Australian Cathy Freeman wore a full body suit in the 2000 Sydney Olympics, and won the gold medal for the 400 m race. Many swimmers in the 2008 Beijing Olympics wore (Speedo) body suits; it might have made a difference in breaking many world records (See [\[link\]](#)). Most elite swimmers (and cyclists) shave their body hair. Such innovations can have the effect of slicing away milliseconds in a race, sometimes making

the difference between a gold and a silver medal. One consequence is that careful and precise guidelines must be continuously developed to maintain the integrity of the sport.



Body suits, such as this LZR Racer Suit, have been credited with many world records after their release in 2008. Smoother “skin” and more compression forces on a swimmer’s body provide at least 10% less drag. (credit: NASA/Kathy Barnstorff)

Some interesting situations connected to Newton’s second law occur when considering the effects of drag forces upon a moving object. For instance, consider a skydiver falling through air under the influence of gravity. The two forces acting on him are the force of gravity and the drag force (ignoring the buoyant force). The downward force of gravity remains constant regardless of the velocity at which the person is moving. However, as the person’s velocity increases, the magnitude of the drag force increases until the magnitude of the drag force is equal to the gravitational force, thus producing a net force of zero. A zero net force means that there is no

acceleration, as given by Newton's second law. At this point, the person's velocity remains constant and we say that the person has reached his *terminal velocity* ( $v_t$ ). Since  $F_D$  is proportional to the speed, a heavier skydiver must go faster for  $F_D$  to equal his weight. Let's see how this works out more quantitatively.

At the terminal velocity,

**Equation:**

$$F_{\text{net}} = mg - F_D = ma = 0.$$

Thus,

**Equation:**

$$mg = F_D.$$

Using the equation for drag force, we have

**Equation:**

$$mg = \frac{1}{2} \rho C A v^2.$$

Solving for the velocity, we obtain

**Equation:**

$$v = \sqrt{\frac{2mg}{\rho C A}}.$$

Assume the density of air is  $\rho = 1.21 \text{ kg/m}^3$ . A 75-kg skydiver descending head first will have an area approximately  $A = 0.18 \text{ m}^2$  and a drag coefficient of approximately  $C = 0.70$ . We find that

**Equation:**

$$\begin{aligned}
 v &= \sqrt{\frac{2(75 \text{ kg})(9.80 \text{ m/s}^2)}{(1.21 \text{ kg/m}^3)(0.70)(0.18 \text{ m}^2)}} \\
 &= 98 \text{ m/s} \\
 &= 350 \text{ km/h.}
 \end{aligned}$$

This means a skydiver with a mass of 75 kg achieves a maximum terminal velocity of about 350 km/h while traveling in a pike (head first) position, minimizing the area and his drag. In a spread-eagle position, that terminal velocity may decrease to about 200 km/h as the area increases. This terminal velocity becomes much smaller after the parachute opens.

**Note:**

**Take-Home Experiment**

This interesting activity examines the effect of weight upon terminal velocity. Gather together some nested coffee filters. Leaving them in their original shape, measure the time it takes for one, two, three, four, and five nested filters to fall to the floor from the same height (roughly 2 m). (Note that, due to the way the filters are nested, drag is constant and only mass varies.) They obtain terminal velocity quite quickly, so find this velocity as a function of mass. Plot the terminal velocity  $v$  versus mass. Also plot  $v^2$  versus mass. Which of these relationships is more linear? What can you conclude from these graphs?

**Example:**

**A Terminal Velocity**

Find the terminal velocity of an 85-kg skydiver falling in a spread-eagle position.

**Strategy**

At terminal velocity,  $F_{\text{net}} = 0$ . Thus the drag force on the skydiver must equal the force of gravity (the person's weight). Using the equation of drag force, we find  $mg = \frac{1}{2} \rho C A v^2$ .

Thus the terminal velocity  $v_t$  can be written as

**Equation:**

$$v_t = \sqrt{\frac{2mg}{\rho C A}}.$$

**Solution**

All quantities are known except the person's projected area. This is an adult (85 kg) falling spread eagle. We can estimate the frontal area as

**Equation:**

$$A = (2 \text{ m})(0.35 \text{ m}) = 0.70 \text{ m}^2.$$

Using our equation for  $v_t$ , we find that

**Equation:**

$$\begin{aligned} v_t &= \sqrt{\frac{2(85 \text{ kg})(9.80 \text{ m/s}^2)}{(1.21 \text{ kg/m}^3)(1.0)(0.70 \text{ m}^2)}} \\ &= 44 \text{ m/s.} \end{aligned}$$

**Discussion**

This result is consistent with the value for  $v_t$  mentioned earlier. The 75-kg skydiver going feet first had a  $v = 98 \text{ m/s}$ . He weighed less but had a smaller frontal area and so a smaller drag due to the air.

The size of the object that is falling through air presents another interesting application of air drag. If you fall from a 5-m high branch of a tree, you will likely get hurt—possibly fracturing a bone. However, a small squirrel does this all the time, without getting hurt. You don't reach a terminal velocity in such a short distance, but the squirrel does.

The following interesting quote on animal size and terminal velocity is from a 1928 essay by a British biologist, J.B.S. Haldane, titled "On Being the Right Size."

*To the mouse and any smaller animal, [gravity] presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, and a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.*

The above quadratic dependence of air drag upon velocity does not hold if the object is very small, is going very slow, or is in a denser medium than air. Then we find that the drag force is proportional just to the velocity. This relationship is given by **Stokes' law**, which states that

**Equation:**

$$F_s = 6\pi r\eta v,$$

where  $r$  is the radius of the object,  $\eta$  is the viscosity of the fluid, and  $v$  is the object's velocity.

**Note:**

Stokes' Law

**Equation:**

$$F_s = 6\pi r\eta v,$$

where  $r$  is the radius of the object,  $\eta$  is the viscosity of the fluid, and  $v$  is the object's velocity.

Good examples of this law are provided by microorganisms, pollen, and dust particles. Because each of these objects is so small, we find that many of these objects travel unaided only at a constant (terminal) velocity.

Terminal velocities for bacteria (size about  $1\text{ }\mu\text{m}$ ) can be about  $2\text{ }\mu\text{m/s}$ . To

move at a greater speed, many bacteria swim using flagella (organelles shaped like little tails) that are powered by little motors embedded in the cell. Sediment in a lake can move at a greater terminal velocity (about  $5 \mu\text{m/s}$ ), so it can take days to reach the bottom of the lake after being deposited on the surface.

If we compare animals living on land with those in water, you can see how drag has influenced evolution. Fishes, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined and migratory species that fly large distances often have particular features such as long necks. Flocks of birds fly in the shape of a spear head as the flock forms a streamlined pattern (see [\[link\]](#)). In humans, one important example of streamlining is the shape of sperm, which need to be efficient in their use of energy.



Geese fly in a V formation during their long migratory travels. This shape reduces drag and energy consumption for individual birds, and also allows them a better way to communicate.  
(credit: Julo, Wikimedia Commons)

**Note:****Galileo's Experiment**

Galileo is said to have dropped two objects of different masses from the Tower of Pisa. He measured how long it took each to reach the ground. Since stopwatches weren't readily available, how do you think he measured their fall time? If the objects were the same size, but with different masses, what do you think he should have observed? Would this result be different if done on the Moon?

**Note:****PhET Explorations: Masses & Springs**

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energy for each spring.

[Masses](#)  
[&](#)  
[Spring](#)  
[s](#)

**Section Summary**

- Drag forces acting on an object moving in a fluid oppose the motion. For larger objects (such as a baseball) moving at a velocity  $v$  in air, the drag force is given by

**Equation:**

$$F_D = \frac{1}{2} C \rho A v^2,$$

where  $C$  is the drag coefficient (typical values are given in [\[link\]](#)),  $A$  is the area of the object facing the fluid, and  $\rho$  is the fluid density.

- For small objects (such as a bacterium) moving in a denser medium (such as water), the drag force is given by Stokes' law,

**Equation:**

$$F_s = 6\pi\eta r v,$$

where  $r$  is the radius of the object,  $\eta$  is the fluid viscosity, and  $v$  is the object's velocity.

## Conceptual Questions

**Exercise:**

**Problem:**

Athletes such as swimmers and bicyclists wear body suits in competition. Formulate a list of pros and cons of such suits.

**Exercise:**

**Problem:**

Two expressions were used for the drag force experienced by a moving object in a liquid. One depended upon the speed, while the other was proportional to the square of the speed. In which types of motion would each of these expressions be more applicable than the other one?

**Exercise:**

**Problem:**

As cars travel, oil and gasoline leaks onto the road surface. If a light rain falls, what does this do to the control of the car? Does a heavy rain make any difference?

**Exercise:**

**Problem:**

Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

**Problems & Exercise****Exercise:****Problem:**

The terminal velocity of a person falling in air depends upon the weight and the area of the person facing the fluid. Find the terminal velocity (in meters per second and kilometers per hour) of an 80.0-kg skydiver falling in a pike (headfirst) position with a surface area of  $0.140 \text{ m}^2$ .

---

**Solution:**

115 m/s; 414 km/hr

**Exercise:****Problem:**

A 60-kg and a 90-kg skydiver jump from an airplane at an altitude of 6000 m, both falling in the pike position. Make some assumption on their frontal areas and calculate their terminal velocities. How long will it take for each skydiver to reach the ground (assuming the time to reach terminal velocity is small)? Assume all values are accurate to three significant digits.

**Exercise:**

**Problem:**

A 560-g squirrel with a surface area of  $930 \text{ cm}^2$  falls from a 5.0-m tree to the ground. Estimate its terminal velocity. (Use a drag coefficient for a horizontal skydiver.) What will be the velocity of a 56-kg person hitting the ground, assuming no drag contribution in such a short distance?

---

**Solution:**

25 m/s; 9.9 m/s

**Exercise:****Problem:**

To maintain a constant speed, the force provided by a car's engine must equal the drag force plus the force of friction of the road (the rolling resistance). (a) What are the magnitudes of drag forces at 70 km/h and 100 km/h for a Toyota Camry? (Drag area is  $0.70 \text{ m}^2$ ) (b) What is the magnitude of drag force at 70 km/h and 100 km/h for a Hummer H2? (Drag area is  $2.44 \text{ m}^2$ ) Assume all values are accurate to three significant digits.

**Exercise:****Problem:**

By what factor does the drag force on a car increase as it goes from 65 to 110 km/h?

---

**Solution:**

2.9

**Exercise:**

**Problem:**

Calculate the speed a spherical rain drop would achieve falling from 5.00 km (a) in the absence of air drag (b) with air drag. Take the size across of the drop to be 4 mm, the density to be  $1.00 \times 10^3 \text{ kg/m}^3$ , and the surface area to be  $\pi r^2$ .

**Exercise:****Problem:**

Using Stokes' law, verify that the units for viscosity are kilograms per meter per second.

---

**Solution:****Equation:**

$$[\eta] = \frac{[F_s]}{[r][v]} = \frac{\text{kg} \cdot \text{m/s}^2}{\text{m} \cdot \text{m/s}} = \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

**Exercise:****Problem:**

Find the terminal velocity of a spherical bacterium (diameter  $2.00 \mu\text{m}$ ) falling in water. You will first need to note that the drag force is equal to the weight at terminal velocity. Take the density of the bacterium to be  $1.10 \times 10^3 \text{ kg/m}^3$ .

**Exercise:**

**Problem:**

Stokes' law describes sedimentation of particles in liquids and can be used to measure viscosity. Particles in liquids achieve terminal velocity quickly. One can measure the time it takes for a particle to fall a certain distance and then use Stokes' law to calculate the viscosity of the liquid. Suppose a steel ball bearing (density  $7.8 \times 10^3 \text{ kg/m}^3$ , diameter 3.0 mm) is dropped in a container of motor oil. It takes 12 s to fall a distance of 0.60 m. Calculate the viscosity of the oil.

---

**Solution:**

0.76 kg/m · s

**Glossary**

drag force

$F_D$ , found to be proportional to the square of the speed of the object; mathematically

**Equation:**

$$F_D \propto v^2$$

**Equation:**

$$F_D = \frac{1}{2} C \rho A v^2,$$

where  $C$  is the drag coefficient,  $A$  is the area of the object facing the fluid, and  $\rho$  is the density of the fluid

Stokes' law

$F_s = 6\pi r \eta v$ , where  $r$  is the radius of the object,  $\eta$  is the viscosity of the fluid, and  $v$  is the object's velocity

## Introduction to Work, Energy, and Energy Resources

class="introduction"

How many  
forms of  
energy can  
you identify  
in this  
photograph  
of a wind  
farm in  
Iowa?  
(credit:  
Jürgen from  
Sandesneben  
, Germany,  
Wikimedia  
Commons)



*Energy* plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is

involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is *conserved*.

**Conservation of energy** (as physicists like to call the principle that energy can neither be created nor destroyed) is based on experiment. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation  $E = mc^2$ ).

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define **energy** as the ability to do work, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.

## Work: The Scientific Definition

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

## What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be displacement in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

**Equation:**

$$W = | \mathbf{F} | (\cos \theta) | \mathbf{d} |,$$

where  $W$  is work,  $\mathbf{d}$  is the displacement of the system, and  $\theta$  is the angle between the force vector  $\mathbf{F}$  and the displacement vector  $\mathbf{d}$ , as in [\[link\]](#). We can also write this as

**Equation:**

$$W = Fd \cos \theta.$$

To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

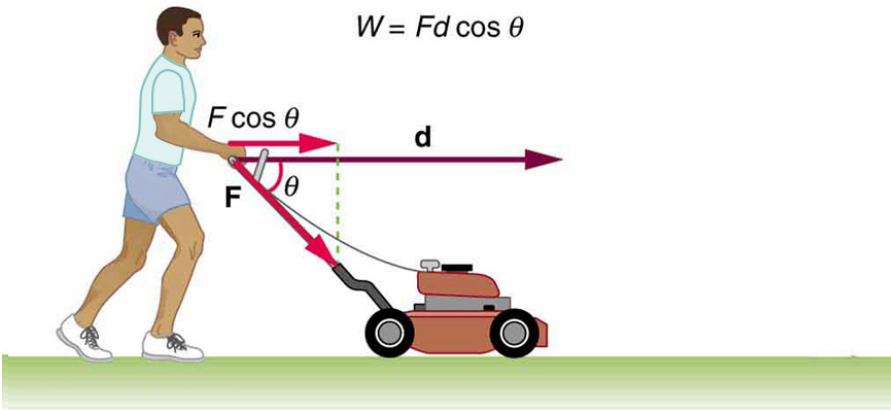
**Note:****What is Work?**

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

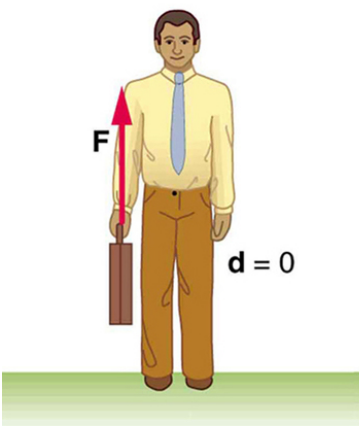
**Equation:**

$$W = Fd \cos \theta,$$

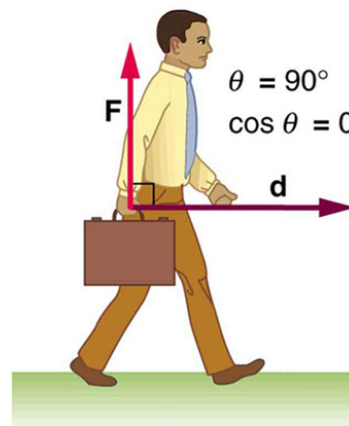
where  $W$  is work,  $F$  is the magnitude of the force on the system,  $d$  is the magnitude of the displacement of the system, and  $\theta$  is the angle between the force vector  $\mathbf{F}$  and the displacement vector  $\mathbf{d}$ .



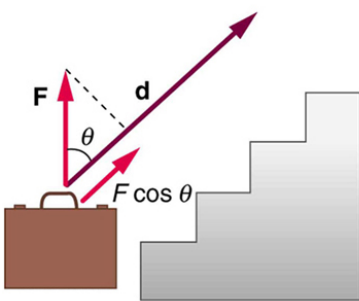
(a)



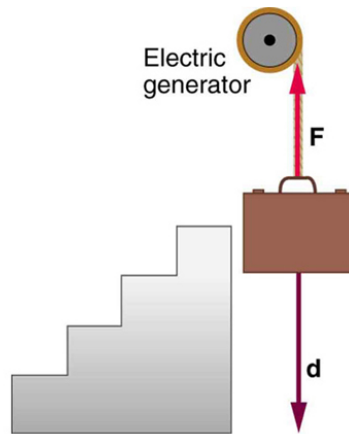
(b)



(c)



(d)



(e)

Examples of work. (a) The work done by the force  $\mathbf{F}$  on this lawn mower is  $Fd \cos \theta$ . Note that  $F \cos \theta$  is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no

displacement. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work is done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force  $\mathbf{F}$  in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because  $\mathbf{F}$  and  $d\mathbf{l}$  are in opposite directions.

To examine what the definition of work means, let us consider the other situations shown in [\[link\]](#). The person holding the briefcase in [\[link\]\(b\)](#) does no work, for example. Here  $d = 0$ , so  $W = 0$ . Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the “briefcase-Earth system”—see [Gravitational Potential Energy](#) for more details). There must be displacement for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in [\[link\]\(c\)](#) does no work on it, because the force is perpendicular to the motion. That is,  $\cos 90^\circ = 0$ , and so  $W = 0$ .

In contrast, when a force exerted on the system has a component in the direction of motion, such as in [\[link\]\(d\)](#), work is done—energy is transferred to the briefcase. Finally, in [\[link\]\(e\)](#), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase’s weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward

on the briefcase, and the displacement downward. This makes  $\theta = 180^\circ$ , and  $\cos 180^\circ = -1$ ; therefore,  $W$  is negative.

## Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and  $1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$ . One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

### Example:

#### Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in [\[link\]](#)(a) if he exerts a constant force of 75.0 N at an angle  $35^\circ$  below the horizontal and pushes the mower 25.0 m on level ground? Convert the amount of work from joules to kilocalories and compare it with this person's average daily intake of 10,000 kJ (about 2400 kcal) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by  $1^\circ\text{C}$ , and is equivalent to 4.184 J, while one *food calorie* (1 kcal) is equivalent to 4184 J.

#### Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation  $W = Fd \cos \theta$ . The force, angle, and displacement are given, so that only the work  $W$  is unknown.

#### Solution

The equation for the work is

#### Equation:

$$W = Fd \cos \theta.$$

Substituting the known values gives

**Equation:**

$$\begin{aligned} W &= (75.0 \text{ N})(25.0 \text{ m}) \cos (35.0^\circ) \\ &= 1536 \text{ J} = 1.54 \times 10^3 \text{ J}. \end{aligned}$$

Converting the work in joules to kilocalories yields

$W = (1536 \text{ J})(1 \text{ kcal}/4184 \text{ J}) = 0.367 \text{ kcal}$ . The ratio of the work done to the daily consumption is

**Equation:**

$$\frac{W}{2400 \text{ kcal}} = 1.53 \times 10^{-4}.$$

**Discussion**

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we “work” all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

**Section Summary**

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work  $W$  that a force  $\mathbf{F}$  does on an object is the product of the magnitude  $F$  of the force, times the magnitude  $d$  of the displacement, times the cosine of the angle  $\theta$  between them. In symbols,

**Equation:**

$$W = Fd \cos \theta.$$

- The SI unit for work and energy is the joule (J), where  $1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$ .
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.

- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

## Conceptual Questions

### Exercise:

#### Problem:

Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.

### Exercise:

#### Problem:

Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

### Exercise:

#### Problem:

Describe a situation in which a force is exerted for a long time but does no work. Explain.

## Problems & Exercises

### Exercise:

#### Problem:

How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.

---

#### Solution:

**Equation:**

$$3.00 \text{ J} = 7.17 \times 10^{-4} \text{ kcal}$$

**Exercise:****Problem:**

A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

**Exercise:****Problem:**

(a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the lift by the gravitational force in this process? (c) What is the total work done on the lift?

---

**Solution:**

(a)  $5.92 \times 10^5 \text{ J}$

(b)  $-5.88 \times 10^5 \text{ J}$

(c) The net force is zero.

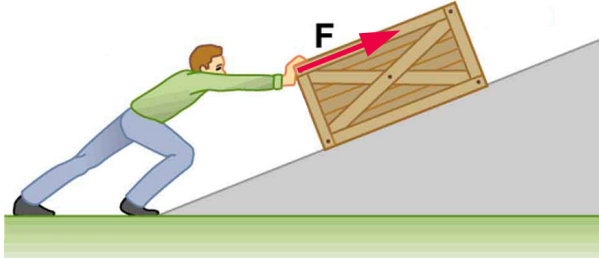
**Exercise:****Problem:**

Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See [\[link\]](#) for the energy content of gasoline.) (a) What is the magnitude of the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?

**Exercise:**

**Problem:**

Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of  $20.0^\circ$  with the horizontal. (See [\[link\]](#).) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate *and* on his body to get up the ramp.



A man pushes a crate up a ramp.

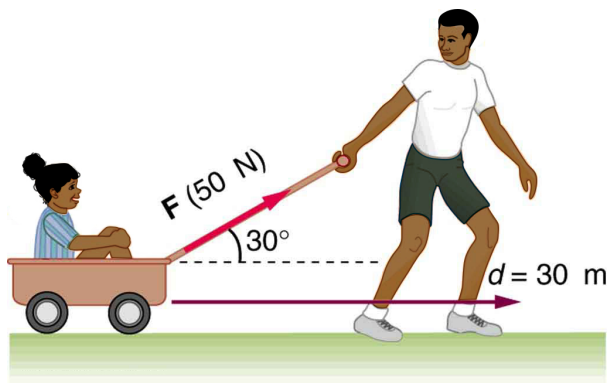
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**Solution:****Equation:**

$$3.14 \times 10^3 \text{ J}$$

**Exercise:****Problem:**

How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in [\[link\]](#)? Assume no friction acts on the wagon.



The boy does work on the system of the wagon and the child when he pulls them as shown.

**Exercise:**

**Problem:**

A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction  $25.0^\circ$  below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?

---

**Solution:**

(a)  $-700 \text{ J}$

(b)  $0$

(c)  $700 \text{ J}$

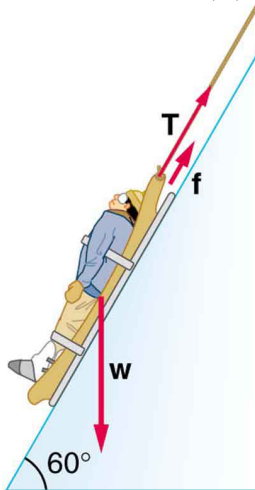
(d)  $38.6 \text{ N}$

(e) 0

### Exercise:

#### Problem:

Suppose the ski patrol lowers a rescue sled and victim, having a total mass of  $90.0\text{ kg}$ , down a  $60.0^\circ$  slope at constant speed, as shown in [\[link\]](#). The coefficient of friction between the sled and the snow is  $0.100$ . (a) How much work is done by friction as the sled moves  $30.0\text{ m}$  along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?



A rescue sled and victim are lowered down a steep slope.

### Glossary

energy

the ability to do work

work

the transfer of energy by a force that causes an object to be displaced;  
the product of the component of the force in the direction of the  
displacement and the magnitude of the displacement

joule

SI unit of work and energy, equal to one newton-meter

## Kinetic Energy and the Work-Energy Theorem

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

## Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if the lawn mower in [\[link\]](#) (a) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in [\[link\]](#) (d) is stored in the briefcase-Earth system and can be recovered at any time, as shown in [\[link\]](#) (e). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

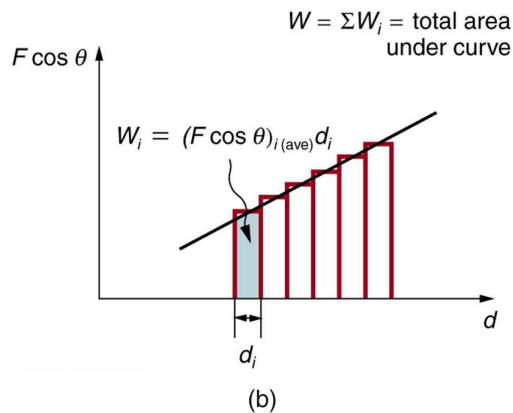
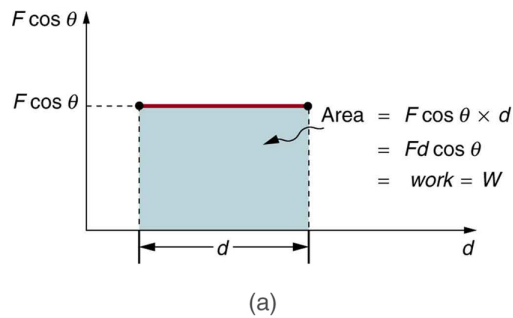
In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

## Net Work and the Work-Energy Theorem

We know from the study of Newton's laws in [Dynamics: Force and Newton's Laws of Motion](#) that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is, **net work** is the work done by the net external force  $\mathbf{F}_{\text{net}}$ . In equation form, this is  $W_{\text{net}} = F_{\text{net}}d \cos \theta$  where  $\theta$  is the angle between the force vector and the displacement vector.

[\[link\]](#)(a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an  $F \cos \theta$  vs.  $d$  graph. In this case,  $F \cos \theta$  is constant. You can see that the area under the graph is  $Fd \cos \theta$ , or the work done. [\[link\]](#)(b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force  $(F \cos \theta)_{i(\text{ave})}$ . The work done is  $(F \cos \theta)_{i(\text{ave})}d_i$  for each strip, and the total work done is the sum of the  $W_i$ . Thus the total work done is the total area under the curve, a useful property to which we shall refer later.

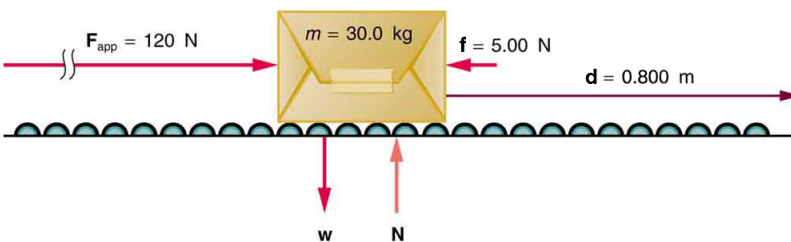


(a) A graph of  $F \cos \theta$  vs.  $d$ , when  $F \cos \theta$  is

constant. The area under the curve represents the work done by the force.

(b) A graph of  $F \cos \theta$  vs.  $d$  in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in [\[link\]](#).



A package on a roller belt is pushed horizontally through a distance  $d$ .

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force  $F_{\text{app}}$  and the horizontal friction force  $f$ . Thus, as expected, the net force is

parallel to the displacement, so that  $\theta = 0^\circ$  and  $\cos \theta = 1$ , and the net work is given by

**Equation:**

$$W_{\text{net}} = F_{\text{net}}d.$$

The effect of the net force  $\mathbf{F}_{\text{net}}$  is to accelerate the package from  $v_0$  to  $v$ . The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See [\[link\]](#).) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting  $F_{\text{net}} = ma$  from Newton's second law gives

**Equation:**

$$W_{\text{net}} = mad.$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take  $d = x - x_0$  and use the equation studied in [Motion Equations for Constant Acceleration in One Dimension](#) for the change in speed over a distance  $d$  if the acceleration has the constant value  $a$ ; namely,  $v^2 = v_0^2 + 2ad$  (note that  $a$  appears in the expression for the net work). Solving for acceleration gives  $a = \frac{v^2 - v_0^2}{2d}$ . When  $a$  is substituted into the preceding expression for  $W_{\text{net}}$ , we obtain

**Equation:**

$$W_{\text{net}} = m \left( \frac{v^2 - v_0^2}{2d} \right) d.$$

The  $d$  cancels, and we rearrange this to obtain

**Equation:**

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity  $\frac{1}{2}mv^2$ . This quantity is our first example of a form of energy.

**Note:**

**The Work-Energy Theorem**

The net work on a system equals the change in the quantity  $\frac{1}{2}mv^2$ .

**Equation:**

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

The quantity  $\frac{1}{2}mv^2$  in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass  $m$  moving at a speed  $v$ . (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

**Equation:**

$$\text{KE} = \frac{1}{2}mv^2,$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in [\[link\]](#), up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50

km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

**Example:**

**Calculating the Kinetic Energy of a Package**

Suppose a 30.0-kg package on the roller belt conveyor system in [\[link\]](#) is moving at 0.500 m/s. What is its kinetic energy?

**Strategy**

Because the mass  $m$  and speed  $v$  are given, the kinetic energy can be calculated from its definition as given in the equation  $\text{KE} = \frac{1}{2}mv^2$ .

**Solution**

The kinetic energy is given by

**Equation:**

$$\text{KE} = \frac{1}{2}mv^2.$$

Entering known values gives

**Equation:**

$$\text{KE} = 0.5(30.0 \text{ kg})(0.500 \text{ m/s})^2,$$

which yields

**Equation:**

$$\text{KE} = 3.75 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 3.75 \text{ J}.$$

**Discussion**

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

**Example:****Determining the Work to Accelerate a Package**

Suppose that you push on the 30.0-kg package in [\[link\]](#) with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

**Strategy and Concept for (a)**

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See [\[link\]](#).) As expected, the net work is the net force times distance.

**Solution for (a)**

The net force is the push force minus friction, or

$F_{\text{net}} = 120 \text{ N} - 5.00 \text{ N} = 115 \text{ N}$ . Thus the net work is

**Equation:**

$$\begin{aligned} W_{\text{net}} &= F_{\text{net}}d = (115 \text{ N})(0.800 \text{ m}) \\ &= 92.0 \text{ N} \cdot \text{m} = 92.0 \text{ J}. \end{aligned}$$

**Discussion for (a)**

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

**Strategy and Concept for (b)**

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

**Solution for (b)**

The applied force does work.

**Equation:**

$$\begin{aligned}
 W_{\text{app}} &= F_{\text{app}}d \cos(0^\circ) = F_{\text{app}}d \\
 &= (120 \text{ N})(0.800 \text{ m}) \\
 &= 96.0 \text{ J}
 \end{aligned}$$

The friction force and displacement are in opposite directions, so that  $\theta = 180^\circ$ , and the work done by friction is

**Equation:**

$$\begin{aligned}
 W_{\text{fr}} &= F_{\text{fr}}d \cos(180^\circ) = -F_{\text{fr}}d \\
 &= -(5.00 \text{ N})(0.800 \text{ m}) \\
 &= -4.00 \text{ J.}
 \end{aligned}$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

**Equation:**

$$\begin{aligned}
 W_{\text{gr}} &= 0, \\
 W_{\text{N}} &= 0, \\
 W_{\text{app}} &= 96.0 \text{ J}, \\
 W_{\text{fr}} &= -4.00 \text{ J.}
 \end{aligned}$$

The total work done as the sum of the work done by each force is then seen to be

**Equation:**

$$W_{\text{total}} = W_{\text{gr}} + W_{\text{N}} + W_{\text{app}} + W_{\text{fr}} = 92.0 \text{ J.}$$

### Discussion for (b)

The calculated total work  $W_{\text{total}}$  as the sum of the work by each force agrees, as expected, with the work  $W_{\text{net}}$  done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

**Example:**

### Determining Speed from Work and Energy

Find the speed of the package in [\[link\]](#) at the end of the push, using work and energy concepts.

#### Strategy

Here the work-energy theorem can be used, because we have just calculated the net work,  $W_{\text{net}}$ , and the initial kinetic energy,  $\frac{1}{2}mv_0^2$ . These calculations allow us to find the final kinetic energy,  $\frac{1}{2}mv^2$ , and thus the final speed  $v$ .

#### Solution

The work-energy theorem in equation form is

#### Equation:

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Solving for  $\frac{1}{2}mv^2$  gives

#### Equation:

$$\frac{1}{2}mv^2 = W_{\text{net}} + \frac{1}{2}mv_0^2.$$

Thus,

#### Equation:

$$\frac{1}{2}mv^2 = 92.0 \text{ J} + 3.75 \text{ J} = 95.75 \text{ J}.$$

Solving for the final speed as requested and entering known values gives

#### Equation:

$$\begin{aligned} v &= \sqrt{\frac{2(95.75 \text{ J})}{m}} = \sqrt{\frac{191.5 \text{ kg}\cdot\text{m}^2/\text{s}^2}{30.0 \text{ kg}}} \\ &= 2.53 \text{ m/s}. \end{aligned}$$

#### Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work

done on the package. This means that the work indeed adds to the energy of the package.

**Example:****Work and Energy Can Reveal Distance, Too**

How far does the package in [\[link\]](#) coast after the push, assuming friction remains constant? Use work and energy considerations.

**Strategy**

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package's kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

**Solution**

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so  $\theta = 180^\circ$ . To reduce the kinetic energy of the package to zero, the work  $W_{\text{fr}}$  by friction must be minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus  $W_{\text{fr}} = -95.75 \text{ J}$ . Furthermore,  $W_{\text{fr}} = f d' \cos \theta = -f d'$ , where  $d'$  is the distance it takes to stop. Thus,

**Equation:**

$$d' = -\frac{W_{\text{fr}}}{f} = -\frac{-95.75 \text{ J}}{5.00 \text{ N}},$$

and so

**Equation:**

$$d' = 19.2 \text{ m}.$$

**Discussion**

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the

force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

## Section Summary

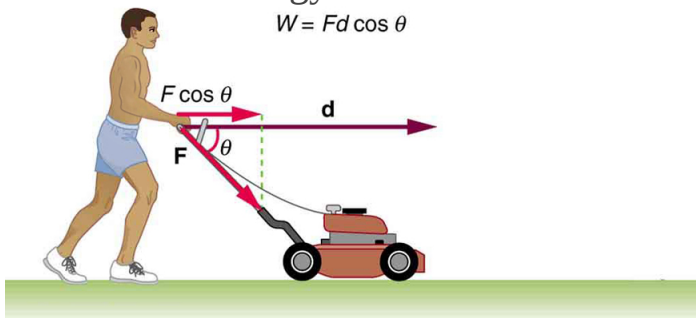
- The net work  $W_{\text{net}}$  is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass  $m$  moving at speed  $v$  is  $\text{KE} = \frac{1}{2}mv^2$ .
- The work-energy theorem states that the net work  $W_{\text{net}}$  on a system changes its kinetic energy,  $W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$ .

## Conceptual Questions

### Exercise:

#### Problem:

The person in [\[link\]](#) does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?



**Exercise:****Problem:**

Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

**Exercise:****Problem:**

When solving for speed in [\[link\]](#), we kept only the positive root. Why?

**Problems & Exercises****Exercise:****Problem:**

Compare the kinetic energy of a 20,000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27,500 km/h.

---

**Solution:**

1/250

**Exercise:****Problem:**

(a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.

**Exercise:****Problem:**

Confirm the value given for the kinetic energy of an aircraft carrier in [\[link\]](#). You will need to look up the definition of a nautical mile (1 knot = 1 nautical mile/h).

---

**Solution:**

$$1.1 \times 10^{10} \text{ J}$$

**Exercise:****Problem:**

(a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

**Exercise:****Problem:**

A car's bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.

---

**Solution:**

$$2.8 \times 10^3 \text{ N}$$

**Exercise:**

**Problem:**

Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the gory old days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?

**Exercise:****Problem:**

Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

---

**Solution:**

102 N

**Glossary**

net work

work done by the net force, or vector sum of all the forces, acting on an object

work-energy theorem

the result, based on Newton's laws, that the net work done on an object is equal to its change in kinetic energy

kinetic energy

the energy an object has by reason of its motion, equal to  $\frac{1}{2}mv^2$  for the translational (i.e., non-rotational) motion of an object of mass  $m$  moving at speed  $v$

## Gravitational Potential Energy

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass  $m$  at height  $h$  on Earth is given by  $PE_g = mgh$ .
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

## Work Done Against Gravity

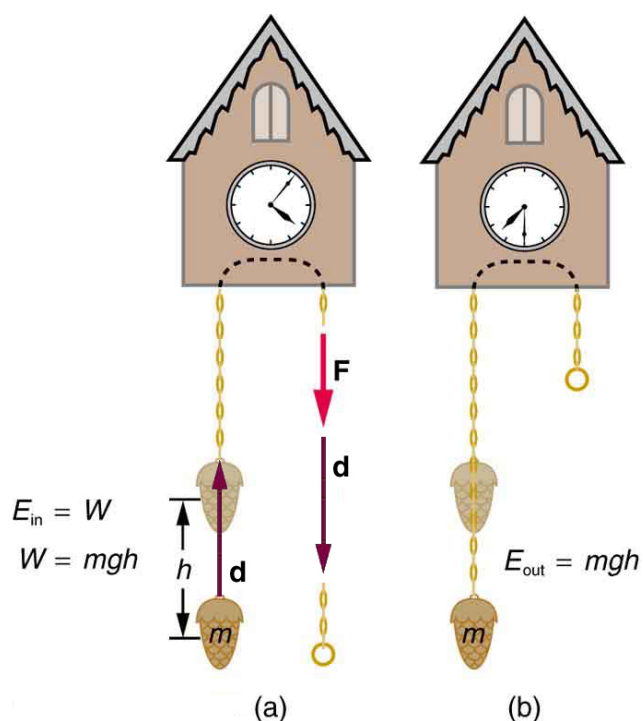
Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass  $m$  through a height  $h$ , such as in [\[link\]](#). If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight  $mg$ . The work done on the mass is then  $W = Fd = mgh$ . We define this to be the **gravitational potential energy** ( $PE_g$ ) put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the  $PE_g$  gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word “system”? Potential energy is a property of a system rather than of a single object—due to its physical position. An object’s gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth’s surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

## Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work

equal to  $mgh$  on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of  $PE_g$  to KE without explicitly considering the intermediate step of work. (See [\[link\]](#).) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.



(a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy  $\Delta PE_g$  to be  
**Equation:**

$$\Delta PE_g = mgh,$$

where, for simplicity, we denote the change in height by  $h$  rather than the usual  $\Delta h$ . Note that  $h$  is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

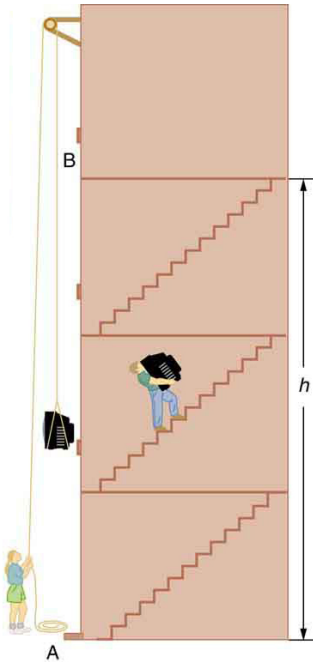
**Equation:**

$$\begin{aligned} mgh &= (0.500 \text{ kg}) (9.80 \text{ m/s}^2) (1.00 \text{ m}) \\ &= 4.90 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 4.90 \text{ J}. \end{aligned}$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work*.

## Using Potential Energy to Simplify Calculations

The equation  $\Delta \text{PE}_g = mgh$  applies for any path that has a change in height of  $h$ , not just when the mass is lifted straight up. (See [\[link\]](#).) It is much easier to calculate  $mgh$  (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position  $h$  of a mass  $m$  is accompanied by a change in gravitational potential energy  $mgh$ , and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.



The change in  
gravitational  
potential energy  
( $\Delta PE_g$ )  
between points  
A and B is  
independent of  
the path.

$\Delta PE_g = mgh$   
for any path  
between the two  
points. Gravity  
is one of a small  
class of forces  
where the work  
done by or  
against the force  
depends only on  
the starting and  
ending points,  
not on the path  
between them.

**Example:****The Force to Stop Falling**

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

**Strategy**

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial  $PE_g$  is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.

**Solution**

The work done on the person by the floor as he stops is given by

**Equation:**

$$W = Fd \cos \theta = -Fd,$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions ( $\cos \theta = \cos 180^\circ = -1$ ). The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height  $h$ :

**Equation:**

$$KE = -\Delta PE_g = -mgh,$$

The distance  $d$  that the person's knees bend is much smaller than the height  $h$  of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work  $W$  done by the floor on the person stops the person and brings the person's kinetic energy to zero:

**Equation:**

$$W = -KE = mgh.$$

Combining this equation with the expression for  $W$  gives

**Equation:**

$$-Fd = mgh.$$

Recalling that  $h$  is negative because the person fell *down*, the force on the knee joints is given by

**Equation:**

$$F = -\frac{mgh}{d} = -\frac{(60.0 \text{ kg})(9.80 \text{ m/s}^2)(-3.00 \text{ m})}{5.00 \times 10^{-3} \text{ m}} = 3.53 \times 10^5 \text{ N}.$$

### Discussion

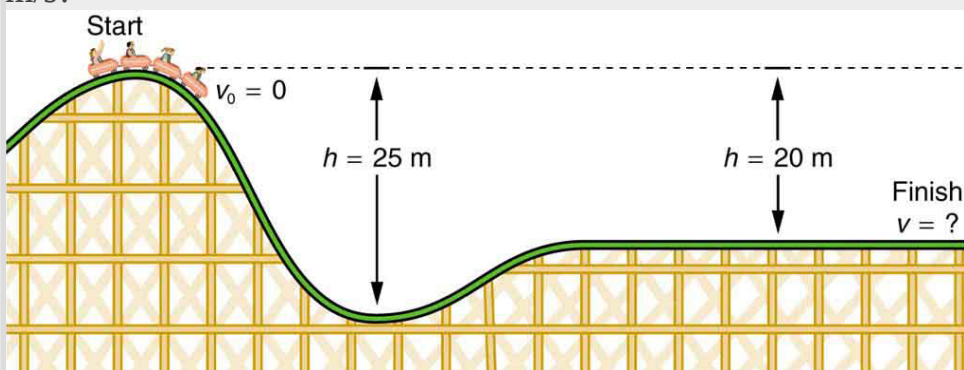
Such a large force (500 times more than the person's weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump. (See [\[link\]](#).)



The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced.  
(credit: Chris Samuel, Flickr)

**Example:****Finding the Speed of a Roller Coaster from its Height**

(a) What is the final speed of the roller coaster shown in [\[link\]](#) if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?



The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all  $\Delta PE_g$  is converted to KE.

**Strategy**

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance  $h$  equals the *gain* in kinetic energy. This can be written in equation form as  $-\Delta PE_g = \Delta KE$ . Using the equations for  $PE_g$  and KE, we can solve for the final speed  $v$ , which is the desired quantity.

**Solution for (a)**

Here the initial kinetic energy is zero, so that  $\Delta KE = \frac{1}{2}mv^2$ . The equation for change in potential energy states that  $\Delta PE_g = mgh$ . Since  $h$  is negative in this case, we will rewrite this as  $\Delta PE_g = -mg |h|$  to show the minus sign clearly. Thus,

**Equation:**

$$-\Delta PE_g = \Delta KE$$

becomes

**Equation:**

$$mg | h | = \frac{1}{2}mv^2.$$

Solving for  $v$ , we find that mass cancels and that

**Equation:**

$$v = \sqrt{2g | h |}.$$

Substituting known values,

**Equation:**

$$\begin{aligned} v &= \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m})} \\ &= 19.8 \text{ m/s.} \end{aligned}$$

**Solution for (b)**

Again  $-\Delta PE_g = \Delta KE$ . In this case there is initial kinetic energy, so

$\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$ . Thus,

**Equation:**

$$mg | h | = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Rearranging gives

**Equation:**

$$\frac{1}{2}mv^2 = mg | h | + \frac{1}{2}mv_0^2.$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

**Equation:**

$$v = \sqrt{2g | h | + v_0^2}.$$

This equation is very similar to the kinematics equation  $v = \sqrt{v_0^2 + 2ad}$ , but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

**Equation:**

$$\begin{aligned} v &= \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m}) + (5.00 \text{ m/s})^2} \\ &= 20.4 \text{ m/s.} \end{aligned}$$

### Discussion and Implications

First, note that mass cancels. This is quite consistent with observations made in [Falling Objects](#) that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of  $h$  at the point of interest.

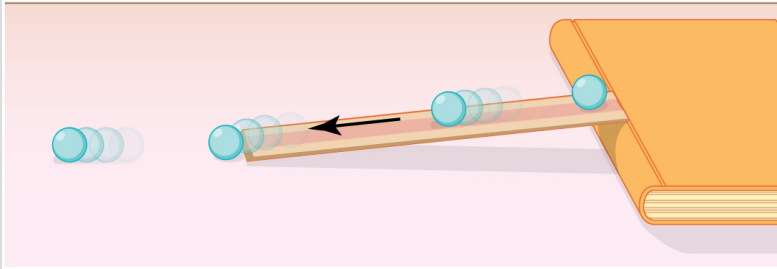
We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

### Note:

**Making Connections: Take-Home Investigation—Converting Potential to Kinetic Energy**

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see [link](#)). Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble

at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.



A marble rolls down a ruler, and its speed on the level surface is measured.

## Section Summary

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy,  $\Delta PE_g$ , is  $\Delta PE_g = mgh$ , with  $h$  being the increase in height and  $g$  the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy,  $\Delta PE_g$ , have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that  $\Delta KE = -\Delta PE_g$ .

## Conceptual Questions

**Exercise:**

**Problem:**

In [\[link\]](#), we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s *uphill* instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that its final speed is the same as its initial speed. Explain in terms of conservation of energy.

**Exercise:****Problem:**

Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

**Problems & Exercises****Exercise:****Problem:**

A hydroelectric power facility (see [\[link\]](#)) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume  $50.0 \text{ km}^3$  (mass  $= 5.00 \times 10^{13} \text{ kg}$ ), given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.



Hydroelectric facility (credit: Denis

**Solution:**

(a)  $1.96 \times 10^{16} \text{ J}$

(b) The ratio of gravitational potential energy in the lake to the energy stored in the bomb is 0.52. That is, the energy stored in the lake is approximately half that in a 9-megaton fusion bomb.

**Exercise:**

**Problem:**

(a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about  $7 \times 10^9 \text{ kg}$  and its center of mass is 36.5 m above the surrounding ground? (b) How does this energy compare with the daily food intake of a person?

**Exercise:**

**Problem:**

Suppose a 350-g kookaburra (a large kingfisher bird) picks up a 75-g snake and raises it 2.5 m from the ground to a branch. (a) How much work did the bird do on the snake? (b) How much work did it do to raise its own center of mass to the branch?

---

**Solution:**

(a) 1.8 J

(b) 8.6 J

**Exercise:**

**Problem:**

In [\[link\]](#), we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that  $\Delta PE \gg KE_i$ . Confirm this statement by taking the ratio of  $\Delta PE$  to  $KE_i$ . (Note that mass cancels.)

**Exercise:**

**Problem:**

A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in [\[link\]](#). Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope, gaining 0.180 m in altitude.



A toy car moves up a sloped track.  
(credit: Leszek Leszczynski, Flickr)

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**Solution:****Equation:**

$$v_f = \sqrt{2gh + v_0^2} = \sqrt{2(9.80 \text{ m/s}^2)(-0.180 \text{ m}) + (2.00 \text{ m/s})^2} = 0.687 \text{ m/s}$$

**Exercise:****Problem:**

In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skies 70.0 m along a 30° slope neglecting friction: (a) Starting from rest. (b) Starting with an initial speed of 2.50 m/s. (c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

**Glossary**

gravitational potential energy

the energy an object has due to its position in a gravitational field

## Conservative Forces and Potential Energy

- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke's law applies.
- Use the work-energy theorem to show how having only conservative forces implies conservation of mechanical energy.

## Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a **potential energy** (PE) for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

### **Note:**

#### Potential Energy and Conservative Forces

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.

A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

We can define a potential energy (PE) for any conservative force. The work done against a conservative force to reach a final configuration

depends on the configuration, not the path followed, and is the potential energy added.

## Potential Energy of a Spring

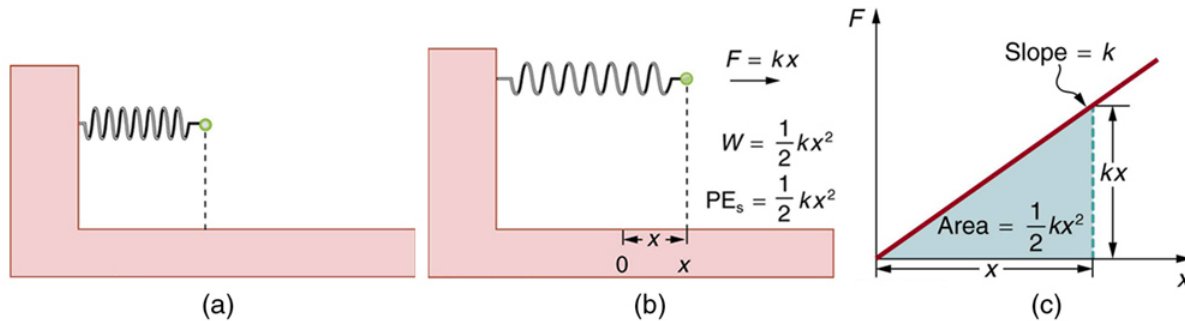
First, let us obtain an expression for the potential energy stored in a spring ( $PE_s$ ). We calculate the work done to stretch or compress a spring that obeys Hooke's law. (Hooke's law was examined in [Elasticity: Stress and Strain](#), and states that the magnitude of force  $F$  on the spring and the resulting deformation  $\Delta L$  are proportional,  $F = k\Delta L$ .) (See [\[link\]](#).) For our spring, we will replace  $\Delta L$  (the amount of deformation produced by a force  $F$ ) by the distance  $x$  that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude  $F = kx$ , where  $k$  is the spring's force constant. The force increases linearly from 0 at the start to  $kx$  in the fully stretched position. The average force is  $kx/2$ . Thus the work done in stretching or compressing the spring is

$W_s = Fd = \left(\frac{kx}{2}\right)x = \frac{1}{2}kx^2$ . Alternatively, we noted in [Kinetic Energy and the Work-Energy Theorem](#) that the area under a graph of  $F$  vs.  $x$  is the work done by the force. In [\[link\]](#)(c) we see that this area is also  $\frac{1}{2}kx^2$ . We therefore define the **potential energy of a spring**,  $PE_s$ , to be

**Equation:**

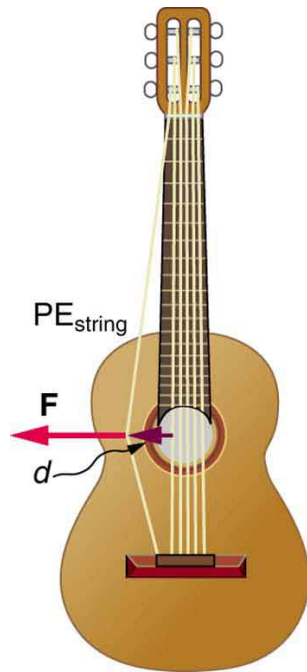
$$PE_s = \frac{1}{2}kx^2,$$

where  $k$  is the spring's force constant and  $x$  is the displacement from its undeformed position. The potential energy represents the work done *on* the spring and the energy stored in it as a result of stretching or compressing it a distance  $x$ . The potential energy of the spring  $PE_s$  does not depend on the path taken; it depends only on the stretch or squeeze  $x$  in the final configuration.



- (a) An undeformed spring has no  $PE_s$  stored in it. (b) The force needed to stretch (or compress) the spring a distance  $x$  has a magnitude  $F = kx$ , and the work done to stretch (or compress) it is  $\frac{1}{2} kx^2$ . Because the force is conservative, this work is stored as potential energy ( $PE_s$ ) in the spring, and it can be fully recovered. (c) A graph of  $F$  vs.  $x$  has a slope of  $k$ , and the area under the graph is  $\frac{1}{2} kx^2$ . Thus the work done or potential energy stored is  $\frac{1}{2} kx^2$ .

The equation  $PE_s = \frac{1}{2} kx^2$  has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of **potential energy** is energy due to position, shape, or configuration. For shape or position deformations, stored energy is  $PE_s = \frac{1}{2} kx^2$ , where  $k$  is the force constant of the particular system and  $x$  is its deformation. Another example is seen in [\[link\]](#) for a guitar string.



Work is done  
to deform the  
guitar string,  
giving it  
potential  
energy.

When  
released, the  
potential  
energy is  
converted to  
kinetic  
energy and  
back to  
potential as  
the string  
oscillates  
back and  
forth. A very  
small  
fraction is  
dissipated as

sound  
energy,  
slowly  
removing  
energy from  
the string.

## Conservation of Mechanical Energy

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

**Equation:**

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \Delta\text{KE}.$$

If only conservative forces act, then

**Equation:**

$$W_{\text{net}} = W_{\text{c}},$$

where  $W_{\text{c}}$  is the total work done by all conservative forces. Thus,

**Equation:**

$$W_{\text{c}} = \Delta\text{KE}.$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is,  $W_{\text{c}} = -\Delta\text{PE}$ . Therefore,

**Equation:**

$$-\Delta PE = \Delta KE$$

or

**Equation:**

$$\Delta KE + \Delta PE = 0.$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

**Equation:**

$$KE + PE = \text{constant}$$

or

(conservative forces only),

$$KE_i + PE_i = KE_f + PE_f$$

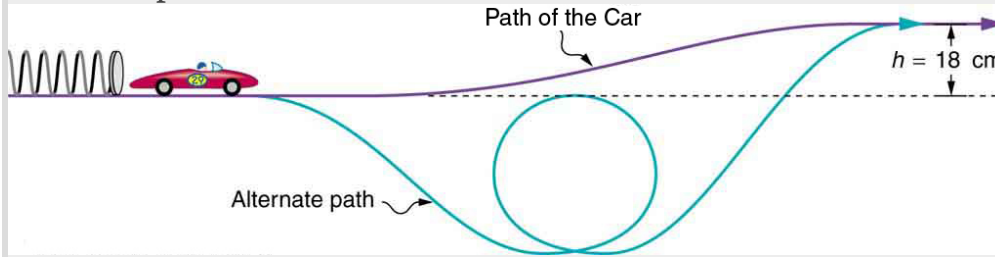
where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical energy**,  $(KE + PE)$ . In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and the various types of PE, with the total energy remaining constant.

**Example:**

**Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car**

A 0.100-kg toy car is propelled by a compressed spring, as shown in [\[link\]](#). The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car

is going before it starts up the slope and (b) how fast it is going at the top of the slope.



A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.

### Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

**Equation:**

$$KE_i + PE_i = KE_f + PE_f$$

or

**Equation:**

$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2,$$

where  $h$  is the height (vertical position) and  $x$  is the compression of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

**Solution for (a)**

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both  $h_i$  and  $h_f$  are zero. Furthermore, the initial speed  $v_i$  is zero and the final compression of the spring  $x_f$  is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

**Equation:**

$$\frac{1}{2} k x_i^2 = \frac{1}{2} m v_f^2.$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

**Equation:**

$$\begin{aligned} v_f &= \sqrt{\frac{k}{m}} x_i \\ &= \sqrt{\frac{250.0 \text{ N/m}}{0.100 \text{ kg}}} (0.0400 \text{ m}) \\ &= 2.00 \text{ m/s.} \end{aligned}$$

### **Solution for (b)**

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

**Equation:**

$$\frac{1}{2} k x_i^2 = \frac{1}{2} m v_f^2 + m g h_f.$$

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for  $v_f$  and substituting known values gives

**Equation:**

$$\begin{aligned}
 v_f &= \sqrt{\frac{kx_i^2}{m} - 2gh_f} \\
 &= \sqrt{\left(\frac{250.0 \text{ N/m}}{0.100 \text{ kg}}\right)(0.0400 \text{ m})^2 - 2(9.80 \text{ m/s}^2)(0.180 \text{ m})} \\
 &= 0.687 \text{ m/s.}
 \end{aligned}$$

### Discussion

Another way to solve this problem is to realize that the car's kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in [\[link\]](#). Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.

### Note:

#### PhET Explorations: Energy Skate Park

Learn about conservation of energy with a skater dude! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and friction as he moves. You can also take the skater to different planets or even space!

[https://phet.colorado.edu/sims/html/energy-skate-park-basics/latest/energy-skate-park-basics\\_en.html](https://phet.colorado.edu/sims/html/energy-skate-park-basics/latest/energy-skate-park-basics_en.html)

## Section Summary

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy (PE) for any conservative force, just as we defined  $PE_g$  for the gravitational force.
- The potential energy of a spring is  $PE_s = \frac{1}{2}kx^2$ , where  $k$  is the spring's force constant and  $x$  is the displacement from its undeformed position.
- Mechanical energy is defined to be  $KE + PE$  for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,

**Equation:**

$$KE + PE = \text{constant}$$

or

$$KE_i + PE_i = KE_f + PE_f$$

where i and f denote initial and final values. This is known as the conservation of mechanical energy.

## Conceptual Questions

**Exercise:**

**Problem:** What is a conservative force?

**Exercise:**

**Problem:**

The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.

**Exercise:**

**Problem:**

Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?

**Exercise:****Problem:**

What is the relationship of potential energy to conservative force?

**Problems & Exercises****Exercise:****Problem:**

A  $5.00 \times 10^5$ -kg subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the force constant  $k$  of the spring?

---

**Solution:****Equation:**

$$7.81 \times 10^5 \text{ N/m}$$

**Exercise:****Problem:**

A pogo stick has a spring with a force constant of  $2.50 \times 10^4$  N/m, which can be compressed 12.0 cm. To what maximum height can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40.0 kg? Explicitly show how you follow the steps in the [Problem-Solving Strategies for Energy](#).

## Glossary

conservative force

a force that does the same work for any given initial and final configuration, regardless of the path followed

potential energy

energy due to position, shape, or configuration

potential energy of a spring

the stored energy of a spring as a function of its displacement; when Hooke's law applies, it is given by the expression  $\frac{1}{2}kx^2$  where  $x$  is the distance the spring is compressed or extended and  $k$  is the spring constant

conservation of mechanical energy

the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

mechanical energy

the sum of kinetic energy and potential energy

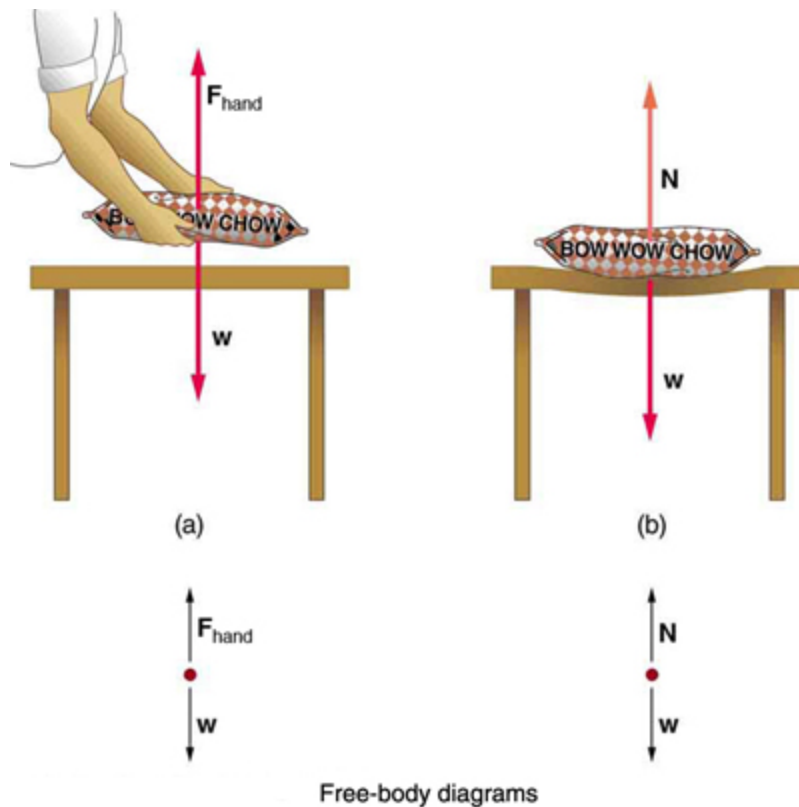
## Normal, Tension, and Other Examples of Forces

- Define normal and tension forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.
- Use trigonometric identities to resolve weight into components.

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. The most important of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

### Normal Force

**Weight** (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in [\[link\]](#)(a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in [\[link\]](#)(b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.



(a) The person holding the bag of dog food must supply an upward force  $\mathbf{F}_{\text{hand}}$  equal in magnitude and opposite in direction to the weight of the food  $\mathbf{w}$ . (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force  $\mathbf{N}$  equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a **normal force** and here is given the symbol  $\mathbf{N}$ . (This is not the unit for force N.) The word *normal* means perpendicular to a

surface. The normal force can be less than the object's weight if the object is on an incline, as you will see in the next example.

**Note:**

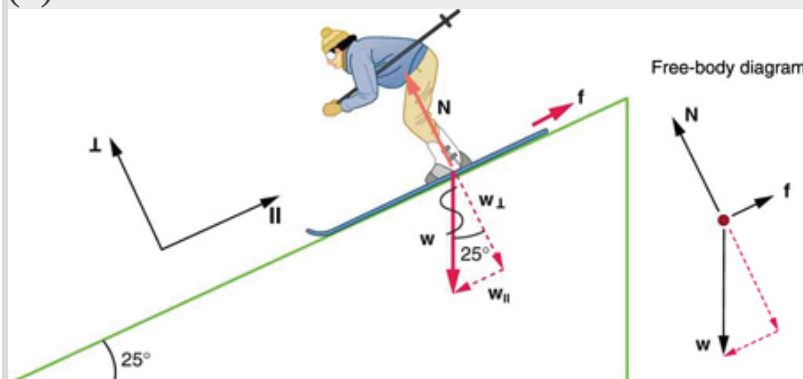
**Common Misconception: Normal Force ( $\mathbf{N}$ ) vs. Newton ( $\text{N}$ )**

In this section we have introduced the quantity normal force, which is represented by the variable  $\mathbf{N}$ . This should not be confused with the symbol for the newton, which is also represented by the letter  $\text{N}$ . These symbols are particularly important to distinguish because the units of a normal force ( $\mathbf{N}$ ) happen to be newtons ( $\text{N}$ ). For example, the normal force  $\mathbf{N}$  that the floor exerts on a chair might be  $\mathbf{N} = 100 \text{ N}$ . One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations! You will encounter more similarities among variables and units as you proceed in physics. Another example of this is the quantity work ( $W$ ) and the unit watts ( $\text{W}$ ).

**Example:**

**Weight on an Incline, a Two-Dimensional Problem**

Consider the skier on a slope shown in [\[link\]](#). Her mass including equipment is  $60.0 \text{ kg}$ . (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is known to be  $45.0 \text{ N}$ ?



Since motion and friction are parallel to the slope, it is most convenient to project all

forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier).

$\mathbf{N}$  is perpendicular to the slope and  $\mathbf{f}$  is parallel to the slope, but  $\mathbf{w}$  has components along both axes, namely  $\mathbf{w}_\perp$  and  $\mathbf{w}_\parallel$ .  $\mathbf{N}$  is equal in magnitude to  $\mathbf{w}_\perp$ , so that there is no motion perpendicular to the slope, but  $f$  is less than  $w_\parallel$ , so that there is a downslope acceleration (along the parallel axis).

### Strategy

This is a two-dimensional problem, since the forces on the skier (the system of interest) are not parallel. The approach we have used in two-dimensional kinematics also works very well here. Choose a convenient coordinate system and project the vectors onto its axes, creating two connected *one*-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Remember that motions along mutually perpendicular axes are independent.) We use the symbols  $\perp$  and  $\parallel$  to represent perpendicular and parallel, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and because friction is always parallel to the surface between two objects. The only external forces acting on the system are the skier's weight, friction, and the support of the slope, respectively labeled  $\mathbf{w}$ ,  $\mathbf{f}$ , and  $\mathbf{N}$  in [\[link\]](#).  $\mathbf{N}$  is always perpendicular to the slope, and  $\mathbf{f}$  is parallel to it. But  $\mathbf{w}$  is not in the direction of either axis, and so the first step we take is to project it into components along the chosen axes, defining  $w_\parallel$  to be the component of weight parallel to the slope and  $w_\perp$  the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

### Solution

The magnitude of the component of the weight parallel to the slope is  $w_\parallel = w \sin(25^\circ) = mg \sin(25^\circ)$ , and the magnitude of the component of

the weight perpendicular to the slope is

$$w_{\perp} = w \cos (25^{\circ}) = mg \cos (25^{\circ}).$$

(a) Neglecting friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the amount of the skier's weight parallel to the slope  $w_{\parallel}$  and friction  $f$ . Using Newton's second law, with subscripts to denote quantities parallel to the slope,

**Equation:**

$$a_{\parallel} = \frac{F_{\text{net}\parallel}}{m}$$

where  $F_{\text{net}\parallel} = w_{\parallel} = mg \sin (25^{\circ})$ , assuming no friction for this part, so that

**Equation:**

$$a_{\parallel} = \frac{F_{\text{net}\parallel}}{m} = \frac{mg \sin (25^{\circ})}{m} = g \sin (25^{\circ})$$

**Equation:**

$$(9.80 \text{ m/s}^2)(0.4226) = 4.14 \text{ m/s}^2$$

is the acceleration.

(b) Including friction. We now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

**Equation:**

$$F_{\text{net}\parallel} = w_{\parallel} - f,$$

and substituting this into Newton's second law,  $a_{\parallel} = \frac{F_{\text{net}\parallel}}{m}$ , gives

**Equation:**

$$a_{\parallel} = \frac{F_{\text{net}\parallel}}{m} = \frac{w_{\parallel} - f}{m} = \frac{mg \sin (25^{\circ}) - f}{m}.$$

We substitute known values to obtain

**Equation:**

$$a_{\parallel} = \frac{(60.0 \text{ kg})(9.80 \text{ m/s}^2)(0.4226) - 45.0 \text{ N}}{60.0 \text{ kg}},$$

which yields

**Equation:**

$$a_{\parallel} = 3.39 \text{ m/s}^2,$$

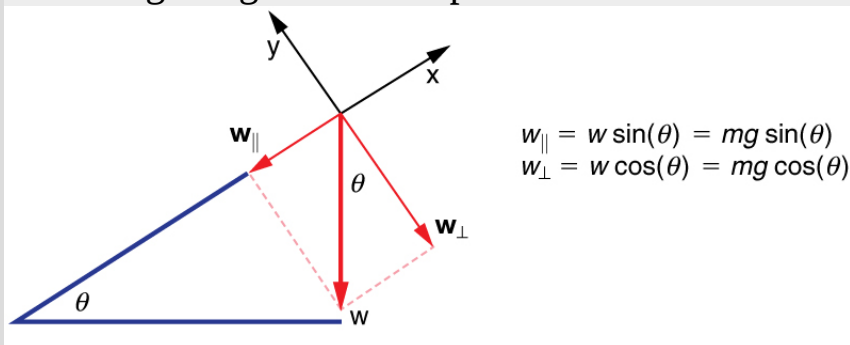
which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

### Discussion

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. In fact, it is a general result that if friction on an incline is negligible, then the acceleration down the incline is  $a = g \sin\theta$ , *regardless of mass*. This is related to the previously discussed fact that all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

### Note:

#### Resolving Weight into Components



An object rests on an incline that makes an

angle  $\theta$  with the horizontal.

When an object rests on an incline that makes an angle  $\theta$  with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane,  $\mathbf{w}_\perp$ , and a force acting parallel to the plane,  $\mathbf{w}_\parallel$ . The perpendicular force of weight,  $\mathbf{w}_\perp$ , is typically equal in magnitude and opposite in direction to the normal force,  $\mathbf{N}$ . The force acting parallel to the plane,  $\mathbf{w}_\parallel$ , causes the object to accelerate down the incline. The force of friction,  $\mathbf{f}$ , opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline is at an angle  $\theta$  to the horizontal, then the magnitudes of the weight components are

**Equation:**

$$w_\parallel = w \sin (\theta) = mg \sin (\theta)$$

and

**Equation:**

$$w_\perp = w \cos (\theta) = mg \cos (\theta).$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the angle  $\theta$  of the incline is the same as the angle formed between  $\mathbf{w}$  and  $\mathbf{w}_\perp$ . Knowing this property, you can use trigonometry to determine the magnitude of the weight components:

**Equation:**

$$\begin{aligned} \cos (\theta) &= \frac{w_\perp}{w} \\ w_\perp &= w \cos (\theta) = mg \cos (\theta) \end{aligned}$$

**Equation:**

$$\begin{aligned} \sin (\theta) &= \frac{w_\parallel}{w} \\ w_\parallel &= w \sin (\theta) = mg \sin (\theta) \end{aligned}$$

### **Note:**

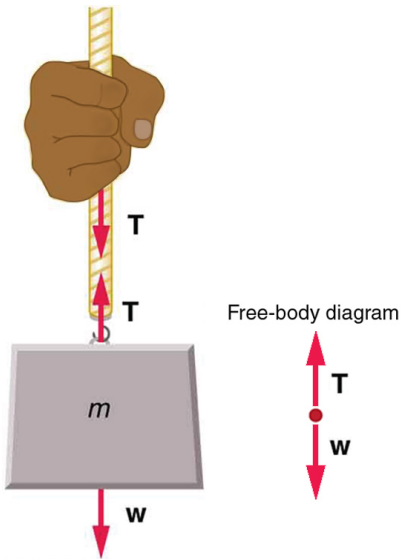
#### **Take-Home Experiment: Force Parallel**

To investigate how a force parallel to an inclined plane changes, find a rubber band, some objects to hang from the end of the rubber band, and a board you can position at different angles. How much does the rubber band stretch when you hang the object from the end of the board? Now place the board at an angle so that the object slides off when placed on the board. How much does the rubber band extend if it is lined up parallel to the board and used to hold the object stationary on the board? Try two more angles. What does this show?

## **Tension**

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word “tension” comes from a Latin word meaning “to stretch.” Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: “You can’t push a rope.” The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in [\[link\]](#).



When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force  $\mathbf{T}$ , that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that carries

the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton's second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus  $\mathbf{F}_{\text{net}} = 0$ . The only external forces acting on the mass are its weight  $\mathbf{w}$  and the tension  $\mathbf{T}$  supplied by the rope. Thus,

**Equation:**

$$F_{\text{net}} = T - w = 0,$$

where  $T$  and  $w$  are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

**Equation:**

$$T = w = mg.$$

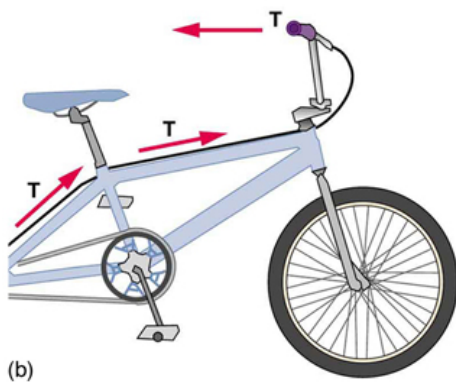
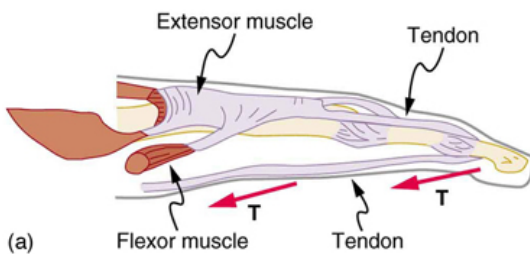
For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

### Equation:

$$T = mg = (5.00 \text{ kg})(9.80 \text{ m/s}^2) = 49.0 \text{ N}.$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in [\[link\]](#) (a) and (b).



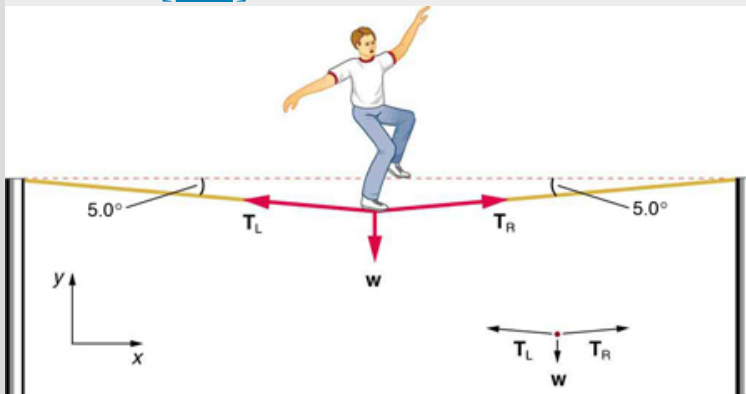
(a) Tendons in the finger carry force  $\mathbf{T}$  from the muscles to other parts of the finger, usually changing the force's direction, but not its magnitude (the

tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension  $\mathbf{T}$  from the handlebars to the brake mechanism. Again, the direction but not the magnitude of  $\mathbf{T}$  is changed.

### Example:

#### What Is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in [\[link\]](#).



The weight of a tightrope walker causes a wire to sag by 5.0 degrees. The system of interest here is the point in the wire at which the tightrope walker is standing.

### Strategy

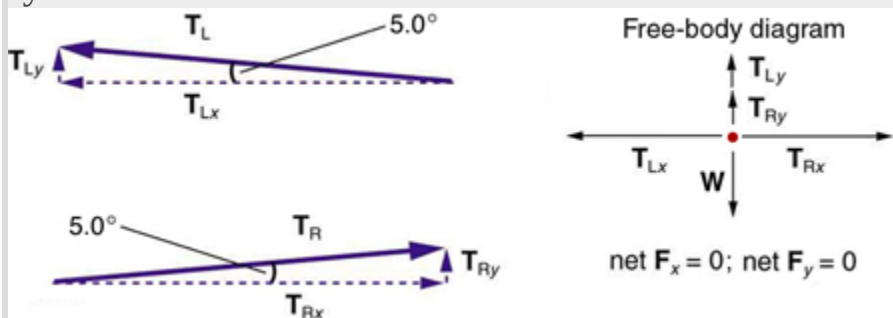
As you can see in the figure, the wire is not perfectly horizontal (it cannot be!), but is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As

usual, forces are vectors represented pictorially by arrows having the same directions as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight  $\mathbf{w}$  and the two tensions  $\mathbf{T}_L$  (left tension) and  $\mathbf{T}_R$  (right tension), as illustrated. It is reasonable to neglect the weight of the wire itself. The net external force is zero since the system is stationary. A little trigonometry can now be used to find the tensions. One conclusion is possible at the outset—we can see from part (b) of the figure that the magnitudes of the tensions  $T_L$  and  $T_R$  must be equal. This is because there is no horizontal acceleration in the rope, and the only forces acting to the left and right are  $T_L$  and  $T_R$ . Thus, the magnitude of those forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case the best coordinate system has one axis horizontal and the other vertical. We call the horizontal the  $x$ -axis and the vertical the  $y$ -axis.

### Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to draw a new free-body diagram showing all of the horizontal and vertical components of each force acting on the system.



When the vectors are projected onto vertical and horizontal axes, their components along those axes must add to zero, since the tightrope walker is stationary. The small angle results in  $T$  being much greater than  $w$ .

Consider the horizontal components of the forces (denoted with a subscript  $x$ ):

**Equation:**

$$F_{\text{net}x} = T_{Lx} - T_{Rx}.$$

The net external horizontal force  $F_{\text{net}x} = 0$ , since the person is stationary. Thus,

**Equation:**

$$\begin{aligned} F_{\text{net}x} = 0 &= T_{Lx} - T_{Rx} \\ T_{Lx} &= T_{Rx}. \end{aligned}$$

Now, observe [\[link\]](#). You can use trigonometry to determine the magnitude of  $T_L$  and  $T_R$ . Notice that:

**Equation:**

$$\begin{aligned} \cos(5.0^\circ) &= \frac{T_{Lx}}{T_L} \\ T_{Lx} &= T_L \cos(5.0^\circ) \\ \cos(5.0^\circ) &= \frac{T_{Rx}}{T_R} \\ T_{Rx} &= T_R \cos(5.0^\circ). \end{aligned}$$

Equating  $T_{Lx}$  and  $T_{Rx}$ :

**Equation:**

$$T_L \cos(5.0^\circ) = T_R \cos(5.0^\circ).$$

Thus,

**Equation:**

$$T_L = T_R = T,$$

as predicted. Now, considering the vertical components (denoted by a subscript  $y$ ), we can solve for  $T$ . Again, since the person is stationary, Newton's second law implies that net  $F_y = 0$ . Thus, as illustrated in the free-body diagram in [\[link\]](#),

**Equation:**

$$F_{\text{net}y} = T_{Ly} + T_{Ry} - w = 0.$$

Observing [\[link\]](#), we can use trigonometry to determine the relationship between  $T_{Ly}$ ,  $T_{Ry}$ , and  $T$ . As we determined from the analysis in the horizontal direction,  $T_L = T_R = T$ :

**Equation:**

$$\begin{aligned}\sin (5.0^\circ) &= \frac{T_{Ly}}{T_L} \\ T_{Ly} = T_L \sin (5.0^\circ) &= T \sin (5.0^\circ) \\ \sin (5.0^\circ) &= \frac{T_{Ry}}{T_R} \\ T_{Ry} = T_R \sin (5.0^\circ) &= T \sin (5.0^\circ).\end{aligned}$$

Now, we can substitute the values for  $T_{Ly}$  and  $T_{Ry}$ , into the net force equation in the vertical direction:

**Equation:**

$$\begin{aligned}F_{\text{net}y} &= T_{Ly} + T_{Ry} - w = 0 \\ F_{\text{net}y} &= T \sin (5.0^\circ) + T \sin (5.0^\circ) - w = 0 \\ 2 T \sin (5.0^\circ) - w &= 0 \\ 2 T \sin (5.0^\circ) &= w\end{aligned}$$

and

**Equation:**

$$T = \frac{w}{2 \sin (5.0^\circ)} = \frac{mg}{2 \sin (5.0^\circ)},$$

so that

**Equation:**

$$T = \frac{(70.0 \text{ kg})(9.80 \text{ m/s}^2)}{2(0.0872)},$$

and the tension is

**Equation:**

$$T = 3900 \text{ N}.$$

**Discussion**

Note that the vertical tension in the wire acts as a normal force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a small fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, and so most of the tension in the wire is not used to support the weight of the tightrope walker.

If we wish to *create* a very large tension, all we have to do is exert a force perpendicular to a flexible connector, as illustrated in [\[link\]](#). As we saw in the last example, the weight of the tightrope walker acted as a force perpendicular to the rope. We saw that the tension in the rope related to the weight of the tightrope walker in the following way:

**Equation:**

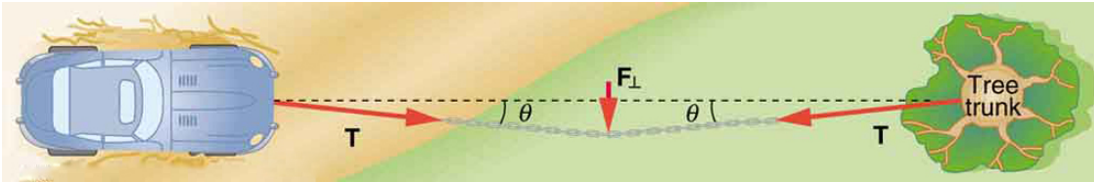
$$T = \frac{w}{2 \sin (\theta)}.$$

We can extend this expression to describe the tension  $T$  created when a perpendicular force ( $\mathbf{F}_\perp$ ) is exerted at the middle of a flexible connector:

**Equation:**

$$T = \frac{F_\perp}{2 \sin (\theta)}.$$

Note that  $\theta$  is the angle between the horizontal and the bent connector. In this case,  $T$  becomes very large as  $\theta$  approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e.,  $\theta = 0$  and  $\sin \theta = 0$ ). (See [\[link\]](#).)



We can create a very large tension in the chain by pushing on it perpendicular to its length, as shown. Suppose we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as nearly straight as possible. The tension in the chain is given by  $T = \frac{F_{\perp}}{2 \sin(\theta)}$ ; since  $\theta$  is small,  $T$  is very large. This situation is analogous to the tightrope walker shown in [\[link\]](#), except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where  $\mathbf{F}_{\perp}$  is applied.



Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly

distributed along the length.

Suspension bridges—such as the Golden Gate Bridge shown in this image—are essentially very heavy flexible connectors. The weight of the bridge is evenly distributed along the length of flexible connectors, usually cables, which take on the characteristic shape.

(credit: Leaflet, Wikimedia Commons)

## Extended Topic: Real Forces and Inertial Frames

There is another distinction among forces in addition to the types already mentioned. Some forces are real, whereas others are not. *Real forces* are those that have some physical origin, such as the gravitational pull. Contrastingly, *fictitious forces* are those that arise simply because an observer is in an accelerating frame of reference, such as one that rotates (like a merry-go-round) or undergoes linear acceleration (like a car slowing down). For example, if a satellite is heading due north above Earth's northern hemisphere, then to an observer on Earth it will appear to experience a force to the west that has no physical origin. Of course, what is happening here is that Earth is rotating toward the east and moves east under the satellite. In Earth's frame this looks like a westward force on the satellite, or it can be interpreted as a violation of Newton's first law (the law of inertia). An **inertial frame of reference** is one in which all forces are real and, equivalently, one in which Newton's laws have the simple forms given in this chapter.

Earth's rotation is slow enough that Earth is nearly an inertial frame. You ordinarily must perform precise experiments to observe fictitious forces and the slight departures from Newton's laws, such as the effect just described. On the large scale, such as for the rotation of weather systems and ocean currents, the effects can be easily observed.

The crucial factor in determining whether a frame of reference is inertial is whether it accelerates or rotates relative to a known inertial frame. Unless stated otherwise, all phenomena discussed in this text are considered in inertial frames.

All the forces discussed in this section are real forces, but there are a number of other real forces, such as lift and thrust, that are not discussed in this section. They are more specialized, and it is not necessary to discuss every type of force. It is natural, however, to ask where the basic simplicity we seek to find in physics is in the long list of forces. Are some more basic than others? Are some different manifestations of the same underlying force? The answer to both questions is yes, as will be seen in the next (extended) section and in the treatment of modern physics later in the text.

**Note:****PhET Explorations: Forces in 1 Dimension**

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

[Forces in  
1  
Dimension](#)

## Section Summary

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts

perpendicular to and away from the surface. It is called a normal force, **N**.

- When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object:

**Equation:**

$$N = mg.$$

- When objects rest on an inclined plane that makes an angle  $\theta$  with the horizontal surface, the weight of the object can be resolved into components that act perpendicular ( $\mathbf{w}_\perp$ ) and parallel ( $\mathbf{w}_\parallel$ ) to the surface of the plane. These components can be calculated using:

**Equation:**

$$w_\parallel = w \sin(\theta) = mg \sin(\theta)$$

**Equation:**

$$w_\perp = w \cos(\theta) = mg \cos(\theta).$$

- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, **T**. When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object:

**Equation:**

$$T = mg.$$

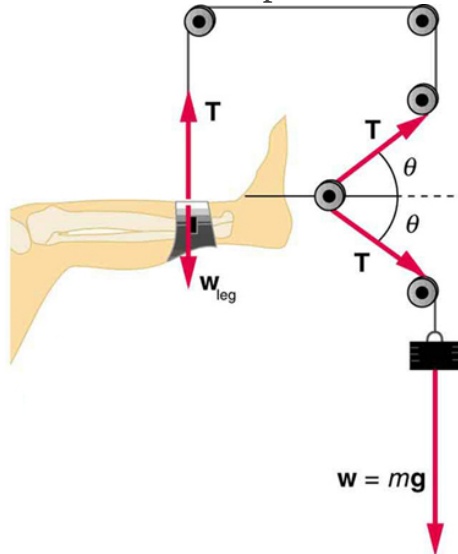
- In any inertial frame of reference (one that is not accelerated or rotated), Newton's laws have the simple forms given in this chapter and all forces are real forces having a physical origin.

## Conceptual Questions

**Exercise:**

**Problem:**

If a leg is suspended by a traction setup as shown in [\[link\]](#), what is the tension in the rope?



A leg is suspended by a traction system in which wires are used to transmit forces. Frictionless pulleys change the direction of the force  $T$  without changing its magnitude.

**Exercise:****Problem:**

In a traction setup for a broken bone, with pulleys and rope available, how might we be able to increase the force along the tibia using the same weight? (See [\[link\]](#).) (Note that the tibia is the shin bone shown in this image.)

## Problem Exercises

### Exercise:

#### Problem:

Two teams of nine members each engage in a tug of war. Each of the first team's members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team's members has an average mass of 73 kg and exerts an average force of 1365 N horizontally. (a) What is magnitude of the acceleration of the two teams? (b) What is the tension in the section of rope between the teams?

---

#### Solution:

- a.  $0.11 \text{ m/s}^2$
- b.  $1.2 \times 10^4 \text{ N}$

### Exercise:

#### Problem:

What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at  $7.50 \text{ m/s}^2$ ? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.

### Exercise:

#### Problem:

(a) Calculate the tension in a vertical strand of spider web if a spider of mass  $8.00 \times 10^{-5} \text{ kg}$  hangs motionless on it. (b) Calculate the tension in a horizontal strand of spider web if the same spider sits motionless in the middle of it much like the tightrope walker in [\[link\]](#). The strand sags at an angle of  $12^\circ$  below the horizontal. Compare this with the tension in the vertical strand (find their ratio).

---

**Solution:**

(a)  $7.84 \times 10^{-4} \text{ N}$

(b)  $1.89 \times 10^{-3} \text{ N}$  . This is 2.41 times the tension in the vertical strand.

**Exercise:****Problem:**

Suppose a 60.0-kg gymnast climbs a rope. (a) What is the tension in the rope if he climbs at a constant speed? (b) What is the tension in the rope if he accelerates upward at a rate of  $1.50 \text{ m/s}^2$ ?

**Exercise:****Problem:**

Show that, as stated in the text, a force  $\mathbf{F}_\perp$  exerted on a flexible medium at its center and perpendicular to its length (such as on the tightrope wire in [\[link\]](#)) gives rise to a tension of magnitude

$$T = \frac{F_\perp}{2 \sin(\theta)}.$$

---

**Solution:**

Newton's second law applied in vertical direction gives

**Equation:**

$$F_y = F - 2T \sin \theta = 0$$

**Equation:**

$$F = 2T \sin \theta$$

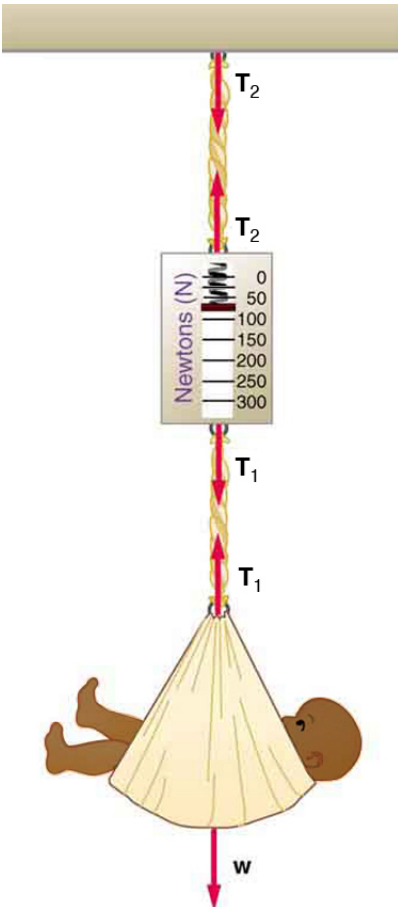
**Equation:**

$$T = \frac{F}{2 \sin \theta}.$$

### Exercise:

#### Problem:

Consider the baby being weighed in [\[link\]](#). (a) What is the mass of the child and basket if a scale reading of 55 N is observed? (b) What is the tension  $T_1$  in the cord attaching the baby to the scale? (c) What is the tension  $T_2$  in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Draw a sketch of the situation indicating the system of interest used to solve each part. The masses of the cords are negligible.



A baby is weighed  
using a spring  
scale.

## Glossary

### inertial frame of reference

a coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious forces that are observed due to an accelerating frame of reference

### normal force

the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

### tension

the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

## Power

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

### What is Power?

*Power*—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in [\[link\]](#).



This powerful rocket on the Space Shuttle *Endeavor* did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** ( $P$ ) as the rate at which work is done.

**Note:****Power**

Power is the rate at which work is done.

**Equation:**

$$P = \frac{W}{t}$$

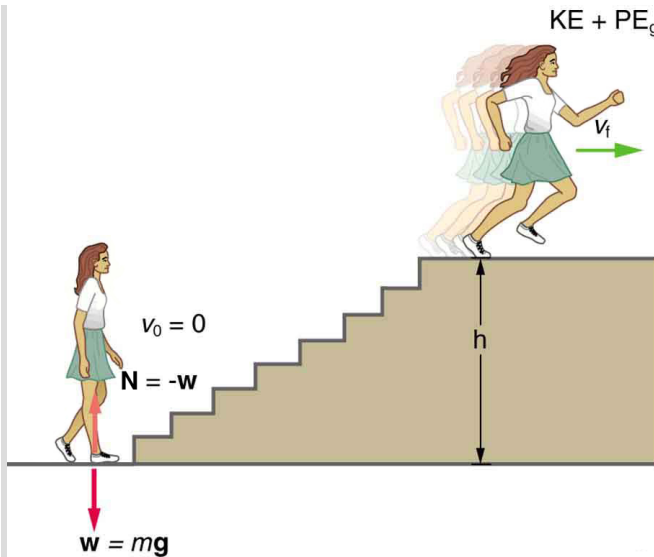
The SI unit for power is the **watt** (W), where 1 watt equals 1 joule/second ( $1 \text{ W} = 1 \text{ J/s}$ ).

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

## Calculating Power from Energy

**Example:****Calculating the Power to Climb Stairs**

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See [\[link\]](#).)



When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

### Strategy and Concept

The work going into mechanical energy is  $W = KE + PE$ . At the bottom of the stairs, we take both  $KE$  and  $PE_g$  as initially zero; thus,

$W = KE_f + PE_g = \frac{1}{2}mv_f^2 + mgh$ , where  $h$  is the vertical height of the stairs. Because all terms are given, we can calculate  $W$  and then divide it by time to get power.

### Solution

Substituting the expression for  $W$  into the definition of power given in the previous equation,  $P = W/t$  yields

### Equation:

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_f^2 + mgh}{t}.$$

Entering known values yields

**Equation:**

$$\begin{aligned} P &= \frac{0.5(60.0 \text{ kg})(2.00 \text{ m/s})^2 + (60.0 \text{ kg})(9.80 \text{ m/s}^2)(3.00 \text{ m})}{3.50 \text{ s}} \\ &= \frac{120 \text{ J} + 1764 \text{ J}}{3.50 \text{ s}} \\ &= 538 \text{ W}. \end{aligned}$$

**Discussion**

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** (1 hp = 746 W)! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

**Note:****Making Connections: Take-Home Investigation—Measure Your Power Rating**

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

**Examples of Power**

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See [\[link\]](#) for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter ( $\text{kW}/\text{m}^2$ ). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is  $10^6$  W of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See [\[link\]](#).)



Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings.

The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Object or Phenomenon	Power in Watts
Supernova (at peak)	$5 \times 10^{37}$
Milky Way galaxy	$10^{37}$
Crab Nebula pulsar	$10^{28}$
The Sun	$4 \times 10^{26}$

Object or Phenomenon	Power in Watts
Volcanic eruption (maximum)	$4 \times 10^{15}$
Lightning bolt	$2 \times 10^{12}$
Nuclear power plant (total electric and heat transfer)	$3 \times 10^9$
Aircraft carrier (total useful and heat transfer)	$10^8$
Dragster (total useful and heat transfer)	$2 \times 10^6$
Car (total useful and heat transfer)	$8 \times 10^4$
Football player (total useful and heat transfer)	$5 \times 10^3$
Clothes dryer	$4 \times 10^3$
Person at rest (all heat transfer)	100

Object or Phenomenon	Power in Watts
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	$10^{-3}$

Power Output or Consumption

## Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is  $P = W/t = E/t$ , where  $E$  is the energy supplied by the electricity company. So the energy consumed over a time  $t$  is

**Equation:**

$$E = Pt.$$

Electricity bills state the energy used in units of **kilowatt-hours** ( $\text{kW} \cdot \text{h}$ ), which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

**Example:****Calculating Energy Costs**

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is \$0.120 per kW · h?

**Strategy**

Cost is based on energy consumed; thus, we must find  $E$  from  $E = Pt$  and then calculate the cost. Because electrical energy is expressed in kW · h, at the start of a problem such as this it is convenient to convert the units into kW and hours.

**Solution**

The energy consumed in kW · h is

**Equation:**

$$\begin{aligned} E &= Pt = (0.200 \text{ kW})(6.00 \text{ h/d})(30.0 \text{ d}) \\ &= 36.0 \text{ kW} \cdot \text{h}, \end{aligned}$$

and the cost is simply given by

**Equation:**

$$\text{cost} = (36.0 \text{ kW} \cdot \text{h})(\$0.120 \text{ per kW} \cdot \text{h}) = \$4.32 \text{ per month.}$$

**Discussion**

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day

usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail in [Thermodynamics](#), the potential for energy to produce useful work has been “degraded” in the energy transformation.

## Section Summary

- Power is the rate at which work is done, or in equation form, for the average power  $P$  for work  $W$  done over a time  $t$ ,  $P = W/t$ .
- The SI unit for power is the watt (W), where  $1 \text{ W} = 1 \text{ J/s}$ .
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where  $1 \text{ hp} = 746 \text{ W}$ .

## Conceptual Questions

### Exercise:

#### Problem:

Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

### Exercise:

**Problem:**

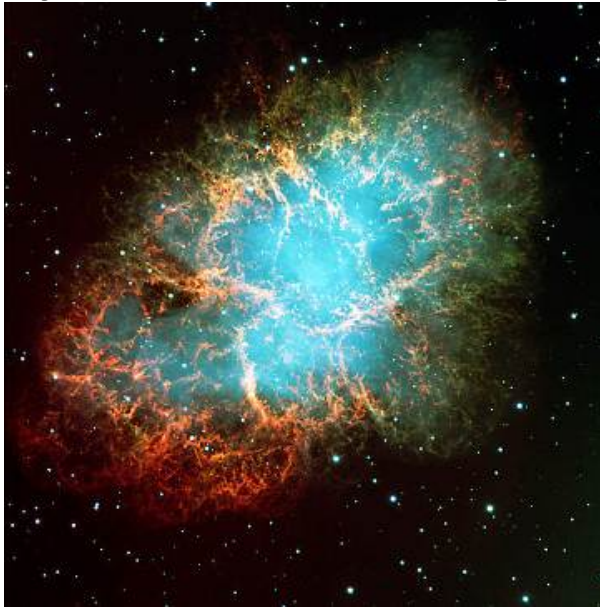
Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?

**Exercise:****Problem:**

A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

**Problems & Exercises****Exercise:****Problem:**

The Crab Nebula (see [\[link\]](#)) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from [\[link\]](#), calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.



Crab Nebula (credit: ESO, via  
Wikimedia Commons)

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**Solution:**

**Equation:**

$$2 \times 10^{-10}$$

**Exercise:**

**Problem:**

Suppose a star 1000 times brighter than our Sun (that is, emitting 1000 times the power) suddenly goes supernova. Using data from [\[link\]](#): (a) By what factor does its power output increase? (b) How many times brighter than our entire Milky Way galaxy is the supernova? (c) Based on your answers, discuss whether it should be possible to observe supernovas in distant galaxies. Note that there are on the order of  $10^{11}$  observable galaxies, the average brightness of which is somewhat less than our own galaxy.

**Exercise:**

**Problem:**

A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?

---

**Solution:**

(a) 40

(b) 8 million

**Exercise:**

**Problem:**

What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is \$0.0900 per  $\text{kW} \cdot \text{h}$ ?

**Exercise:**

**Problem:**

A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is \$0.110 per  $\text{kW} \cdot \text{h}$ ?

---

**Solution:**

\$149

**Exercise:**

**Problem:**

(a) What is the average power consumption in watts of an appliance that uses 5.00  $\text{kW} \cdot \text{h}$  of energy per day? (b) How many joules of energy does this appliance consume in a year?

**Exercise:**

**Problem:**

(a) What is the average useful power output of a person who does  $6.00 \times 10^6 \text{ J}$  of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

---

**Solution:**

(a) 208 W

(b) 141 s

**Exercise:**

**Problem:**

A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

**Exercise:**

**Problem:**

(a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp = 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m-high hill in the process?

---

**Solution:**

(a) 3.20 s

(b) 4.04 s

**Exercise:**

**Problem:**

(a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is \$0.0900 per kW · h?

**Exercise:**

**Problem:**

(a) What is the available energy content, in joules, of a battery that operates a 2.00-W electric clock for 18 months? (b) How long can a battery that can supply  $8.00 \times 10^4$  J run a pocket calculator that consumes energy at the rate of  $1.00 \times 10^{-3}$  W?

---

**Solution:**

(a)  $9.46 \times 10^7$  J

(b) 2.54 y

**Exercise:****Problem:**

(a) How long would it take a  $1.50 \times 10^5$ -kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)

**Exercise:****Problem:**

Calculate the power output needed for a 950-kg car to climb a  $2.00^\circ$  slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N. Explicitly show how you follow the steps in the [Problem-Solving Strategies for Energy](#).

---

**Solution:**

Identify knowns:  $m = 950$  kg, slope angle  $\theta = 2.00^\circ$ ,  $v = 30.0$  m/s,  $f = 600$  N

Identify unknowns: power  $P$  of the car, force  $F$  that car applies to road

Solve for unknown:

$$P = \frac{W}{t} = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv,$$

where  $F$  is parallel to the incline and must oppose the resistive forces and the force of gravity:

$$F = f + w = 600 \text{ N} + mg \sin \theta$$

Insert this into the expression for power and solve:

$$\begin{aligned} P &= (f + mg \sin \theta)v \\ &= \left[ 600 \text{ N} + (950 \text{ kg}) \left( 9.80 \text{ m/s}^2 \right) \sin 2^\circ \right] (30.0 \text{ m/s}) \\ &= 2.77 \times 10^4 \text{ W} \end{aligned}$$

About 28 kW (or about 37 hp) is reasonable for a car to climb a gentle incline.

### **Exercise:**

#### **Problem:**

(a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be  $4.00 \times 10^{26} \text{ W}$ .) (b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of  $1.30 \text{ kW/m}^2$  reaches Earth's surface. Calculate the area in  $\text{km}^2$  of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs ( $1.05 \times 10^{20} \text{ J}$ )? Australia's energy needs ( $5.4 \times 10^{18} \text{ J}$ )? China's energy needs ( $6.3 \times 10^{19} \text{ J}$ )? (These energy consumption values are from 2006.)

## Glossary

power

the rate at which work is done

watt

(W) SI unit of power, with  $1 \text{ W} = 1 \text{ J/s}$

horsepower

an older non-SI unit of power, with  $1 \text{ hp} = 746 \text{ W}$

kilowatt-hour

(kW · h) unit used primarily for electrical energy provided by electric utility companies

## Conservation of Energy

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

## Law of Conservation of Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of conservation of energy** can be stated as follows:

*Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.*

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy ( $KE + PE$ ) and energy transferred via work done by nonconservative forces ( $W_{nc}$ ). But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

## Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called other energy (OE). Then we can state the conservation of energy in equation form as

**Equation:**

$$KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f.$$

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is  $KE$ , work done by a conservative force is represented by  $PE$ , work done by nonconservative forces is  $W_{nc}$ , and

all other energies are included as OE. This equation applies to all previous examples; in those situations OE was constant, and so it subtracted out and was not directly considered.

**Note:**

**Making Connections: Usefulness of the Energy Conservation Principle**

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does OE play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of OE).

## **Some of the Many Forms of Energy**

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons.

Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

[\[link\]](#) gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

**Note:**

**Problem-Solving Strategies for Energy**

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on—continue to be relevant here.

**Step 1.** Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

**Step 2.** Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

**Step 3.** If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is

**Equation:**

$$KE_i + PE_i = KE_f + PE_f.$$

**Step 4.** If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.

**Equation:**

$$KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f.$$

In most problems, one or more of the terms is zero, simplifying its solution. Do not calculate  $W_c$ , the work done by conservative forces; it is already incorporated in the PE terms.

**Step 5.** You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose  $h = 0$  at either the initial or final point, so that  $PE_g$  is zero there. Then solve for the unknown in the customary manner.

**Step 6.** *Check the answer to see if it is reasonable.* Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

## Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see [\[link\]](#)) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.



Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Object/phenomenon	Energy in joules
Big Bang	$10^{68}$
Energy released in a supernova	$10^{44}$
Fusion of all the hydrogen in Earth's oceans	$10^{34}$
Annual world energy use	$4 \times 10^{20}$

Object/phenomenon	Energy in joules
Large fusion bomb (9 megaton)	$3.8 \times 10^{16}$
1 kg hydrogen (fusion to helium)	$6.4 \times 10^{14}$
1 kg uranium (nuclear fission)	$8.0 \times 10^{13}$
Hiroshima-size fission bomb (10 kiloton)	$4.2 \times 10^{13}$
90,000-ton aircraft carrier at 30 knots	$1.1 \times 10^{10}$
1 barrel crude oil	$5.9 \times 10^9$
1 ton TNT	$4.2 \times 10^9$
1 gallon of gasoline	$1.2 \times 10^8$
Daily home electricity use (developed countries)	$7 \times 10^7$
Daily adult food intake (recommended)	$1.2 \times 10^7$

Object/phenomenon	Energy in joules
1000-kg car at 90 km/h	$3.1 \times 10^5$
1 g fat (9.3 kcal)	$3.9 \times 10^4$
ATP hydrolysis reaction	$3.2 \times 10^4$
1 g carbohydrate (4.1 kcal)	$1.7 \times 10^4$
1 g protein (4.1 kcal)	$1.7 \times 10^4$
Tennis ball at 100 km/h	22
Mosquito ( $10^{-2}$ g at 0.5 m/s)	$1.3 \times 10^{-6}$
Single electron in a TV tube beam	$4.0 \times 10^{-15}$
Energy to break one DNA strand	$10^{-19}$

Energy of Various Objects and Phenomena

## Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The **efficiency**  $\text{Eff}$  of an energy conversion process is defined as

**Equation:**

$$\text{Efficiency}(\text{Eff}) = \frac{\text{useful energy or work output}}{\text{total energy input}} = \frac{W_{\text{out}}}{E_{\text{in}}}.$$

[\[link\]](#) lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

Activity/device	Efficiency (%) <a href="#">[footnote]</a> Representative values
Cycling and climbing	20
Swimming, surface	2
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30

Activity/device	Efficiency (%) <sup>[footnote]</sup> Representative values
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90
Solar cell	10

## Efficiency of the Human Body and Mechanical Devices

### Note:

#### PhET Explorations: Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energies for each spring.

[https://phet.colorado.edu/sims/mass-spring-lab/mass-spring-lab\\_en.html](https://phet.colorado.edu/sims/mass-spring-lab/mass-spring-lab_en.html)

## Section Summary

- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as

$KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f$ , where  $OE$  is all **other forms of energy** besides mechanical energy.

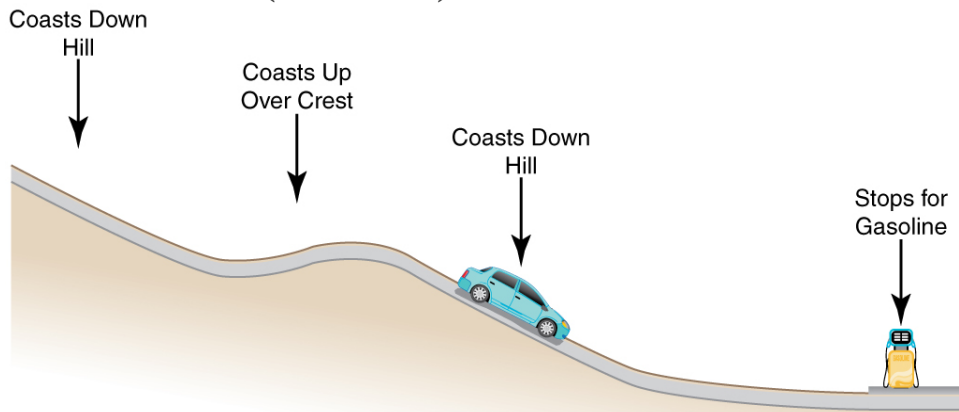
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.
- The efficiency  $Eff$  of a machine or human is defined to be  $Eff = \frac{W_{out}}{E_{in}}$ , where  $W_{out}$  is useful work output and  $E_{in}$  is the energy consumed.

## Conceptual Questions

### Exercise:

#### Problem:

Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See [\[link\]](#).)



A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.

**Exercise:****Problem:**

Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.

**Exercise:****Problem:**

Do devices with efficiencies of less than one violate the law of conservation of energy? Explain.

**Exercise:****Problem:**

List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.

**Exercise:**

**Problem:** List the energy conversions that occur when riding a bicycle.

**Problems & Exercises****Exercise:****Problem:**

Using values from [\[link\]](#), how many DNA molecules could be broken by the energy carried by a single electron in the beam of an old-fashioned TV tube? (These electrons were not dangerous in themselves, but they did create dangerous x rays. Later model tube TVs had shielding that absorbed x rays before they escaped and exposed viewers.)

---

**Solution:**

$4 \times 10^4$  molecules

**Exercise:****Problem:**

Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.

---

**Solution:**

Equating  $\Delta PE_g$  and  $\Delta KE$ , we obtain

$$v = \sqrt{2gh + v_0^2} = \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m}) + (15.0 \text{ m/s})^2} = 24.8 \text{ m/s}$$

**Exercise:****Problem:**

If the energy in fusion bombs were used to supply the energy needs of the world, how many of the 9-megaton variety would be needed for a year's supply of energy (using data from [\[link\]](#))? This is not as far-fetched as it may sound—there are thousands of nuclear bombs, and their energy can be trapped in underground explosions and converted to electricity, as natural geothermal energy is.

**Exercise:****Problem:**

(a) Use of hydrogen fusion to supply energy is a dream that may be realized in the next century. Fusion would be a relatively clean and almost limitless supply of energy, as can be seen from [\[link\]](#). To illustrate this, calculate how many years the present energy needs of the world could be supplied by one millionth of the oceans' hydrogen fusion energy. (b) How does this time compare with historically significant events, such as the duration of stable economic systems?

---

**Solution:**

(a)  $25 \times 10^6$  years

(b) This is much, much longer than human time scales.

## Glossary

law of conservation of energy

the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

electrical energy

the energy carried by a flow of charge

chemical energy

the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

radiant energy

the energy carried by electromagnetic waves

nuclear energy

energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

thermal energy

the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

efficiency

a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

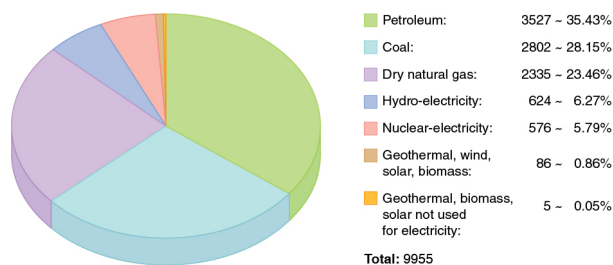
## World Energy Use

- Describe the distinction between renewable and nonrenewable energy sources.
- Explain why the inevitable conversion of energy to less useful forms makes it necessary to conserve energy resources.

Energy is an important ingredient in all phases of society. We live in a very interdependent world, and access to adequate and reliable energy resources is crucial for economic growth and for maintaining the quality of our lives. But current levels of energy consumption and production are not sustainable. About 40% of the world's energy comes from oil, and much of that goes to transportation uses. Oil prices are dependent as much upon new (or foreseen) discoveries as they are upon political events and situations around the world. The U.S., with 4.5% of the world's population, consumes 24% of the world's oil production per year; 66% of that oil is imported!

## Renewable and Nonrenewable Energy Sources

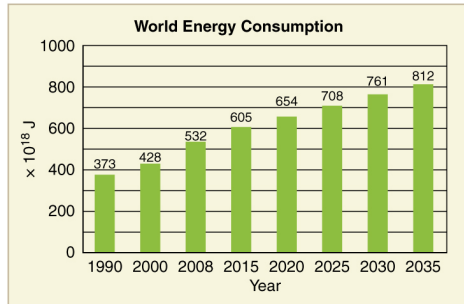
The principal energy resources used in the world are shown in [\[link\]](#). The fuel mix has changed over the years but now is dominated by oil, although natural gas and solar contributions are increasing. **Renewable forms of energy** are those sources that cannot be used up, such as water, wind, solar, and biomass. About 85% of our energy comes from nonrenewable **fossil fuels**—oil, natural gas, coal. The likelihood of a link between global warming and fossil fuel use, with its production of carbon dioxide through combustion, has made, in the eyes of many scientists, a shift to non-fossil fuels of utmost importance—but it will not be easy.



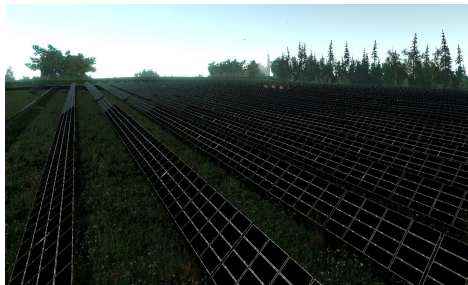
World energy consumption by source, in billions of kilowatt-hours: 2006. (credit: KVDP)

## The World's Growing Energy Needs

World energy consumption continues to rise, especially in the developing countries. (See [\[link\]](#).) Global demand for energy has tripled in the past 50 years and might triple again in the next 30 years. While much of this growth will come from the rapidly booming economies of China and India, many of the developed countries, especially those in Europe, are hoping to meet their energy needs by expanding the use of renewable sources. Although presently only a small percentage, renewable energy is growing very fast, especially wind energy. For example, Germany plans to meet 20% of its electricity and 10% of its overall energy needs with renewable resources by the year 2020. (See [\[link\]](#).) Energy is a key constraint in the rapid economic growth of China and India. In 2003, China surpassed Japan as the world's second largest consumer of oil. However, over 1/3 of this is imported. Unlike most Western countries, coal dominates the commercial energy resources of China, accounting for 2/3 of its energy consumption. In 2009 China surpassed the United States as the largest generator of CO<sub>2</sub>. In India, the main energy resources are biomass (wood and dung) and coal. Half of India's oil is imported. About 70% of India's electricity is generated by highly polluting coal. Yet there are sizeable strides being made in renewable energy. India has a rapidly growing wind energy base, and it has the largest solar cooking program in the world.



Past and projected world energy use  
(source: Based on data from U.S.  
Energy Information Administration,  
2011)



Solar cell arrays at a power plant in  
Steindorf, Germany (credit: Michael  
Betke, Flickr)

[\[link\]](#) displays the 2006 commercial energy mix by country for some of the prime energy users in the world. While non-renewable sources dominate, some countries get a sizeable percentage of their electricity from renewable resources. For example, about 67% of New Zealand's electricity demand is met by hydroelectric. Only 10% of the U.S. electricity is generated by renewable resources, primarily hydroelectric. It is difficult to determine total contributions of renewable energy in some countries with a large rural population, so these percentages in this table are left blank.

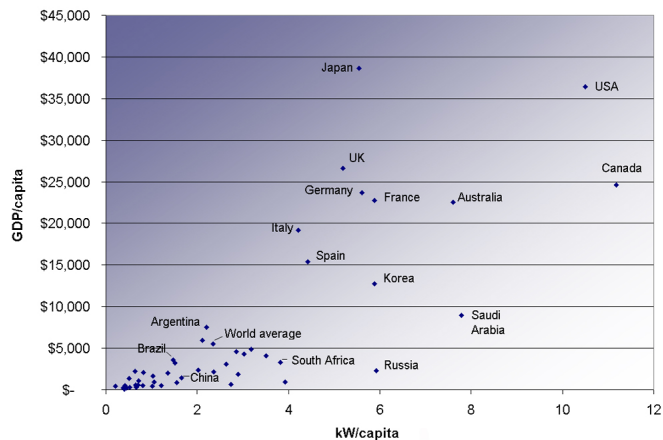
Country	Consumption, in EJ (10 <sup>18</sup> J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables
Australia	5.4	34%	17%	44%	0%	3%	1%

Country	Consumption, in EJ ( $10^{18}$ J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables
Brazil	9.6	48%	7%	5%	1%	35%	2%
China	63	22%	3%	69%	1%	6%	
Egypt	2.4	50%	41%	1%	0%	6%	
Germany	16	37%	24%	24%	11%	1%	3%
India	15	34%	7%	52%	1%	5%	
Indonesia	4.9	51%	26%	16%	0%	2%	3%
Japan	24	48%	14%	21%	12%	4%	1%
New Zealand	0.44	32%	26%	6%	0%	11%	19%
Russia	31	19%	53%	16%	5%	6%	
U.S.	105	40%	23%	22%	8%	3%	1%
<b>World</b>	<b>432</b>	<b>39%</b>	<b>23%</b>	<b>24%</b>	<b>6%</b>	<b>6%</b>	<b>2%</b>

Energy Consumption—Selected Countries (2006)

### Energy and Economic Well-being

The last two columns in this table examine the energy and electricity use per capita. Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (gross domestic product) per capita, are matched by higher levels of energy consumption per capita. This is borne out in [\[link\]](#). Increased efficiency of energy use will change this dependency. A global problem is balancing energy resource development against the harmful effects upon the environment in its extraction and use.



Power consumption per capita versus GDP per capita for various countries. Note the increase in energy usage with increasing GDP. (2007, credit: Frank van Mierlo, Wikimedia Commons)

## Conserving Energy

As we finish this chapter on energy and work, it is relevant to draw some distinctions between two sometimes misunderstood terms in the area of energy use. As has been mentioned elsewhere, the “law of the conservation of energy” is a very useful principle in analyzing physical processes. It is a statement that cannot be proven from basic principles, but is a very good bookkeeping device, and no exceptions have ever been found. It states that the total amount of energy in an isolated system will always remain constant. Related to this principle, but remarkably different from it, is the important philosophy of energy conservation. This concept has to do with seeking to decrease the amount of energy used by an individual or group through (1) reduced activities (e.g., turning down thermostats, driving fewer kilometers) and/or (2) increasing conversion efficiencies in the performance of a particular task—such as developing and using more efficient room heaters, cars that have greater miles-per-gallon ratings, energy-efficient compact fluorescent lights, etc.

Since energy in an isolated system is not destroyed or created or generated, one might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat transfer to the environment and conversion to energy forms no longer useful for doing work. To state it in another way, the potential for energy to produce useful work has been “degraded” in the energy transformation. (This will be discussed in more detail in [Thermodynamics](#).)

## Section Summary

- The relative use of different fuels to provide energy has changed over the years, but fuel use is currently dominated by oil, although natural gas and solar contributions are increasing.
- Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources.
- The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power.
- Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (Gross Domestic Product) per capita, are matched by higher levels of energy consumption per capita.
- Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes.

## Conceptual Questions

### Exercise:

#### Problem:

What is the difference between energy conservation and the law of conservation of energy? Give some examples of each.

### Exercise:

#### Problem:

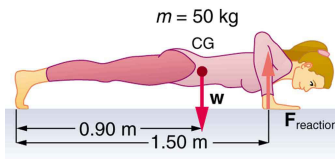
If the efficiency of a coal-fired electrical generating plant is 35%, then what do we mean when we say that energy is a conserved quantity?

## Problems & Exercises

### Exercise:

#### Problem: Integrated Concepts

(a) Calculate the force the woman in [\[link\]](#) exerts to do a push-up at constant speed, taking all data to be known to three digits. (b) How much work does she do if her center of mass rises 0.240 m? (c) What is her useful power output if she does 25 push-ups in 1 min? (Should work done lowering her body be included? See the discussion of useful work in [Work, Energy, and Power in Humans](#).



Forces involved in doing push-ups. The woman's weight acts as a force exerted downward on her center of gravity (CG).

---

### Solution:

(a) 294 N

(b) 118 J

(c) 49.0 W

### Exercise:

#### Problem: Integrated Concepts

A 75.0-kg cross-country skier is climbing a  $3.0^\circ$  slope at a constant speed of 2.00 m/s and encounters air resistance of 25.0 N. Find his power output for work done against the gravitational force and air resistance. (b) What average force does he exert backward on the snow to accomplish this? (c) If he continues to exert this force and to experience the same air resistance when he reaches a level area, how long will it take him to reach a velocity of 10.0 m/s?

**Exercise:****Problem: Integrated Concepts**

The 70.0-kg swimmer in [\[link\]](#) starts a race with an initial velocity of 1.25 m/s and exerts an average force of 80.0 N backward with his arms during each 1.80 m long stroke. (a) What is his initial acceleration if water resistance is 45.0 N? (b) What is the subsequent average resistance force from the water during the 5.00 s it takes him to reach his top velocity of 2.50 m/s? (c) Discuss whether water resistance seems to increase linearly with velocity.

---

**Solution:**

(a)  $0.500 \text{ m/s}^2$

(b) 62.5 N

(c) Assuming the acceleration of the swimmer decreases linearly with time over the 5.00 s interval, the frictional force must therefore be increasing linearly with time, since  $f = F - ma$ . If the acceleration decreases linearly with time, the velocity will contain a term dependent on time squared ( $t^2$ ). Therefore, the water resistance will not depend linearly on the velocity.

**Exercise:****Problem: Integrated Concepts**

A toy gun uses a spring with a force constant of 300 N/m to propel a 10.0-g steel ball. If the spring is compressed 7.00 cm and friction is negligible: (a) How much force is needed to compress the spring? (b) To what maximum height can the ball be shot? (c) At what angles above the horizontal may a child aim to hit a target 3.00 m away at the same height as the gun? (d) What is the gun's maximum range on level ground?

**Exercise:****Problem: Integrated Concepts**

(a) What force must be supplied by an elevator cable to produce an acceleration of  $0.800 \text{ m/s}^2$  against a 200-N frictional force, if the mass of the loaded elevator is 1500 kg? (b) How much work is done by the cable in lifting the elevator 20.0 m? (c) What is the final speed of the elevator if it starts from rest? (d) How much work went into thermal energy?

---

**Solution:**

(a)  $16.1 \times 10^3 \text{ N}$

(b)  $3.22 \times 10^5 \text{ J}$

(c) 5.66 m/s

(d) 4.00 kJ

**Exercise:****Problem: Unreasonable Results**

A car advertisement claims that its 900-kg car accelerated from rest to 30.0 m/s and drove 100 km, gaining 3.00 km in altitude, on 1.0 gal of gasoline. The average force of friction including air resistance was 700 N. Assume all values are known to three significant figures. (a) Calculate the car's efficiency. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

**Exercise:****Problem: Unreasonable Results**

Body fat is metabolized, supplying 9.30 kcal/g, when dietary intake is less than needed to fuel metabolism. The manufacturers of an exercise bicycle claim that you can lose 0.500 kg of fat per day by vigorously exercising for 2.00 h per day on their machine. (a) How many kcal are supplied by the metabolization of 0.500 kg of fat? (b) Calculate the kcal/min that you would have to utilize to metabolize fat at the rate of 0.500 kg in 2.00 h. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

---

**Solution:**

(a)  $4.65 \times 10^3$  kcal

(b) 38.8 kcal/min

(c) This power output is higher than the highest value on [\[link\]](#), which is about 35 kcal/min (corresponding to 2415 watts) for sprinting.

(d) It would be impossible to maintain this power output for 2 hours (imagine sprinting for 2 hours!).

**Exercise:****Problem: Construct Your Own Problem**

Consider a person climbing and descending stairs. Construct a problem in which you calculate the long-term rate at which stairs can be climbed considering the mass of the person, his ability to generate power with his legs, and the height of a single stair step. Also consider why the same person can descend stairs at a faster rate for a nearly unlimited time in spite of the fact that very similar forces are exerted going down as going up. (This points to a fundamentally different process for descending versus climbing stairs.)

**Exercise:****Problem: Construct Your Own Problem**

Consider humans generating electricity by pedaling a device similar to a stationary bicycle. Construct a problem in which you determine the number of people it would take to replace a large electrical generation facility. Among the things to consider are the power output that is reasonable using the legs, rest time, and the need for electricity 24 hours per day. Discuss the practical implications of your results.

**Exercise:****Problem: Integrated Concepts**

A 105-kg basketball player crouches down 0.400 m while waiting to jump. After exerting a force on the floor through this 0.400 m, his feet leave the floor and his center of gravity rises 0.950 m above its normal standing erect position. (a) Using energy considerations, calculate his velocity when he leaves the floor. (b) What average force did he exert on the floor? (Do not neglect the force to support his weight as well as that to accelerate him.) (c) What was his power output during the acceleration phase?

---

**Solution:**

(a) 4.32 m/s

(b)  $3.47 \times 10^3$  N

(c) 8.93 kW

## **Glossary**

renewable forms of energy

those sources that cannot be used up, such as water, wind, solar, and biomass

fossil fuels

oil, natural gas, and coal

## Introduction to Statics and Torque

class="introduction"

On a short time scale, rocks like these in Australia's Kings Canyon are static, or motionless relative to the Earth.

(credit:  
freeaussiestock.com  
)



What might desks, bridges, buildings, trees, and mountains have in common—at least in the eyes of a physicist? The answer is that they are ordinarily motionless relative to the Earth. Furthermore, their acceleration is zero because they remain motionless. That means they also have something in common with a car moving at a constant velocity, because anything with

a constant velocity also has an acceleration of zero. Now, the important part—Newton’s second law states that net  $F$  , and so the net external force is zero for all stationary objects and for all objects moving at constant velocity. There are forces acting, but they are balanced. That is, they are in *equilibrium*.

**Note:**

**Statics**

Statics is the study of forces in equilibrium, a large group of situations that makes up a special case of Newton’s second law. We have already considered a few such situations; in this chapter, we cover the topic more thoroughly, including consideration of such possible effects as the rotation and deformation of an object by the forces acting on it.

How can we guarantee that a body is in equilibrium and what can we learn from systems that are in equilibrium? There are actually two conditions that must be satisfied to achieve equilibrium. These conditions are the topics of the first two sections of this chapter.

## The First Condition for Equilibrium

- State the first condition of equilibrium.
- Explain static equilibrium.
- Explain dynamic equilibrium.

The first condition necessary to achieve equilibrium is the one already mentioned: the net external force on the system must be zero. Expressed as an equation, this is simply

**Equation:**

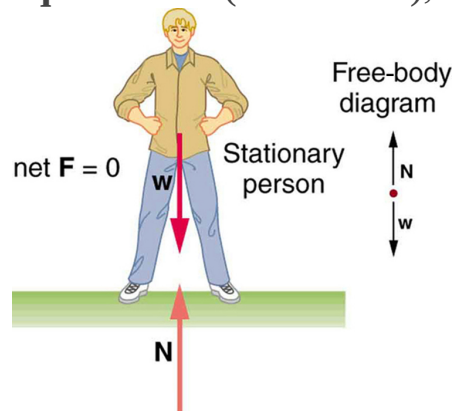
$$\text{net } \mathbf{F} = 0$$

Note that if net  $F$  is zero, then the net external force in *any* direction is zero. For example, the net external forces along the typical x- and y-axes are zero. This is written as

**Equation:**

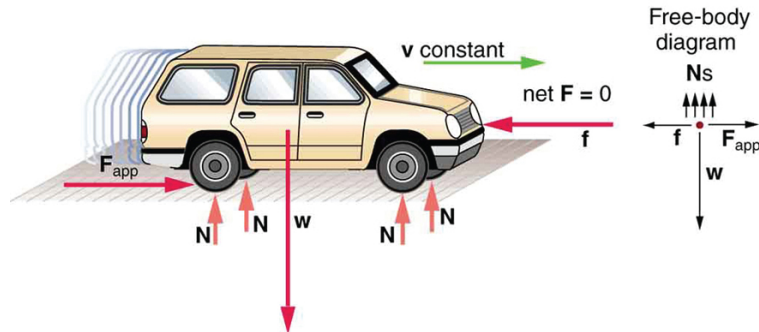
$$\text{net } F_x = 0 \text{ and } F_y = 0$$

[\[link\]](#) and [\[link\]](#) illustrate situations where net  $F = 0$  for both **static equilibrium** (motionless), and **dynamic equilibrium** (constant velocity).



This motionless person is in static equilibrium. The forces acting on him add up to zero. Both

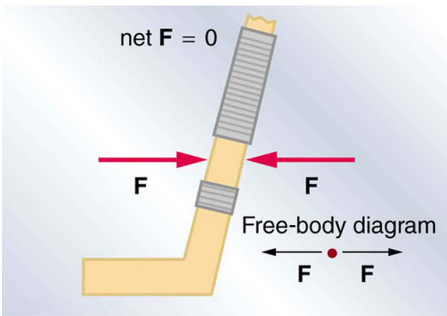
forces are vertical in this case.



This car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force  $F_{\text{app}}$  between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.

However, it is not sufficient for the net external force of a system to be zero for a system to be in equilibrium. Consider the two situations illustrated in [\[link\]](#) and [\[link\]](#) where forces are applied to an ice hockey stick lying flat on ice. The net external force is zero in both situations shown in the figure; but in one case, equilibrium is achieved, whereas in the other, it is not. In [\[link\]](#), the ice hockey stick remains motionless. But in [\[link\]](#), with the same forces applied in different places, the stick experiences accelerated rotation. Therefore, we know that the point at which a force is applied is another factor in determining whether or not equilibrium is achieved. This will be explored further in the next section.

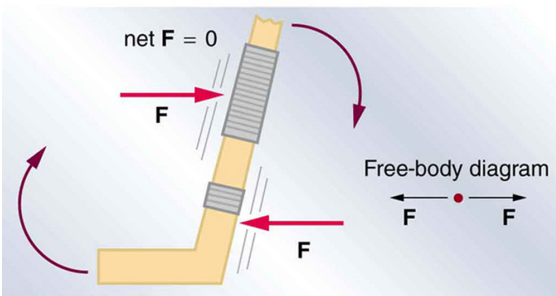
Equilibrium: remains stationary



An ice hockey stick lying flat on ice with two equal and opposite horizontal forces applied to it. Friction is negligible, and the gravitational force is balanced by the support of the ice (a normal force). Thus, net  $F = 0$ .

Equilibrium is achieved, which is static equilibrium in this case.

Nonequilibrium: rotation accelerates



The same forces are applied at other points and the stick

rotates—in fact, it experiences an accelerated rotation. Here net  $F = 0$  but the system is *not* at equilibrium. Hence, the net  $F = 0$  is a necessary—but not sufficient—condition for achieving equilibrium.

**Note:**

PhET Explorations: Torque

Investigate how torque causes an object to rotate. Discover the relationships between angular acceleration, moment of inertia, angular momentum and torque.

[Torqu  
e](#)

## Section Summary

- Statics is the study of forces in equilibrium.
- Two conditions must be met to achieve equilibrium, which is defined to be motion without linear or rotational acceleration.
- The first condition necessary to achieve equilibrium is that the net external force on the system must be zero, so that net  $\mathbf{F} = 0$ .

## Conceptual Questions

**Exercise:****Problem:**

What can you say about the velocity of a moving body that is in dynamic equilibrium? Draw a sketch of such a body using clearly labeled arrows to represent all external forces on the body.

**Exercise:****Problem:**

Under what conditions can a rotating body be in equilibrium? Give an example.

**Glossary****static equilibrium**

a state of equilibrium in which the net external force and torque acting on a system is zero

**dynamic equilibrium**

a state of equilibrium in which the net external force and torque on a system moving with constant velocity are zero

## The Second Condition for Equilibrium

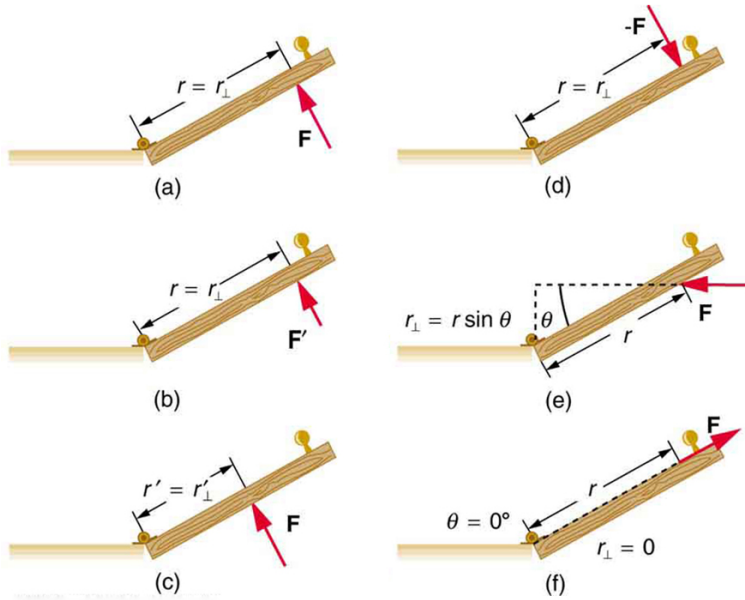
- State the second condition that is necessary to achieve equilibrium.
- Explain torque and the factors on which it depends.
- Describe the role of torque in rotational mechanics.

### **Note:**

#### **Torque**

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a constant angular velocity). A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it. To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.

Several familiar factors determine how effective you are in opening the door. See [\[link\]](#). First of all, the larger the force, the more effective it is in opening the door—obviously, the harder you push, the more rapidly the door opens. Also, the point at which you push is crucial. If you apply your force too close to the hinges, the door will open slowly, if at all. Most people have been embarrassed by making this mistake and bumping up against a door when it did not open as quickly as expected. Finally, the direction in which you push is also important. The most effective direction is perpendicular to the door—we push in this direction almost instinctively.



Torque is the turning or twisting effectiveness of a force, illustrated here for door rotation on its hinges (as viewed from overhead). Torque has both magnitude and direction. (a) Counterclockwise torque is produced by this force, which means that the door will rotate in a counterclockwise due to  $\mathbf{F}$ . Note that  $r_{\perp}$  is the perpendicular distance of the pivot from the line of action of the force. (b) A smaller counterclockwise torque is produced by a smaller force  $\mathbf{F}'$  acting at the same distance from the hinges (the pivot point). (c) The same force as in (a) produces a smaller counterclockwise torque when applied at a smaller distance from the hinges. (d) The same force as in (a), but acting in the opposite direction, produces a clockwise torque. (e) A smaller counterclockwise torque is produced by the same magnitude force acting at the same point

but in a different direction. Here,  $\theta$  is less than  $90^\circ$ . (f) Torque is zero here since the force just pulls on the hinges, producing no rotation. In this case,  $\theta = 0^\circ$ .

The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque.

**Torque** is the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time). In equation form, the magnitude of torque is defined to be

**Equation:**

$$\tau = rF \sin \theta$$

where  $\tau$  (the Greek letter tau) is the symbol for torque,  $r$  is the distance from the pivot point to the point where the force is applied,  $F$  is the magnitude of the force, and  $\theta$  is the angle between the force and the vector directed from the point of application to the pivot point, as seen in [\[link\]](#) and [\[link\]](#). An alternative expression for torque is given in terms of the **perpendicular lever arm**  $r_\perp$  as shown in [\[link\]](#) and [\[link\]](#), which is defined as

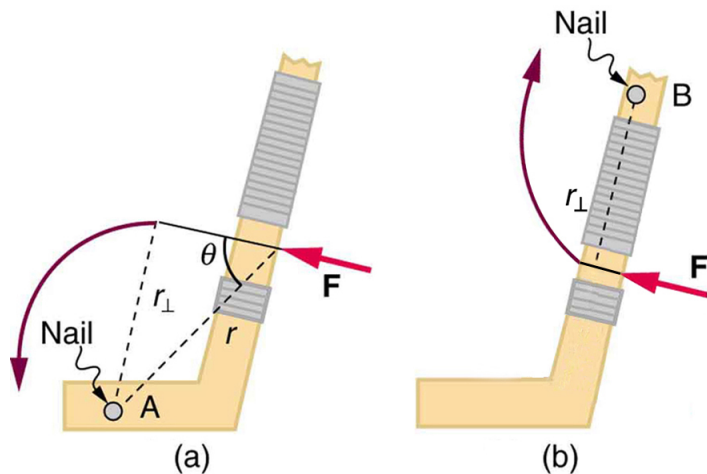
**Equation:**

$$r_\perp = r \sin \theta$$

so that

**Equation:**

$$\tau = r_\perp F.$$



A force applied to an object can produce a torque, which depends on the location of the pivot point. (a) The three factors  $r$ ,  $F$ , and  $\theta$  for pivot point A on a body are shown here— $r$  is the distance from the chosen pivot point to the point where the force  $F$  is applied, and  $\theta$  is the angle between  $\mathbf{F}$  and the vector directed from the point of application to the pivot point. If the object can rotate around point A, it will rotate counterclockwise. This means that torque is counterclockwise relative to pivot A. (b) In this case, point B is the pivot point. The torque from the applied force will cause a clockwise rotation around point B, and so it is a clockwise torque relative to B.

The perpendicular lever arm  $r_{\perp}$  is the shortest distance from the pivot point to the line along which  $\mathbf{F}$  acts; it is shown as a dashed line in [\[link\]](#) and [\[link\]](#). Note that the line segment that defines the distance  $r_{\perp}$  is perpendicular to  $\mathbf{F}$ , as its name implies. It is sometimes easier to find or

visualize  $r_{\perp}$  than to find both  $r$  and  $\theta$ . In such cases, it may be more convenient to use  $\tau = r_{\perp}F$  rather than  $\tau = rF \sin \theta$  for torque, but both are equally valid.

The **SI unit of torque** is newtons times meters, usually written as  $\text{N} \cdot \text{m}$ . For example, if you push perpendicular to the door with a force of 40 N at a distance of 0.800 m from the hinges, you exert a torque of  $32 \text{ N} \cdot \text{m}$  ( $0.800 \text{ m} \times 40 \text{ N} \times \sin 90^\circ$ ) relative to the hinges. If you reduce the force to 20 N, the torque is reduced to  $16 \text{ N} \cdot \text{m}$ , and so on.

The torque is always calculated with reference to some chosen pivot point. For the same applied force, a different choice for the location of the pivot will give you a different value for the torque, since both  $r$  and  $\theta$  depend on the location of the pivot. Any point in any object can be chosen to calculate the torque about that point. The object may not actually pivot about the chosen “pivot point.”

Note that for rotation in a plane, torque has two possible directions. Torque is either clockwise or counterclockwise relative to the chosen pivot point, as illustrated for points B and A, respectively, in [\[link\]](#). If the object can rotate about point A, it will rotate counterclockwise, which means that the torque for the force is shown as counterclockwise relative to A. But if the object can rotate about point B, it will rotate clockwise, which means the torque for the force shown is clockwise relative to B. Also, the magnitude of the torque is greater when the lever arm is longer.

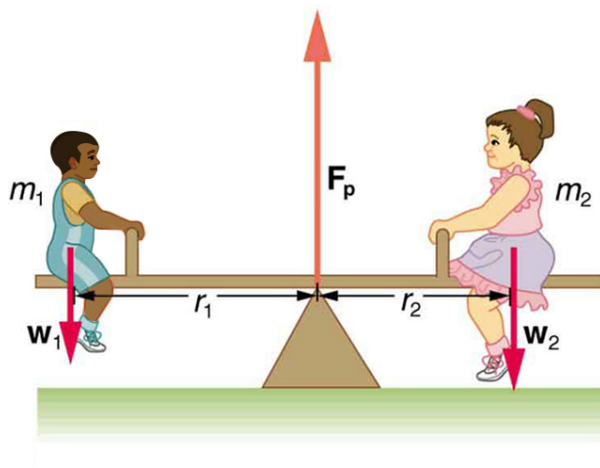
Now, *the second condition necessary to achieve equilibrium is that the net external torque on a system must be zero.* An external torque is one that is created by an external force. You can choose the point around which the torque is calculated. The point can be the physical pivot point of a system or any other point in space—but it must be the same point for all torques. If the second condition (net external torque on a system is zero) is satisfied for one choice of pivot point, it will also hold true for any other choice of pivot point in or out of the system of interest. (This is true only in an inertial frame of reference.) The second condition necessary to achieve equilibrium is stated in equation form as

**Equation:**

$$\text{net } \tau = 0$$

where net means total. Torques, which are in opposite directions are assigned opposite signs. A common convention is to call counterclockwise (ccw) torques positive and clockwise (cw) torques negative.

When two children balance a seesaw as shown in [\[link\]](#), they satisfy the two conditions for equilibrium. Most people have perfect intuition about seesaws, knowing that the lighter child must sit farther from the pivot and that a heavier child can keep a lighter one off the ground indefinitely.



Two children balancing a seesaw satisfy both conditions for equilibrium. The lighter child sits farther from the pivot to create a torque equal in magnitude to that of the heavier child.

**Example:****She Saw Torques On A Seesaw**

The two children shown in [\[link\]](#) are balanced on a seesaw of negligible mass. (This assumption is made to keep the example simple—more

involved examples will follow.) The first child has a mass of 26.0 kg and sits 1.60 m from the pivot.(a) If the second child has a mass of 32.0 kg, how far is she from the pivot? (b) What is  $F_p$ , the supporting force exerted by the pivot?

### Strategy

Both conditions for equilibrium must be satisfied. In part (a), we are asked for a distance; thus, the second condition (regarding torques) must be used, since the first (regarding only forces) has no distances in it. To apply the second condition for equilibrium, we first identify the system of interest to be the seesaw plus the two children. We take the supporting pivot to be the point about which the torques are calculated. We then identify all external forces acting on the system.

### Solution (a)

The three external forces acting on the system are the weights of the two children and the supporting force of the pivot. Let us examine the torque produced by each. Torque is defined to be

### Equation:

$$\tau = rF \sin \theta.$$

Here  $\theta = 90^\circ$ , so that  $\sin \theta = 1$  for all three forces. That means  $r_\perp = r$  for all three. The torques exerted by the three forces are first,

### Equation:

$$\tau_1 = r_1 w_1$$

second,

### Equation:

$$\tau_2 = -r_2 w_2$$

and third,

### Equation:

$$\begin{aligned}\tau_p &= r_p F_p \\ &= 0 \cdot F_p \\ &= 0.\end{aligned}$$

Note that a minus sign has been inserted into the second equation because this torque is clockwise and is therefore negative by convention. Since  $F_p$  acts directly on the pivot point, the distance  $r_p$  is zero. A force acting on the pivot cannot cause a rotation, just as pushing directly on the hinges of a door will not cause it to rotate. Now, the second condition for equilibrium is that the sum of the torques on both children is zero. Therefore

**Equation:**

$$\tau_2 = -\tau_1,$$

or

**Equation:**

$$r_2 w_2 = r_1 w_1.$$

Weight is mass times the acceleration due to gravity. Entering  $mg$  for  $w$ , we get

**Equation:**

$$r_2 m_2 g = r_1 m_1 g.$$

Solve this for the unknown  $r_2$ :

**Equation:**

$$r_2 = r_1 \frac{m_1}{m_2}.$$

The quantities on the right side of the equation are known; thus,  $r_2$  is

**Equation:**

$$r_2 = (1.60 \text{ m}) \frac{26.0 \text{ kg}}{32.0 \text{ kg}} = 1.30 \text{ m}.$$

As expected, the heavier child must sit closer to the pivot (1.30 m versus 1.60 m) to balance the seesaw.

**Solution (b)**

This part asks for a force  $F_p$ . The easiest way to find it is to use the first condition for equilibrium, which is

**Equation:**

$$\text{net } \mathbf{F} = 0.$$

The forces are all vertical, so that we are dealing with a one-dimensional problem along the vertical axis; hence, the condition can be written as

**Equation:**

$$\text{net } F_y = 0$$

where we again call the vertical axis the  $y$ -axis. Choosing upward to be the positive direction, and using plus and minus signs to indicate the directions of the forces, we see that

**Equation:**

$$F_p - w_1 - w_2 = 0.$$

This equation yields what might have been guessed at the beginning:

**Equation:**

$$F_p = w_1 + w_2.$$

So, the pivot supplies a supporting force equal to the total weight of the system:

**Equation:**

$$F_p = m_1 g + m_2 g.$$

Entering known values gives

**Equation:**

$$\begin{aligned} F_p &= (26.0 \text{ kg})(9.80 \text{ m/s}^2) + (32.0 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 568 \text{ N.} \end{aligned}$$

### Discussion

The two results make intuitive sense. The heavier child sits closer to the pivot. The pivot supports the weight of the two children. Part (b) can also be solved using the second condition for equilibrium, since both distances are known, but only if the pivot point is chosen to be somewhere other than the location of the seesaw's actual pivot!

Several aspects of the preceding example have broad implications. First, the choice of the pivot as the point around which torques are calculated simplified the problem. Since  $F_p$  is exerted on the pivot point, its lever arm is zero. Hence, the torque exerted by the supporting force  $F_p$  is zero relative to that pivot point. The second condition for equilibrium holds for any choice of pivot point, and so we choose the pivot point to simplify the solution of the problem.

Second, the acceleration due to gravity canceled in this problem, and we were left with a ratio of masses. *This will not always be the case.* Always enter the correct forces—do not jump ahead to enter some ratio of masses.

Third, the weight of each child is distributed over an area of the seesaw, yet we treated the weights as if each force were exerted at a single point. This is not an approximation—the distances  $r_1$  and  $r_2$  are the distances to points directly below the **center of gravity** of each child. As we shall see in the next section, the mass and weight of a system can act as if they are located at a single point.

Finally, note that the concept of torque has an importance beyond static equilibrium. *Torque plays the same role in rotational motion that force plays in linear motion.* We will examine this in the next chapter.

**Note:**

**Take-Home Experiment**

Take a piece of modeling clay and put it on a table, then mash a cylinder down into it so that a ruler can balance on the round side of the cylinder while everything remains still. Put a penny 8 cm away from the pivot. Where would you need to put two pennies to balance? Three pennies?

## Section Summary

- The second condition assures those torques are also balanced. Torque is the rotational equivalent of a force in producing a rotation and is

defined to be

**Equation:**

$$\tau = rF \sin \theta$$

where  $\tau$  is torque,  $r$  is the distance from the pivot point to the point where the force is applied,  $F$  is the magnitude of the force, and  $\theta$  is the angle between  $\mathbf{F}$  and the vector directed from the point where the force acts to the pivot point. The perpendicular lever arm  $r_{\perp}$  is defined to be

**Equation:**

$$r_{\perp} = r \sin \theta$$

so that

**Equation:**

$$\tau = r_{\perp} F.$$

- The perpendicular lever arm  $r_{\perp}$  is the shortest distance from the pivot point to the line along which  $F$  acts. The SI unit for torque is newton-meter (N·m). The second condition necessary to achieve equilibrium is that the net external torque on a system must be zero:

**Equation:**

$$\text{net } \tau = 0$$

By convention, counterclockwise torques are positive, and clockwise torques are negative.

## Conceptual Questions

**Exercise:**

**Problem:**

What three factors affect the torque created by a force relative to a specific pivot point?

**Exercise:****Problem:**

A wrecking ball is being used to knock down a building. One tall unsupported concrete wall remains standing. If the wrecking ball hits the wall near the top, is the wall more likely to fall over by rotating at its base or by falling straight down? Explain your answer. How is it most likely to fall if it is struck with the same force at its base? Note that this depends on how firmly the wall is attached at its base.

**Exercise:****Problem:**

Mechanics sometimes put a length of pipe over the handle of a wrench when trying to remove a very tight bolt. How does this help? (It is also hazardous since it can break the bolt.)

**Problems & Exercises****Exercise:****Problem:**

(a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?

---

**Solution:**

a) 46.8 N·m

b) It does not matter at what height you push. The torque depends on only the magnitude of the force applied and the perpendicular distance of the force's application from the hinges. (Children don't have a tougher time opening a door because they push lower than adults, they have a tougher time because they don't push far enough from the hinges.)

**Exercise:**

**Problem:**

When tightening a bolt, you push perpendicularly on a wrench with a force of 165 N at a distance of 0.140 m from the center of the bolt. (a) How much torque are you exerting in newton  $\times$  meters (relative to the center of the bolt)? (b) Convert this torque to footpounds.

**Exercise:**

**Problem:**

Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 17.5 N at a distance of 0.600 m from the hinges, and the second child pushes at a distance of 0.450 m. What force must the second child exert to keep the door from moving? Assume friction is negligible.

---

**Solution:**

23.3 N

**Exercise:**

**Problem:**

Use the second condition for equilibrium (net  $\tau = 0$ ) to calculate  $F_p$  in [\[link\]](#), employing any data given or solved for in part (a) of the example.

**Exercise:**

**Problem:**

Repeat the seesaw problem in [\[link\]](#) with the center of mass of the seesaw 0.160 m to the left of the pivot (on the side of the lighter child) and assuming a mass of 12.0 kg for the seesaw. The other data given in the example remain unchanged. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium.

---

**Solution:**

Given:

**Equation:**

$$\begin{aligned}m_1 &= 26.0 \text{ kg}, m_2 = 32.0 \text{ kg}, m_s = 12.0 \text{ kg}, \\r_1 &= 1.60 \text{ m}, r_s = 0.160 \text{ m}, \text{ find (a) } r_2, \text{ (b) } F_p\end{aligned}$$

a) Since children are balancing:

**Equation:**

$$\begin{aligned}\text{net } \tau_{\text{cw}} &= -\text{net } \tau_{\text{ccw}} \\ \Rightarrow w_1 r_1 + m_s g r_s &= w_2 r_2\end{aligned}$$

So, solving for  $r_2$  gives:

**Equation:**

$$\begin{aligned}r_2 &= \frac{w_1 r_1 + m_s g r_s}{w_2} = \frac{m_1 g r_1 + m_s g r_s}{m_2 g} = \frac{m_1 r_1 + m_s r_s}{m_2} \\ &= \frac{(26.0 \text{ kg})(1.60 \text{ m}) + (12.0 \text{ kg})(0.160 \text{ m})}{32.0 \text{ kg}} \\ &= 1.36 \text{ m}\end{aligned}$$

b) Since the children are not moving:

**Equation:**

$$\text{net } F = 0 = F_p - w_1 - w_2 - w_s$$

$$\Rightarrow F_p = w_1 + w_2 + w_s$$

So that

**Equation:**

$$F_p = (26.0 \text{ kg} + 32.0 \text{ kg} + 12.0 \text{ kg})(9.80 \text{ m/s}^2)$$

$$= 686 \text{ N}$$

## Glossary

torque

turning or twisting effectiveness of a force

perpendicular lever arm

the shortest distance from the pivot point to the line along which **F** lies

SI units of torque

newton times meters, usually written as N·m

center of gravity

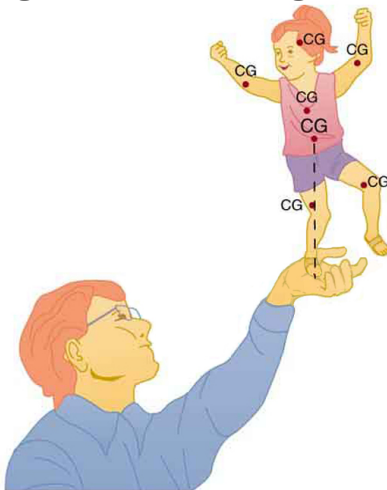
the point where the total weight of the body is assumed to be concentrated

## Stability

- State the types of equilibrium.
- Describe stable and unstable equilibriums.
- Describe neutral equilibrium.

It is one thing to have a system in equilibrium; it is quite another for it to be stable. The toy doll perched on the man's hand in [\[link\]](#), for example, is not in stable equilibrium. There are *three types of equilibrium: stable, unstable, and neutral*. Figures throughout this module illustrate various examples.

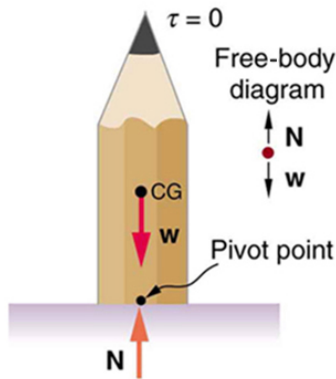
[\[link\]](#) presents a balanced system, such as the toy doll on the man's hand, which has its center of gravity (cg) directly over the pivot, so that the torque of the total weight is zero. This is equivalent to having the torques of the individual parts balanced about the pivot point, in this case the hand. The cgs of the arms, legs, head, and torso are labeled with smaller type.



A man balances a  
toy doll on one  
hand.

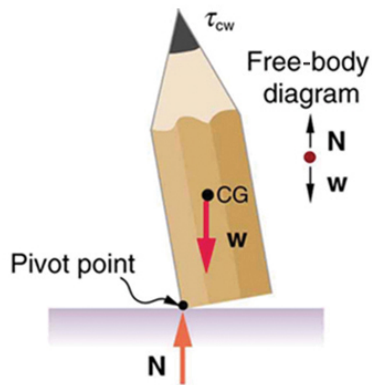
A system is said to be in **stable equilibrium** if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite to the direction of the displacement. For example, a marble at the bottom of a bowl will experience a *restoring* force when displaced from its equilibrium

position. This force moves it back toward the equilibrium position. Most systems are in stable equilibrium, especially for small displacements. For another example of stable equilibrium, see the pencil in [\[link\]](#).

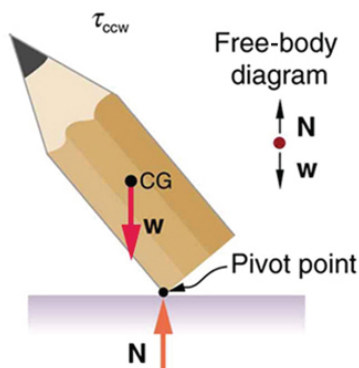


This pencil is in the condition of equilibrium. The net force on the pencil is zero and the total torque about any pivot is zero.

A system is in **unstable equilibrium** if, when displaced, it experiences a net force or torque in the *same* direction as the displacement from equilibrium. A system in unstable equilibrium accelerates away from its equilibrium position if displaced even slightly. An obvious example is a ball resting on top of a hill. Once displaced, it accelerates away from the crest. See the next several figures for examples of unstable equilibrium.

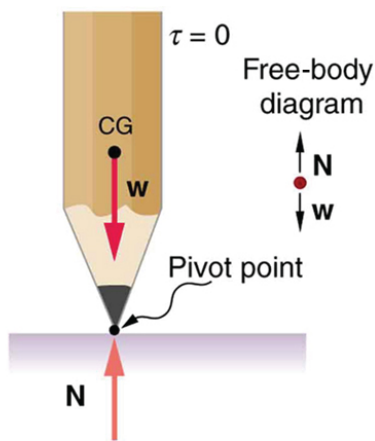


If the pencil is displaced slightly to the side (counterclockwise), it is no longer in equilibrium. Its weight produces a clockwise torque that returns the pencil to its equilibrium position.

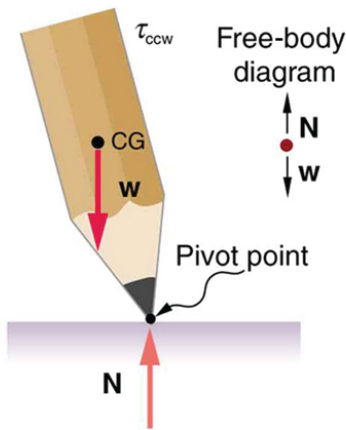


If the pencil is

displaced too far,  
the torque caused  
by its weight  
changes direction  
to  
counterclockwise  
and causes the  
displacement to  
increase.

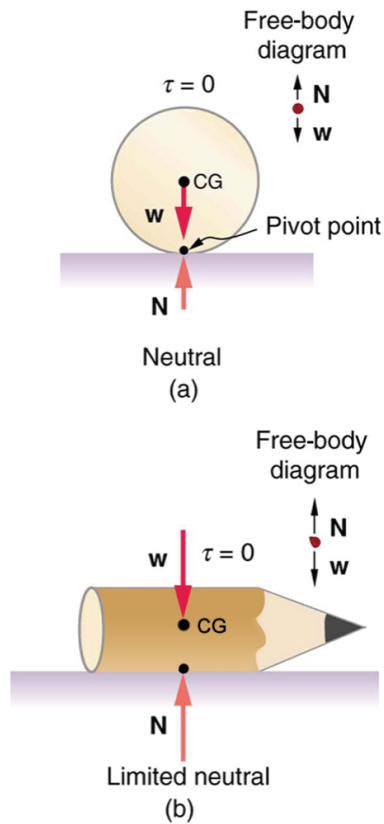


This figure shows  
unstable  
equilibrium,  
although both  
conditions for  
equilibrium are  
satisfied.



If the pencil is displaced even slightly, a torque is created by its weight that is in the same direction as the displacement, causing the displacement to increase.

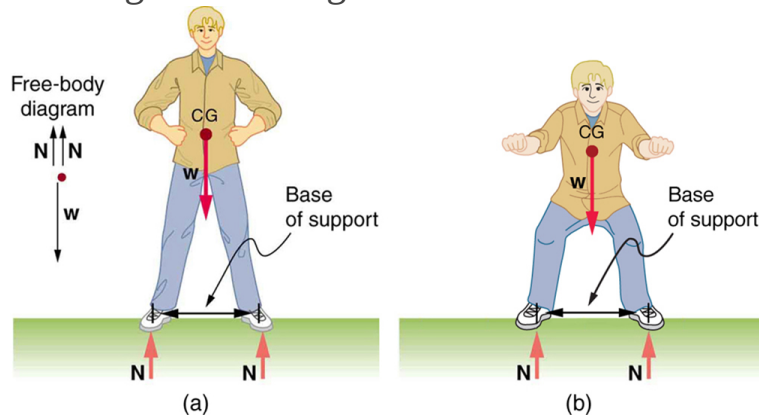
A system is in **neutral equilibrium** if its equilibrium is independent of displacements from its original position. A marble on a flat horizontal surface is an example. Combinations of these situations are possible. For example, a marble on a saddle is stable for displacements toward the front or back of the saddle and unstable for displacements to the side. [\[link\]](#) shows another example of neutral equilibrium.



(a) Here we see neutral equilibrium. The cg of a sphere on a flat surface lies directly above the point of support, independent of the position on the surface. The sphere is therefore in equilibrium in any location, and if displaced, it will remain put. (b) Because it has a circular cross section, the pencil

is in neutral equilibrium for displacements perpendicular to its length.

When we consider how far a system in stable equilibrium can be displaced before it becomes unstable, we find that some systems in stable equilibrium are more stable than others. The pencil in [\[link\]](#) and the person in [\[link\]](#)(a) are in stable equilibrium, but become unstable for relatively small displacements to the side. The critical point is reached when the cg is no longer *above* the base of support. Additionally, since the cg of a person's body is above the pivots in the hips, displacements must be quickly controlled. This control is a central nervous system function that is developed when we learn to hold our bodies erect as infants. For increased stability while standing, the feet should be spread apart, giving a larger base of support. Stability is also increased by lowering one's center of gravity by bending the knees, as when a football player prepares to receive a ball or braces themselves for a tackle. A cane, a crutch, or a walker increases the stability of the user, even more as the base of support widens. Usually, the cg of a female is lower (closer to the ground) than a male. Young children have their center of gravity between their shoulders, which increases the challenge of learning to walk.



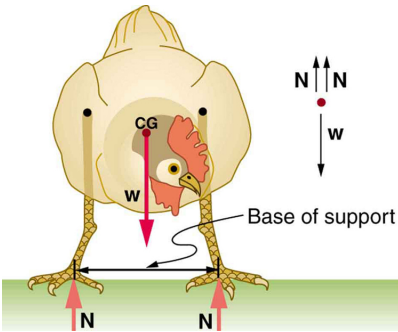
(a) The center of gravity of an adult is above the hip joints (one of the main

pivots in the body) and lies between two narrowly-separated feet. Like a pencil standing on its eraser, this person is in stable equilibrium in relation to sideways displacements, but relatively small displacements take his cg outside the base of support and make him unstable.

Humans are less stable relative to forward and backward displacements because the feet are not very long. Muscles are used extensively to balance the body in the front-to-back direction. (b) While bending in the manner shown, stability is increased by lowering the center of gravity. Stability is also increased if the base is expanded by placing the feet farther apart.

Animals such as chickens have easier systems to control. [\[link\]](#) shows that the cg of a chicken lies below its hip joints and between its widely separated and broad feet. Even relatively large displacements of the chicken's cg are stable and result in restoring forces and torques that return the cg to its equilibrium position with little effort on the chicken's part. Not all birds are like chickens, of course. Some birds, such as the flamingo, have balance systems that are almost as sophisticated as that of humans.

[\[link\]](#) shows that the cg of a chicken is below the hip joints and lies above a broad base of support formed by widely-separated and large feet. Hence, the chicken is in very stable equilibrium, since a relatively large displacement is needed to render it unstable. The body of the chicken is supported from above by the hips and acts as a pendulum between the hips. Therefore, the chicken is stable for front-to-back displacements as well as for side-to-side displacements.



The center of gravity of a chicken is below the hip joints. The chicken is in stable equilibrium. The body of the chicken is supported from above by the hips and acts as a pendulum between them.

Engineers and architects strive to achieve extremely stable equilibriums for buildings and other systems that must withstand wind, earthquakes, and other forces that displace them from equilibrium. Although the examples in this section emphasize gravitational forces, the basic conditions for equilibrium are the same for all types of forces. The net external force must be zero, and the net torque must also be zero.

**Note:**

**Take-Home Experiment**

Stand straight with your heels, back, and head against a wall. Bend forward from your waist, keeping your heels and bottom against the wall, to touch your toes. Can you do this without toppling over? Explain why and what

you need to do to be able to touch your toes without losing your balance. Is it easier for a woman to do this?

## Section Summary

- A system is said to be in stable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite the direction of the displacement.
- A system is in unstable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in the same direction as the displacement from equilibrium.
- A system is in neutral equilibrium if its equilibrium is independent of displacements from its original position.

## Conceptual Questions

### Exercise:

#### Problem:

A round pencil lying on its side as in [\[link\]](#) is in neutral equilibrium relative to displacements perpendicular to its length. What is its stability relative to displacements parallel to its length?

### Exercise:

#### Problem:

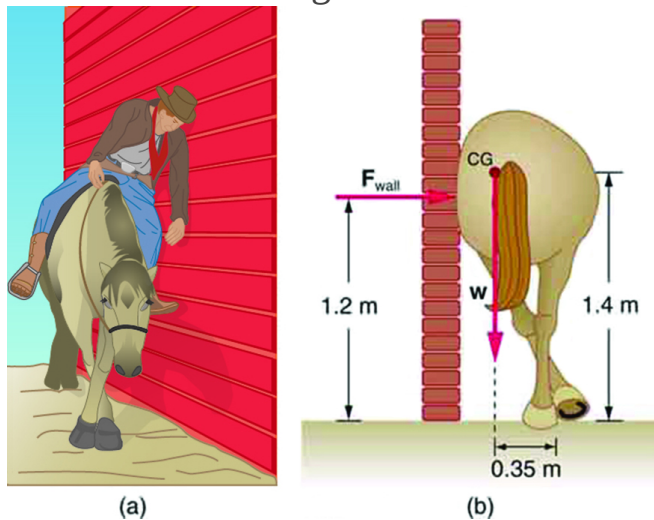
Explain the need for tall towers on a suspension bridge to ensure stable equilibrium.

## Problems & Exercises

### Exercise:

**Problem:**

Suppose a horse leans against a wall as in [\[link\]](#). Calculate the force exerted on the wall assuming that force is horizontal while using the data in the schematic representation of the situation. Note that the force exerted on the wall is equal in magnitude and opposite in direction to the force exerted on the horse, keeping it in equilibrium. The total mass of the horse and rider is 500 kg. Take the data to be accurate to three digits.

**Solution:**

$$F_{\text{wall}} = 1.43 \times 10^3 \text{ N}$$

**Exercise:****Problem:**

Two children of mass 20.0 kg and 30.0 kg sit balanced on a seesaw with the pivot point located at the center of the seesaw. If the children are separated by a distance of 3.00 m, at what distance from the pivot point is the small child sitting in order to maintain the balance?

**Exercise:**

**Problem:**

(a) Calculate the magnitude and direction of the force on each foot of the horse in [\[link\]](#) (two are on the ground), assuming the center of mass of the horse is midway between the feet. The total mass of the horse and rider is 500kg. (b) What is the minimum coefficient of friction between the hooves and ground? Note that the force exerted by the wall is horizontal.

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**Solution:**

a)  $2.55 \times 10^3$  N,  $16.3^\circ$  to the left of vertical (i.e., toward the wall)

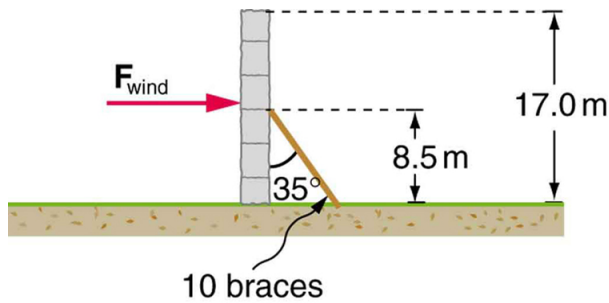
b) 0.292

**Exercise:****Problem:**

A person carries a plank of wood 2.00 m long with one hand pushing down on it at one end with a force  $F_1$  and the other hand holding it up at .500 m from the end of the plank with force  $F_2$ . If the plank has a mass of 20.0 kg and its center of gravity is at the middle of the plank, what are the magnitudes of the forces  $F_1$  and  $F_2$ ?

**Exercise:****Problem:**

A 17.0-m-high and 11.0-m-long wall under construction and its bracing are shown in [\[link\]](#). The wall is in stable equilibrium without the bracing but can pivot at its base. Calculate the force exerted by each of the 10 braces if a strong wind exerts a horizontal force of 650 N on each square meter of the wall. Assume that the net force from the wind acts at a height halfway up the wall and that all braces exert equal forces parallel to their lengths. Neglect the thickness of the wall.



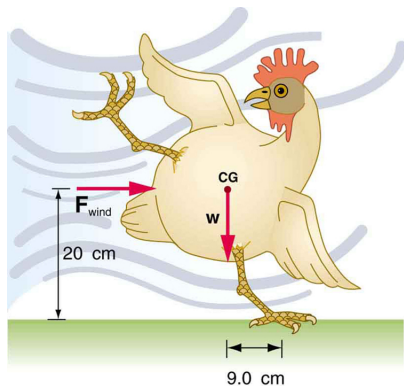
**Solution:**

$$F_B = 2.12 \times 10^4 \text{ N}$$

**Exercise:**

**Problem:**

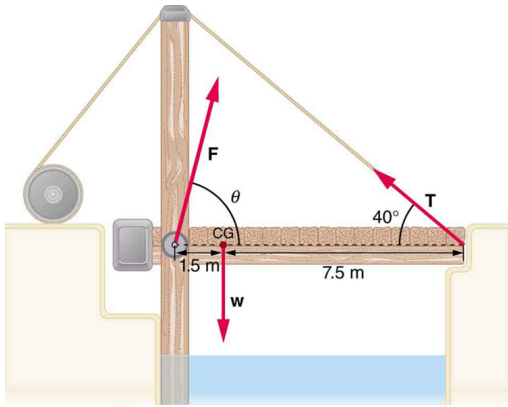
(a) What force must be exerted by the wind to support a 2.50-kg chicken in the position shown in [\[link\]](#)? (b) What is the ratio of this force to the chicken's weight? (c) Does this support the contention that the chicken has a relatively stable construction?



**Exercise:**

**Problem:**

Suppose the weight of the drawbridge in [\[link\]](#) is supported entirely by its hinges and the opposite shore, so that its cables are slack. (a) What fraction of the weight is supported by the opposite shore if the point of support is directly beneath the cable attachments? (b) What is the direction and magnitude of the force the hinges exert on the bridge under these circumstances? The mass of the bridge is 2500 kg.



A small drawbridge,  
showing the forces on the  
hinges ( $F$ ), its weight ( $w$ ),  
and the tension in its  
wires ( $T$ ).

---

**Solution:**

a) 0.167, or about one-sixth of the weight is supported by the opposite shore.

b)  $F = 2.0 \times 10^4 \text{ N}$ , straight up.

**Exercise:**

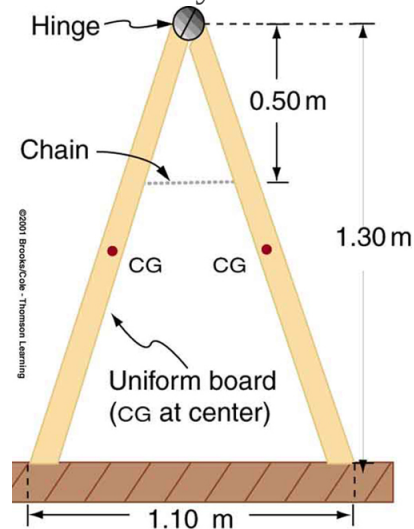
**Problem:**

Suppose a 900-kg car is on the bridge in [\[link\]](#) with its center of mass halfway between the hinges and the cable attachments. (The bridge is supported by the cables and hinges only.) (a) Find the force in the cables. (b) Find the direction and magnitude of the force exerted by the hinges on the bridge.

**Exercise:**

**Problem:**

A sandwich board advertising sign is constructed as shown in [\[link\]](#). The sign's mass is 8.00 kg. (a) Calculate the tension in the chain assuming no friction between the legs and the sidewalk. (b) What force is exerted by each side on the hinge?



A sandwich board  
advertising sign  
demonstrates  
tension.

---

**Solution:**

a) 21.6 N

b) 21.6 N

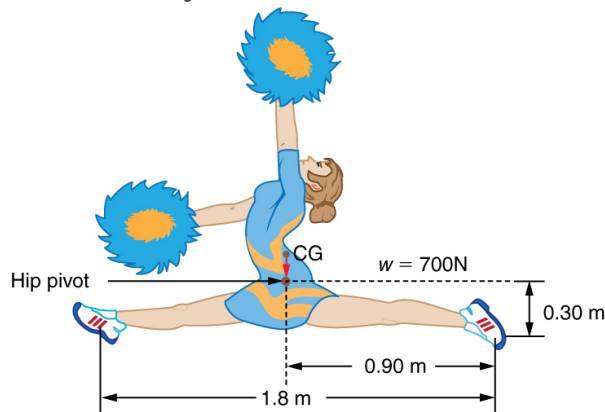
**Exercise:****Problem:**

(a) What minimum coefficient of friction is needed between the legs and the ground to keep the sign in [\[link\]](#) in the position shown if the chain breaks? (b) What force is exerted by each side on the hinge?

## Exercise:

### Problem:

A gymnast is attempting to perform splits. From the information given in [\[link\]](#), calculate the magnitude and direction of the force exerted on each foot by the floor.



A gymnast performs full split.  
The center of gravity and the  
various distances from it are  
shown.

---

### Solution:

350 N directly upwards

## Glossary

neutral equilibrium

a state of equilibrium that is independent of a system's displacements from its original position

stable equilibrium

a system, when displaced, experiences a net force or torque in a direction opposite to the direction of the displacement

unstable equilibrium

a system, when displaced, experiences a net force or torque in the same direction as the displacement from equilibrium

## Applications of Statics, Including Problem-Solving Strategies

- Discuss the applications of Statics in real life.
- State and discuss various problem-solving strategies in Statics.

Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We begin with a discussion of problem-solving strategies specifically used for statics. Since statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in [Problem-Solving Strategies](#), still apply.

### Note:

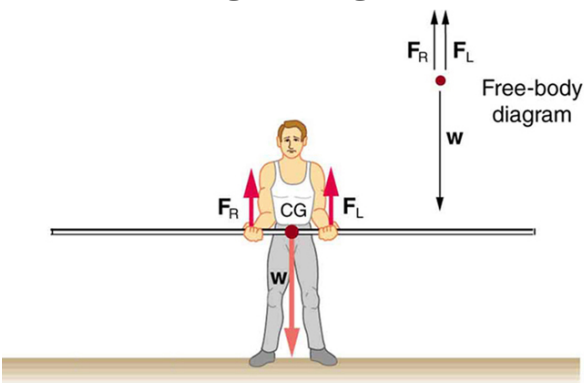
#### Problem-Solving Strategy: Static Equilibrium Situations

1. The first step is to determine whether or not the system is in **static equilibrium**. This condition is always the case when the *acceleration of the system is zero and accelerated rotation does not occur*.
2. It is particularly important to *draw a free body diagram for the system of interest*. Carefully label all forces, and note their relative magnitudes, directions, and points of application whenever these are known.
3. Solve the problem by applying either or both of the conditions for equilibrium (represented by the equations  $\text{net } F = 0$  and  $\text{net } \tau = 0$ , depending on the list of known and unknown factors. If the second condition is involved, *choose the pivot point to simplify the solution*. Any pivot point can be chosen, but the most useful ones cause torques by unknown forces to be zero. (Torque is zero if the force is applied at the pivot (then  $r = 0$ ), or along a line through the pivot point (then  $\theta = 0$ )). Always choose a convenient coordinate system for projecting forces.
4. *Check the solution to see if it is reasonable* by examining the magnitude, direction, and units of the answer. The importance of this last step never diminishes, although in unfamiliar applications, it is usually more difficult to judge reasonableness. These judgments become progressively easier with experience.

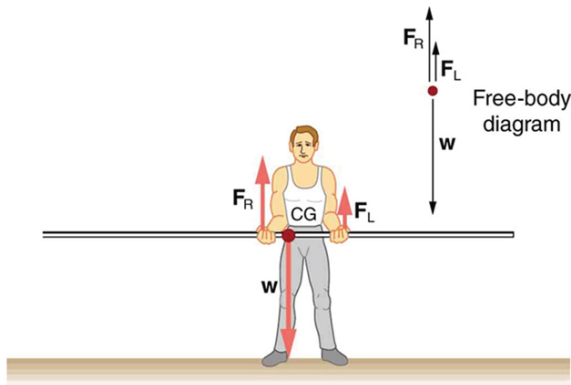
Now let us apply this problem-solving strategy for the pole vaulter shown in the three figures below. The pole is uniform and has a mass of 5.00 kg. In [\[link\]](#), the pole's cg lies halfway between the vaulter's hands. It seems reasonable that the force exerted by each hand is equal to half the weight of the pole, or 24.5 N. This obviously satisfies the first condition for equilibrium (net  $F = 0$ ). The second condition (net  $\tau = 0$ ) is also satisfied, as we can see by choosing the cg to be the pivot point. The weight exerts no torque about a pivot point located at the cg, since it is applied at that point and its lever arm is zero. The equal forces exerted by the hands are equidistant from the chosen pivot, and so they exert equal and opposite torques. Similar arguments hold for other systems where supporting forces are exerted symmetrically about the cg. For example, the four legs of a uniform table each support one-fourth of its weight.

In [\[link\]](#), a pole vaulter holding a pole with its cg halfway between his hands is shown. Each hand exerts a force equal to half the weight of the pole,  $F_R = F_L = w/2$ . (b) The pole vaulter moves the pole to his left, and the forces that the hands exert are no longer equal. See [\[link\]](#). If the pole is held with its cg to the left of the person, then he must push down with his right hand and up with his left. The forces he exerts are larger here because they are in opposite directions and the cg is at a long distance from either hand.

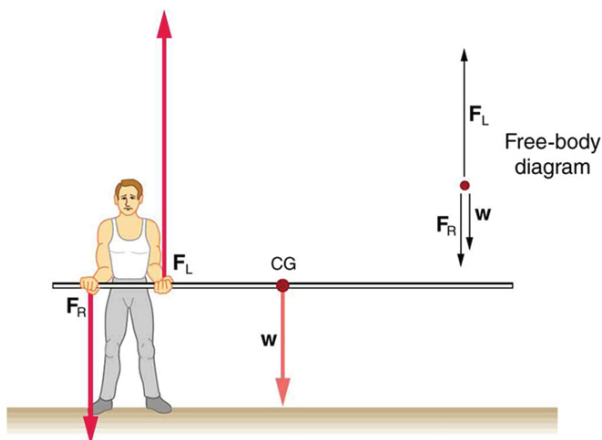
Similar observations can be made using a meter stick held at different locations along its length.



A pole vaulter holds a pole horizontally with both hands.



A pole vaulter is holding a pole horizontally with both hands. The center of gravity is near his right hand.



A pole vaulter is holding a pole horizontally with both hands. The center of gravity is to the left side of the vaulter.

If the pole vaulter holds the pole as shown in [\[link\]](#), the situation is not as simple. The total force he exerts is still equal to the weight of the pole, but it is not evenly divided between his hands. (If  $F_L = F_R$ , then the torques about the cg would not be equal since the lever arms are different.)

Logically, the right hand should support more weight, since it is closer to the cg. In fact, if the right hand is moved directly under the cg, it will support all the weight. This situation is exactly analogous to two people carrying a load; the one closer to the cg carries more of its weight. Finding the forces  $F_L$  and  $F_R$  is straightforward, as the next example shows.

If the pole vaulter holds the pole from near the end of the pole ([\[link\]](#)), the direction of the force applied by the right hand of the vaulter reverses its direction.

### **Example:**

#### **What Force Is Needed to Support a Weight Held Near Its CG?**

For the situation shown in [\[link\]](#), calculate: (a)  $F_R$ , the force exerted by the right hand, and (b)  $F_L$ , the force exerted by the left hand. The hands are 0.900 m apart, and the cg of the pole is 0.600 m from the left hand.

#### **Strategy**

[\[link\]](#) includes a free body diagram for the pole, the system of interest. There is not enough information to use the first condition for equilibrium (net  $F = 0$ ), since two of the three forces are unknown and the hand forces cannot be assumed to be equal in this case. There is enough information to use the second condition for equilibrium (net  $\tau = 0$ ) if the pivot point is chosen to be at either hand, thereby making the torque from that hand zero. We choose to locate the pivot at the left hand in this part of the problem, to eliminate the torque from the left hand.

#### **Solution for (a)**

There are now only two nonzero torques, those from the gravitational force ( $\tau_w$ ) and from the push or pull of the right hand ( $\tau_R$ ). Stating the second condition in terms of clockwise and counterclockwise torques,

#### **Equation:**

$$\text{net } \tau_{\text{cw}} = -\text{net } \tau_{\text{ccw}}.$$

or the algebraic sum of the torques is zero.

Here this is

**Equation:**

$$\tau_R = -\tau_w$$

since the weight of the pole creates a counterclockwise torque and the right hand counters with a clockwise torque. Using the definition of torque,  $\tau = rF \sin \theta$ , noting that  $\theta = 90^\circ$ , and substituting known values, we obtain

**Equation:**

$$(0.900 \text{ m})(F_R) = (0.600 \text{ m})(mg).$$

Thus,

**Equation:**

$$\begin{aligned} F_R &= (0.667)(5.00 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 32.7 \text{ N.} \end{aligned}$$

**Solution for (b)**

The first condition for equilibrium is based on the free body diagram in the figure. This implies that by Newton's second law:

**Equation:**

$$F_L + F_R - mg = 0$$

From this we can conclude:

**Equation:**

$$F_L + F_R = w = mg$$

Solving for  $F_L$ , we obtain

**Equation:**

$$\begin{aligned}
 F_L &= mg - F_R \\
 &= mg - 32.7 \text{ N} \\
 &= (5.00 \text{ kg}) (9.80 \text{ m/s}^2) - 32.7 \text{ N} \\
 &= 16.3 \text{ N}
 \end{aligned}$$

### Discussion

$F_L$  is seen to be exactly half of  $F_R$ , as we might have guessed, since  $F_L$  is applied twice as far from the cg as  $F_R$ .

If the pole vaulter holds the pole as he might at the start of a run, shown in [\[link\]](#), the forces change again. Both are considerably greater, and one force reverses direction.

### Note:

#### Take-Home Experiment

This is an experiment to perform while standing in a bus or a train. Stand facing sideways. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Now stand facing forward. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Why is it easier and safer to stand facing sideways rather than forward? Note: For your safety (and those around you), make sure you are holding onto something while you carry out this activity!

### Note:

#### PhET Explorations: Balancing Act

Play with objects on a teeter totter to learn about balance. Test what you've learned by trying the Balance Challenge game.

[https://phet.colorado.edu/sims/html/balancing-act/latest/balancing-act\\_en.html](https://phet.colorado.edu/sims/html/balancing-act/latest/balancing-act_en.html)

## Summary

- Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We have discussed the problem-solving strategies specifically useful for statics. Statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in [Problem-Solving Strategies](#), still apply.

## Conceptual Questions

### Exercise:

#### Problem:

When visiting some countries, you may see a person balancing a load on the head. Explain why the center of mass of the load needs to be directly above the person's neck vertebrae.

## Problems & Exercises

### Exercise:

#### Problem:

To get up on the roof, a person (mass 70.0 kg) places a 6.00-m aluminum ladder (mass 10.0 kg) against the house on a concrete pad with the base of the ladder 2.00 m from the house. The ladder rests against a plastic rain gutter, which we can assume to be frictionless. The center of mass of the ladder is 2 m from the bottom. The person is standing 3 m from the bottom. What are the magnitudes of the forces on the ladder at the top and bottom?

### Exercise:

**Problem:**

In [\[link\]](#), the cg of the pole held by the pole vaulter is 2.00 m from the left hand, and the hands are 0.700 m apart. Calculate the force exerted by (a) his right hand and (b) his left hand. (c) If each hand supports half the weight of the pole in [\[link\]](#), show that the second condition for equilibrium (net  $\tau = 0$ ) is satisfied for a pivot other than the one located at the center of gravity of the pole. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium described above.

**Glossary**

static equilibrium

equilibrium in which the acceleration of the system is zero and accelerated rotation does not occur

## Introduction to One-Dimensional Kinematics

class="introduction"

The motion  
of an  
American  
kestrel  
through the  
air can be  
described by  
the bird's  
displacement  
, speed,  
velocity, and  
acceleration.  
When it flies  
in a straight  
line without  
any change  
in direction,  
its motion is  
said to be  
one  
dimensional.  
(credit: Vince  
Maidens,  
Wikimedia  
Commons)



Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with **kinematics** which is defined as the *study of motion without considering its causes*. The word “kinematics” comes from a Greek term meaning motion and is related to other English words such as “cinema” (movies) and “kinesiology” (the study of human motion). In one-dimensional kinematics and [Two-Dimensional Kinematics](#) we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In [Two-Dimensional Kinematics](#), we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion); for example, that of a car rounding a curve.

## Displacement

- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.



These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

## Position

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For

example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. (See [\[link\]](#).) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See [\[link\]](#).)

## Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as **displacement**. The word “displacement” implies that an object has moved, or has been displaced.

### Note:

#### Displacement

Displacement is the *change in position* of an object:

#### Equation:

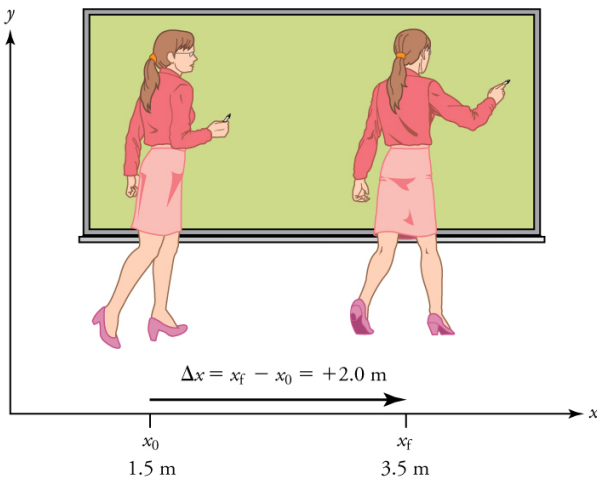
$$\Delta x = x_f - x_0,$$

where  $\Delta x$  is displacement,  $x_f$  is the final position, and  $x_0$  is the initial position.

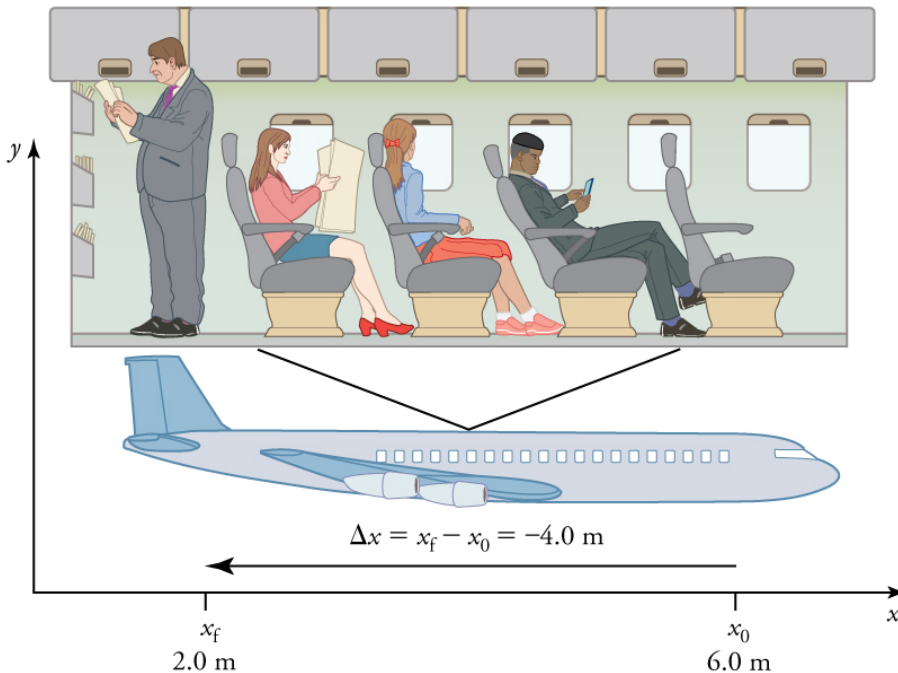
In this text the upper case Greek letter  $\Delta$  (delta) always means “change in” whatever quantity follows it; thus,  $\Delta x$  means *change in position*. Always solve for displacement by subtracting initial position  $x_0$  from final position  $x_f$ .

Note that the SI unit for displacement is the meter (m) (see [Physical Quantities and Units](#)), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are

used in a problem, you may need to convert them into meters to complete the calculation.



A professor paces left and right while lecturing. Her position relative to Earth is given by  $x$ . The  $+2.0 \text{ m}$  displacement of the professor relative to Earth is represented by an arrow pointing to the right.



A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by  $x$ . The  $-4.0\text{-m}$  displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in [\[link\]](#).

Note that displacement has a direction as well as a magnitude. The professor's displacement is  $2.0 \text{ m}$  to the right, and the airline passenger's displacement is  $4.0 \text{ m}$  toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is  $x_0 = 1.5 \text{ m}$  and her final position is  $x_f = 3.5 \text{ m}$ . Thus her displacement is

**Equation:**

$$\Delta x = x_f - x_0 = 3.5 \text{ m} - 1.5 \text{ m} = +2.0 \text{ m}.$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is  $x_0 = 6.0$  m and his final position is  $x_f = 2.0$  m, so his displacement is

**Equation:**

$$\Delta x = x_f - x_0 = 2.0 \text{ m} - 6.0 \text{ m} = -4.0 \text{ m}.$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative  $x$  direction in our coordinate system.

## Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

### Note:

#### Misconception Alert: Distance Traveled vs. Magnitude of Displacement

It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The

displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

### Exercise:

#### Check Your Understanding

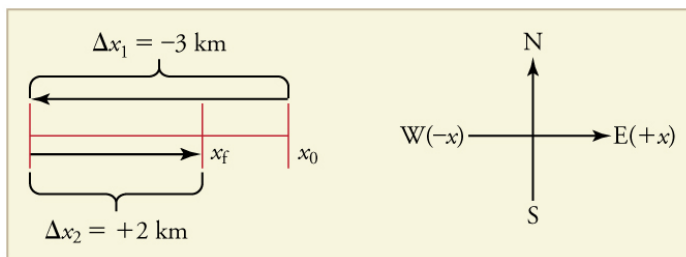
##### Problem:

A cyclist rides 3 km west and then turns around and rides 2 km east.

(a) What is her displacement? (b) What distance does she ride? (c)

What is the magnitude of her displacement?

##### Solution:



(a) The rider's displacement is  $\Delta x = x_f - x_0 = -1 \text{ km}$ . (The displacement is negative because we take east to be positive and west to be negative.)

(b) The distance traveled is  $3 \text{ km} + 2 \text{ km} = 5 \text{ km}$ .

(c) The magnitude of the displacement is  $1 \text{ km}$ .

### Section Summary

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.

- In symbols, displacement  $\Delta x$  is defined to be  
**Equation:**

$$\Delta x = x_f - x_0,$$

where  $x_0$  is the initial position and  $x_f$  is the final position. In this text, the Greek letter  $\Delta$  (delta) always means “change in” whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.

- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

## Conceptual Questions

### Exercise:

#### Problem:

Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

### Exercise:

#### Problem:

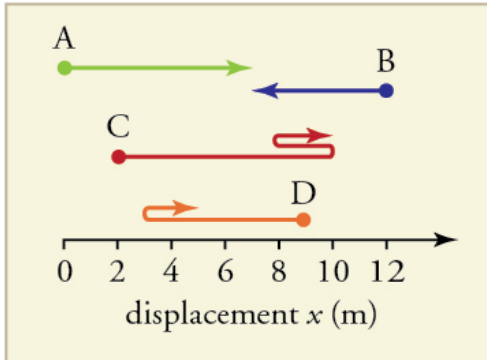
Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

### Exercise:

#### Problem:

Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to  $50 \mu\text{m/s}$  ( $50 \times 10^{-6} \text{ m/s}$ ) have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

## Problems & Exercises



### Exercise:

#### Problem:

Find the following for path A in [\[link\]](#): (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

---

#### Solution:

(a) 7 m

(b) 7 m

(c) +7 m

### Exercise:

#### Problem:

Find the following for path B in [\[link\]](#): (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

### Exercise:

**Problem:**

Find the following for path C in [\[link\]](#): (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

---

**Solution:**

(a) 13 m

(b) 9 m

(c) +9 m

**Exercise:****Problem:**

Find the following for path D in [\[link\]](#): (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

**Glossary**

kinematics

the study of motion without considering its causes

position

the location of an object at a particular time

displacement

the change in position of an object

distance

the magnitude of displacement between two positions

distance traveled

the total length of the path traveled between two positions

## Time, Velocity, and Speed

- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.



The motion of these racing snails can be described by their speeds and their velocities.  
(credit: tobitasflickr, Flickr)

There is more to motion than distance and displacement. Questions such as, “How long does a foot race take?” and “What was the runner’s speed?” cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

## Time

As discussed in [Physical Quantities and Units](#), the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in

some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple—**time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time**  $\Delta t$  is the difference between the ending time and beginning time,

**Equation:**

$$\Delta t = t_f - t_0,$$

where  $\Delta t$  is the change in time or elapsed time,  $t_f$  is the time at the end of the motion, and  $t_0$  is the time at the beginning of the motion. (As usual, the delta symbol,  $\Delta$ , means the change in the quantity that follows it.)

Life is simpler if the beginning time  $t_0$  is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If  $t_0 = 0$ , then  $\Delta t = t_f \equiv t$ .

In this text, for simplicity's sake,

- motion starts at time equal to zero ( $t_0 = 0$ )
- the symbol  $t$  is used for elapsed time unless otherwise specified ( $\Delta t = t_f \equiv t$ )

## Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

### Note:

#### Average Velocity

**Average velocity** is *displacement (change in position) divided by the time of travel*,

#### Equation:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0},$$

where  $\bar{v}$  is the *average* (indicated by the bar over the  $v$ ) velocity,  $\Delta x$  is the change in position (or displacement), and  $x_f$  and  $x_0$  are the final and beginning positions at times  $t_f$  and  $t_0$ , respectively. If the starting time  $t_0$  is taken to be zero, then the average velocity is simply

#### Equation:

$$\bar{v} = \frac{\Delta x}{t}.$$

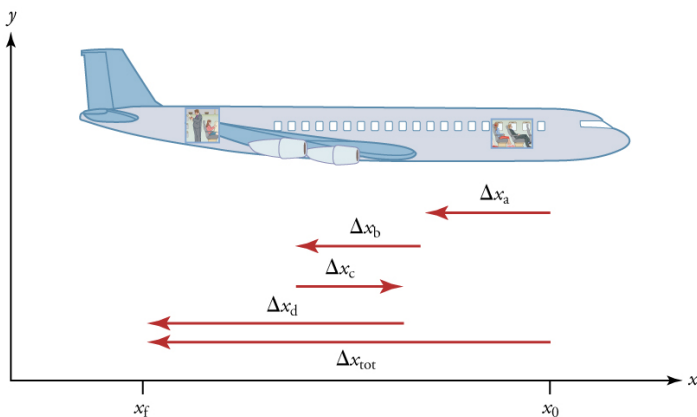
Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move  $-4$  m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

#### Equation:

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4 \text{ m}}{5 \text{ s}} = -0.8 \text{ m/s}.$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.



A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the

direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.)

**Instantaneous velocity**  $v$  is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity,  $v$ , at a precise instant  $t$  can involve taking a limit, a calculus operation beyond the scope of this text.

However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

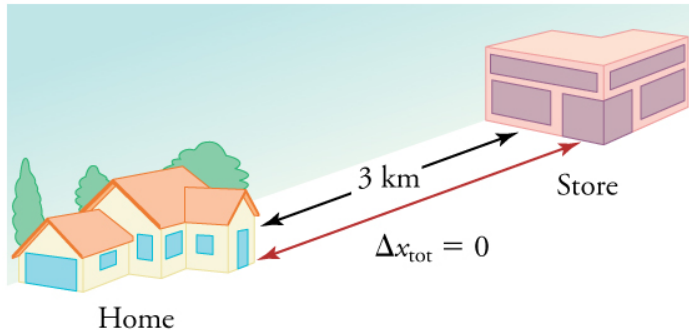
## Speed

In everyday language, most people use the terms “speed” and “velocity” interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

**Instantaneous speed** is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of  $-3.0$  m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was  $3.0$  m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is  $40$  km/h due north. Your instantaneous speed at that instant would be  $40$  km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

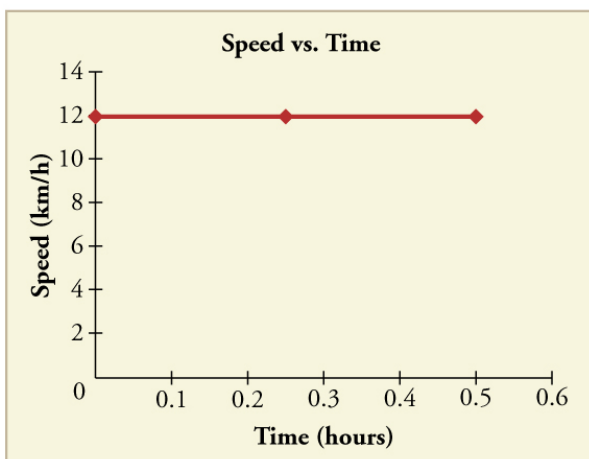
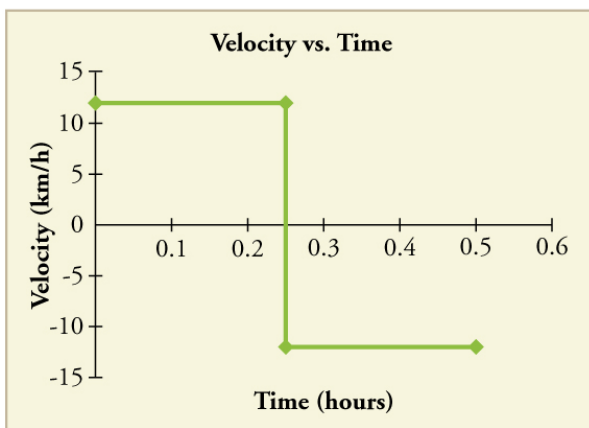
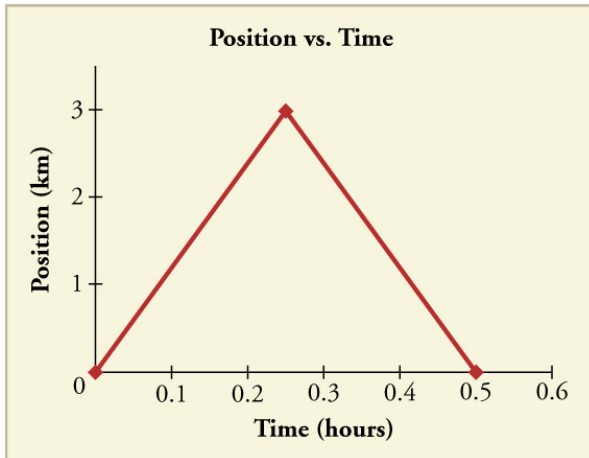
We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car’s odometer shows the total distance traveled was  $6$  km, then your average speed was  $12$  km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero.

(Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.



During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in [\[link\]](#). (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)



Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

**Note:****Making Connections: Take-Home Investigation—Getting a Sense of Speed**

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

**Exercise:****Check Your Understanding****Problem:**

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

**Solution:**

(a) The average velocity of the train is zero because  $x_f = x_0$ ; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

**Equation:**

$$\frac{\text{distance}}{\text{time}} = \frac{80 \text{ miles}}{105 \text{ minutes}}$$

**Equation:**

$$\frac{80 \text{ miles}}{105 \text{ minutes}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ meter}}{3.28 \text{ feet}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 20 \text{ m/s}$$

## Section Summary

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is

**Equation:**

$$\Delta t = t_f - t_0,$$

where  $t_f$  is the final time and  $t_0$  is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just  $t$ .

- Average velocity  $\bar{v}$  is defined as displacement divided by the travel time. In symbols, average velocity is

**Equation:**

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0}.$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity  $v$  is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

## Conceptual Questions

**Exercise:****Problem:**

Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

**Exercise:****Problem:**

There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

**Exercise:****Problem:**

Does a car's odometer measure position or displacement? Does its speedometer measure speed or velocity?

**Exercise:****Problem:**

If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

**Exercise:****Problem:**

How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

**Problems & Exercises****Exercise:**

**Problem:**

(a) Calculate Earth's average speed relative to the Sun. (b) What is its average velocity over a period of one year?

---

**Solution:**

(a)  $3.0 \times 10^4$  m/s

(b) 0 m/s

**Exercise:****Problem:**

A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference. (b) What is its average velocity over one revolution?

**Exercise:****Problem:**

The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

---

**Solution:**

$2 \times 10^7$  years

**Exercise:**

**Problem:**

Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

**Exercise:****Problem:**

On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

---

**Solution:**

$$34.689 \text{ m/s} = 124.88 \text{ km/h}$$

**Exercise:****Problem:**

Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by  $3.84 \times 10^6 \text{ m}$  (1%)?

**Exercise:**

**Problem:**

A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed? (b) If the straight-line distance from her home to the university is 10.3 km in a direction  $25.0^\circ$  south of east, what was her average velocity? (c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

---

**Solution:**

(a) 40.0 km/h

(b) 34.3 km/h,  $25^\circ$  S of E.

(c) average speed = 3.20 km/h,  $\bar{v} = 0$ .

**Exercise:****Problem:**

The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

**Exercise:**

**Problem:**

Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light ( $3.00 \times 10^8$  m/s).

---

**Solution:**

384,000 km

**Exercise:****Problem:**

A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

**Exercise:****Problem:**

The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit  $1.06 \times 10^{-10}$  m in diameter. (a) If the average speed of the electron in this orbit is known to be  $2.20 \times 10^6$  m/s, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity?

---

**Solution:**

(a)  $6.61 \times 10^{15} \text{ rev/s}$

(b) 0 m/s

## Glossary

average speed

distance traveled divided by time during which motion occurs

average velocity

displacement divided by time over which displacement occurs

instantaneous velocity

velocity at a specific instant, or the average velocity over an infinitesimal time interval

instantaneous speed

magnitude of the instantaneous velocity

time

change, or the interval over which change occurs

model

simplified description that contains only those elements necessary to describe the physics of a physical situation

elapsed time

the difference between the ending time and beginning time

## Acceleration

- Define and distinguish between instantaneous acceleration, average acceleration, and deceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.



A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

### **Note:**

#### **Average Acceleration**

**Average Acceleration** is the rate at which velocity changes,

#### **Equation:**

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0},$$

where  $\bar{a}$  is average acceleration,  $v$  is velocity, and  $t$  is time. (The bar over the  $a$  means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are  $\text{m/s}^2$ , meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

**Note:**

**Acceleration as a Vector**

Acceleration is a vector in the same direction as the *change* in velocity,  $\Delta v$ . Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration**.

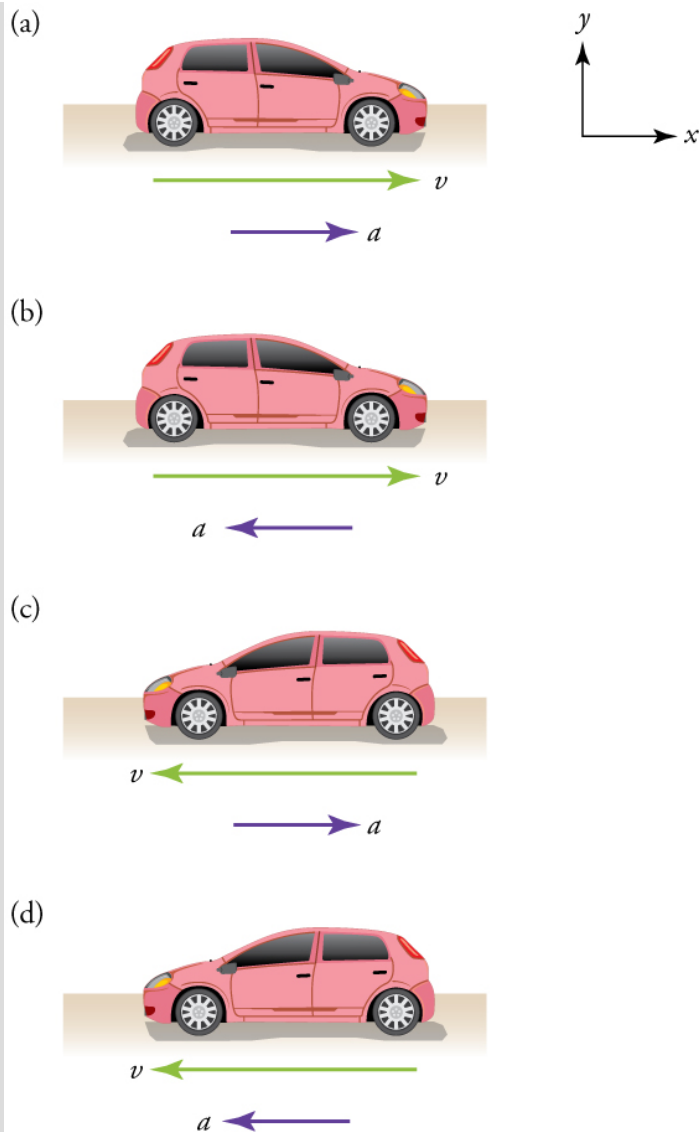


A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

**Note:**

**Misconception Alert: Deceleration vs. Negative Acceleration**

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration *in the negative direction in the chosen coordinate system*. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider [\[link\]](#).



(a) This car is speeding up as it moves toward the right. It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving

toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right.

However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (*not* decelerating).

### Example:

#### Calculating Acceleration: A Racehorse Leaves the Gate

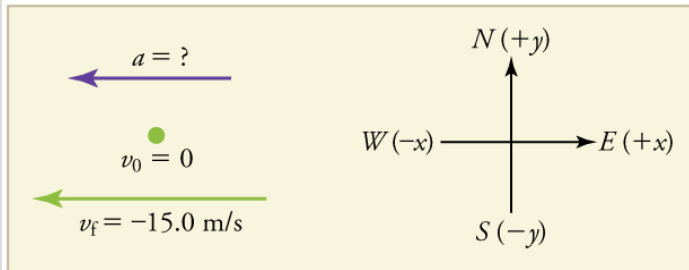
A racehorse coming out of the gate accelerates from rest to a velocity of  $15.0 \text{ m/s}$  due west in  $1.80 \text{ s}$ . What is its average acceleration?



(credit: Jon Sullivan, PD  
Photo.org)

### Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.



We can solve this problem by identifying  $\Delta v$  and  $\Delta t$  from the given information and then calculating the average acceleration directly from the equation  $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$ .

### Solution

1. Identify the knowns.  $v_0 = 0$ ,  $v_f = -15.0 \text{ m/s}$  (the negative sign indicates direction toward the west),  $\Delta t = 1.80 \text{ s}$ .
2. Find the change in velocity. Since the horse is going from zero to  $-15.0 \text{ m/s}$ , its change in velocity equals its final velocity:

$$\Delta v = v_f = -15.0 \text{ m/s}.$$

3. Plug in the known values ( $\Delta v$  and  $\Delta t$ ) and solve for the unknown  $\bar{a}$ .

### Equation:

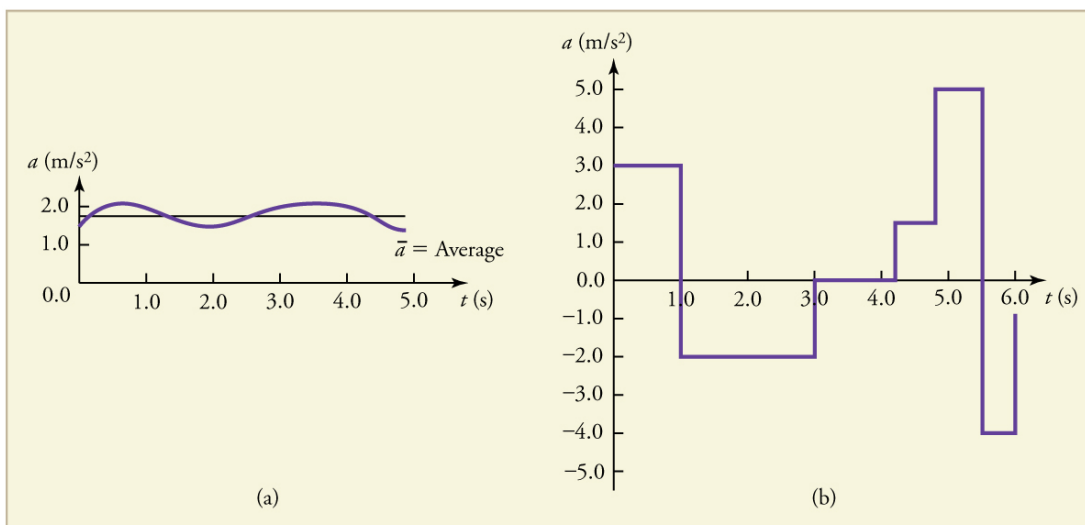
$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0 \text{ m/s}}{1.80 \text{ s}} = -8.33 \text{ m/s}^2.$$

### Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of  $8.33 \text{ m/s}^2$  due west means that the horse increases its velocity by  $8.33 \text{ m/s}$  due west each second, that is,  $8.33$  meters per second per second, which we write as  $8.33 \text{ m/s}^2$ . This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

## Instantaneous Acceleration

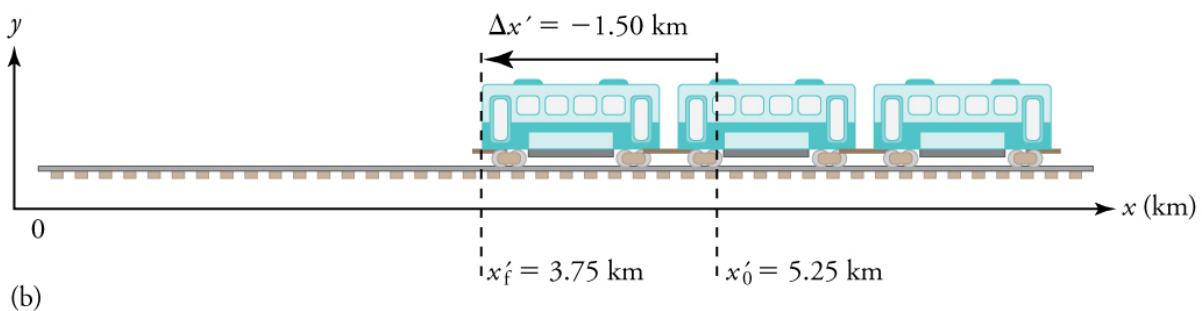
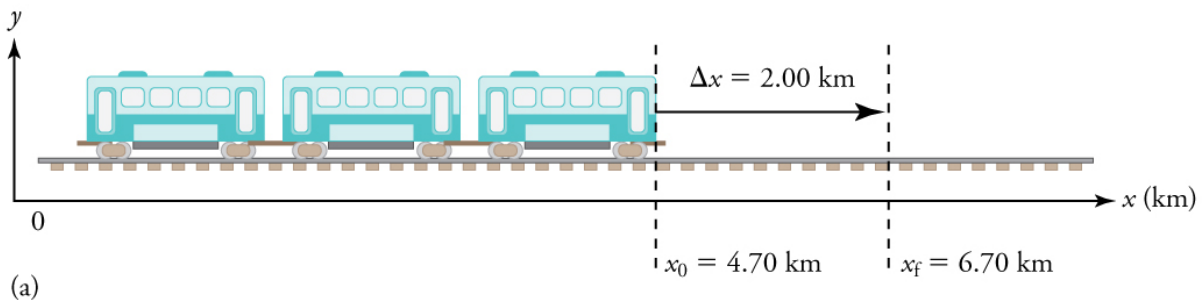
**Instantaneous acceleration**  $a$ , or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity in [Time, Velocity, and Speed](#)—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. [\[link\]](#) shows graphs of instantaneous acceleration versus time for two very different motions. In [\[link\]](#)(a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In [\[link\]](#)(b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of  $+3.0 \text{ m/s}^2$  and  $-2.0 \text{ m/s}^2$ , respectively.



Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the

acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in [\[link\]](#). In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.



One-dimensional motion of a subway train considered in [\[link\]](#), [\[link\]](#), [\[link\]](#), [\[link\]](#), [\[link\]](#), and [\[link\]](#). Here we have chosen the  $x$ -axis so that  $+$  means to the right and  $-$  means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from  $x_0$  to  $x_f$ . Its displacement  $\Delta x$  is  $+2.0$  km. (b) The train moves to the left from  $x'_0$  to  $x'_f$ . Its displacement  $\Delta x'$  is

–1.5 km. (Note that the prime symbol (') is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

**Example:****Calculating Displacement: A Subway Train**

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of [\[link\]](#)?

**Strategy**

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation  $\Delta x = x_f - x_0$ . This is straightforward since the initial and final positions are given.

**Solution**

1. Identify the knowns. In the figure we see that  $x_f = 6.70$  km and  $x_0 = 4.70$  km for part (a), and  $x'_f = 3.75$  km and  $x'_0 = 5.25$  km for part (b).
2. Solve for displacement in part (a).

**Equation:**

$$\Delta x = x_f - x_0 = 6.70 \text{ km} - 4.70 \text{ km} = +2.00 \text{ km}$$

3. Solve for displacement in part (b).

**Equation:**

$$\Delta x' = x'_f - x'_0 = 3.75 \text{ km} - 5.25 \text{ km} = -1.50 \text{ km}$$

**Discussion**

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

**Example:****Comparing Distance Traveled with Displacement: A Subway Train**

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in [\[link\]](#)?

**Strategy**

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in [\[link\]](#). Distance traveled is the total length of the path traveled between the two positions. (See [Displacement](#).) In the case of the subway train shown in [\[link\]](#), the distance traveled is the same as the distance between the initial and final positions of the train.

**Solution**

1. The displacement for part (a) was  $+2.00$  km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.
2. The displacement for part (b) was  $-1.5$  km. Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

**Discussion**

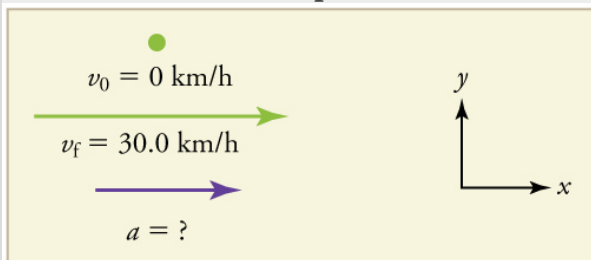
Distance is a scalar. It has magnitude but no sign to indicate direction.

**Example:****Calculating Acceleration: A Subway Train Speeding Up**

Suppose the train in [\[link\]](#)(a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

**Strategy**

It is worth it at this point to make a simple sketch:



This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

**Solution**

1. Identify the knowns.  $v_0 = 0$  (the train starts at rest),  $v_f = 30.0 \text{ km/h}$ , and  $\Delta t = 20.0 \text{ s}$ .
2. Calculate  $\Delta v$ . Since the train starts from rest, its change in velocity is  $\Delta v = +30.0 \text{ km/h}$ , where the plus sign means velocity to the right.
3. Plug in known values and solve for the unknown,  $\bar{a}$ .

**Equation:**

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0 \text{ km/h}}{20.0 \text{ s}}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See [Physical Quantities and Units](#) for more guidance.)

**Equation:**

$$\bar{a} = \left( \frac{+30 \text{ km/h}}{20.0 \text{ s}} \right) \left( \frac{10^3 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = 0.417 \text{ m/s}^2$$

**Discussion**

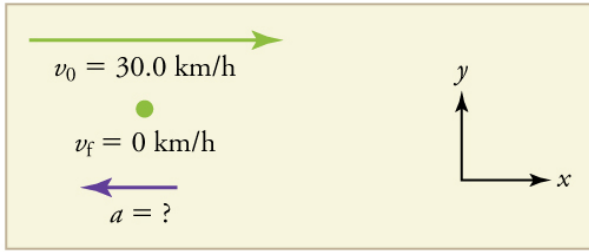
The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

**Example:**

**Calculate Acceleration: A Subway Train Slowing Down**

Now suppose that at the end of its trip, the train in [\[link\]](#)(a) slows to a stop from a speed of  $30.0 \text{ km/h}$  in  $8.00 \text{ s}$ . What is its average acceleration while stopping?

**Strategy**



In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

### Solution

1. Identify the knowns.  $v_0 = 30.0 \text{ km/h}$ ,  $v_f = 0 \text{ km/h}$  (the train is stopped, so its velocity is 0), and  $\Delta t = 8.00 \text{ s}$ .
2. Solve for the change in velocity,  $\Delta v$ .

### Equation:

$$\Delta v = v_f - v_0 = 0 - 30.0 \text{ km/h} = -30.0 \text{ km/h}$$

3. Plug in the knowns,  $\Delta v$  and  $\Delta t$ , and solve for  $\bar{a}$ .

### Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0 \text{ km/h}}{8.00 \text{ s}}$$

4. Convert the units to meters and seconds.

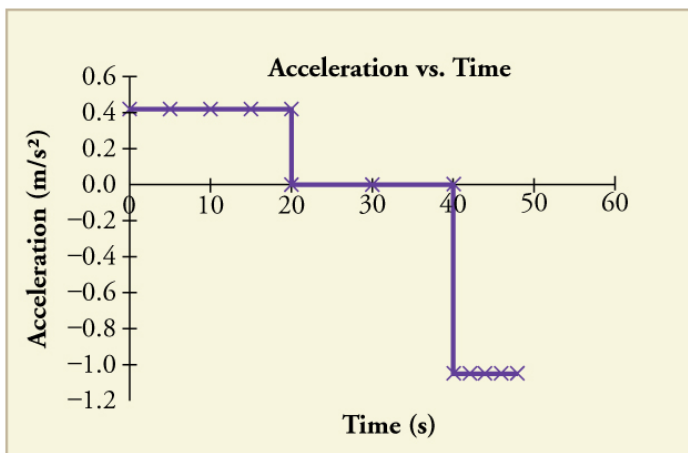
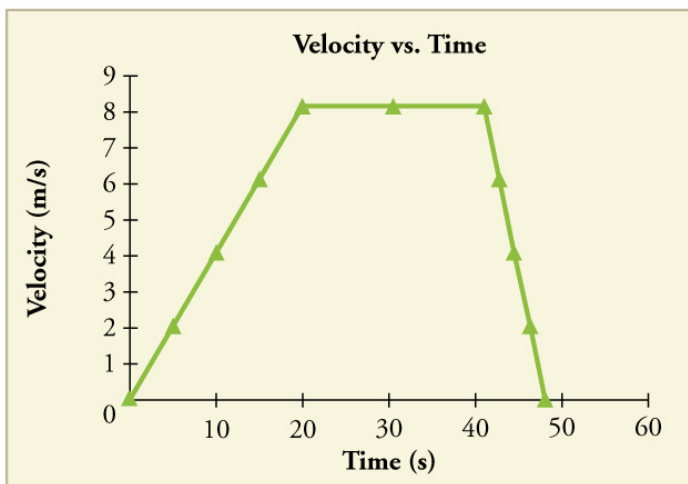
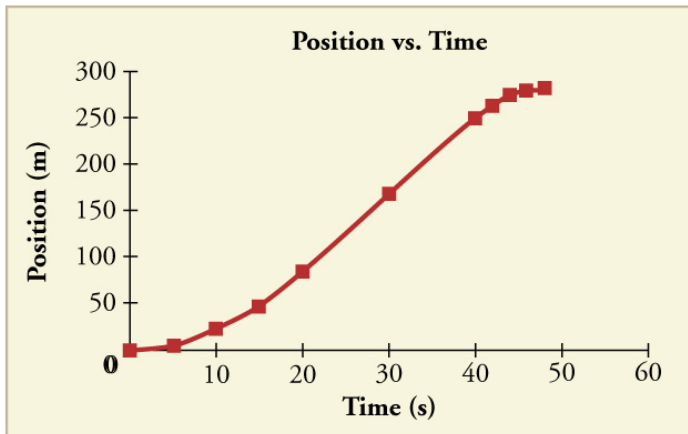
### Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \left( \frac{-30.0 \text{ km/h}}{8.00 \text{ s}} \right) \left( \frac{10^3 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = -1.04 \text{ m/s}^2.$$

### Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the *change* in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains in [\[link\]](#) and [\[link\]](#) are displayed in [\[link\]](#). (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)



(a) Position of the train over time.

Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity

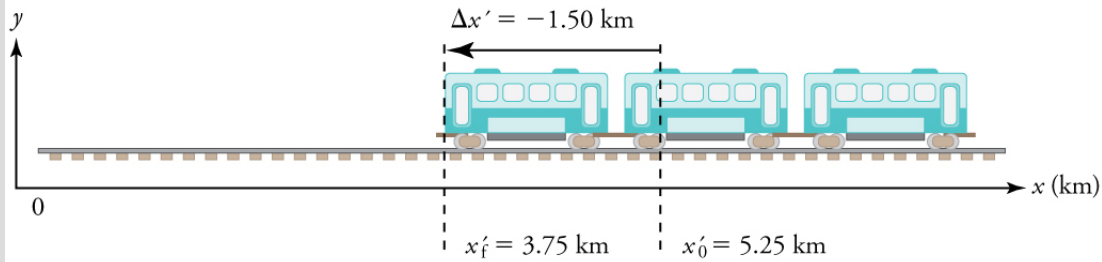
of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey.

(c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

### **Example:**

#### **Calculating Average Velocity: The Subway Train**

What is the average velocity of the train in part b of [\[link\]](#), and shown again below, if it takes 5.00 min to make its trip?



### Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

### Solution

1. Identify the knowns.  $x'_f = 3.75$  km,  $x'_0 = 5.25$  km,  $\Delta t = 5.00$  min.
2. Determine displacement,  $\Delta x'$ . We found  $\Delta x'$  to be  $-1.5$  km in [\[link\]](#).
3. Solve for average velocity.

### Equation:

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \frac{-1.50 \text{ km}}{5.00 \text{ min}}$$

4. Convert units.

### Equation:

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \left( \frac{-1.50 \text{ km}}{5.00 \text{ min}} \right) \left( \frac{60 \text{ min}}{1 \text{ h}} \right) = -18.0 \text{ km/h}$$

### Discussion

The negative velocity indicates motion to the left.

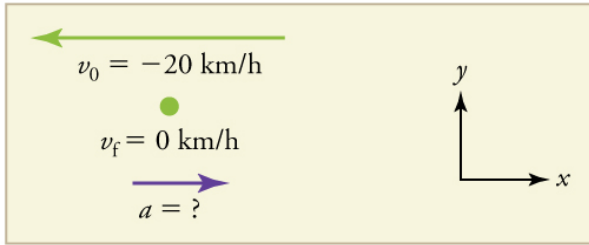
### Example:

#### Calculating Deceleration: The Subway Train

Finally, suppose the train in [\[link\]](#) slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

### Strategy

Once again, let's draw a sketch:



As before, we must find the change in velocity and the change in time to calculate average acceleration.

### Solution

1. Identify the knowns.  $v_0 = -20 \text{ km/h}$ ,  $v_f = 0 \text{ km/h}$ ,  $\Delta t = 10.0 \text{ s}$ .
2. Calculate  $\Delta v$ . The change in velocity here is actually positive, since

### Equation:

$$\Delta v = v_f - v_0 = 0 - (-20 \text{ km/h}) = +20 \text{ km/h}.$$

3. Solve for  $\bar{a}$ .

### Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0 \text{ km/h}}{10.0 \text{ s}}$$

4. Convert units.

### Equation:

$$\bar{a} = \left( \frac{+20.0 \text{ km/h}}{10.0 \text{ s}} \right) \left( \frac{10^3 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = +0.556 \text{ m/s}^2$$

### Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the *change* in velocity, which is positive here. As in [\[link\]](#), this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

## Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in [\[link\]](#), where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in [\[link\]](#) is sped up by an acceleration to the left. In that case, both  $v$  and  $a$  are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

**Exercise:**

**Check Your Understanding**

**Problem:**

An airplane lands on a runway traveling east. Describe its acceleration.

---

**Solution:**

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

**Note:**

**PhET Explorations: Moving Man Simulation**

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

<https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/>

## Section Summary

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration**  $\bar{a}$  is

**Equation:**

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}.$$

- The SI unit for acceleration is  $\text{m/s}^2$ .
- Acceleration is a vector, and thus has both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration  $a$  is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

## Conceptual Questions

**Exercise:**

**Problem:**

Is it possible for speed to be constant while acceleration is not zero?  
Give an example of such a situation.

**Exercise:**

**Problem:**

Is it possible for velocity to be constant while acceleration is not zero?  
Explain.

**Exercise:**

**Problem:**

Give an example in which velocity is zero yet acceleration is not.

**Exercise:****Problem:**

If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

**Exercise:****Problem:**

Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

**Problems & Exercises****Exercise:****Problem:**

A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

---

**Solution:**

$$4.29 \text{ m/s}^2$$

**Exercise:****Problem: Professional Application**

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration.

Express each in multiples of  $g$  ( $9.80 \text{ m/s}^2$ ) by taking its ratio to the acceleration of gravity.

**Exercise:**

**Problem:**

A commuter backs her car out of her garage with an acceleration of  $1.40 \text{ m/s}^2$ . (a) How long does it take her to reach a speed of  $2.00 \text{ m/s}$ ? (b) If she then brakes to a stop in  $0.800 \text{ s}$ , what is her deceleration?

---

**Solution:**

(a)  $1.43 \text{ s}$

(b)  $-2.50 \text{ m/s}^2$

**Exercise:**

**Problem:**

Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of  $6.50 \text{ km/s}$  in  $60.0 \text{ s}$  (the actual speed and time are classified). What is its average acceleration in  $\text{m/s}^2$  and in multiples of  $g$  ( $9.80 \text{ m/s}^2$ )?

## Glossary

acceleration

the rate of change in velocity; the change in velocity over time

average acceleration

the change in velocity divided by the time over which it changes

instantaneous acceleration

acceleration at a specific point in time

deceleration

acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

## Motion Equations for Constant Acceleration in One Dimension

- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.



Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

**Notation:  $t$ ,  $x$ ,  $v$ ,  $a$**

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is  $\Delta t = t_f - t_0$ , taking  $t_0 = 0$  means that  $\Delta t = t_f$ , the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is,  $x_0$  is *the initial position* and  $v_0$  is *the initial velocity*. We put no subscripts on the final values. That is,  $t$  is *the final time*,  $x$  is *the final position*, and  $v$  is *the final velocity*. This gives a simpler expression for elapsed time—now,  $\Delta t = t$ . It also simplifies the expression for displacement, which is now  $\Delta x = x - x_0$ . Also, it simplifies the expression for change in velocity, which is now  $\Delta v = v - v_0$ . To summarize, using the simplified notation, with the initial time taken to be zero,

**Equation:**

$$\begin{aligned}\Delta t &= t \\ \Delta x &= x - x_0 \\ \Delta v &= v - v_0\end{aligned}$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

**Equation:**

$$\bar{a} = a = \text{constant},$$

so we use the symbol  $a$  for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration is constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in

motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

**Note:**

Solving for Displacement ( $\Delta x$ ) and Final Position ( $x$ ) from Average Velocity when Acceleration ( $a$ ) is Constant

To get our first two new equations, we start with the definition of average velocity:

**Equation:**

$$\bar{v} = \frac{\Delta x}{\Delta t}.$$

Substituting the simplified notation for  $\Delta x$  and  $\Delta t$  yields

**Equation:**

$$\bar{v} = \frac{x - x_0}{t}.$$

Solving for  $x$  yields

**Equation:**

$$x = x_0 + \bar{v}t,$$

where the average velocity is

**Equation:**

$$\bar{v} = \frac{v_0 + v}{2} \text{ (constant } a\text{)}.$$

The equation  $\bar{v} = \frac{v_0 + v}{2}$  reflects the fact that, when acceleration is constant,  $\bar{v}$  is just the simple average of the initial and final velocities. For example, if

you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation  $\bar{v} = \frac{v_0 + v}{2}$  to check this, we see that

**Equation:**

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30 \text{ km/h} + 60 \text{ km/h}}{2} = 45 \text{ km/h},$$

which seems logical.

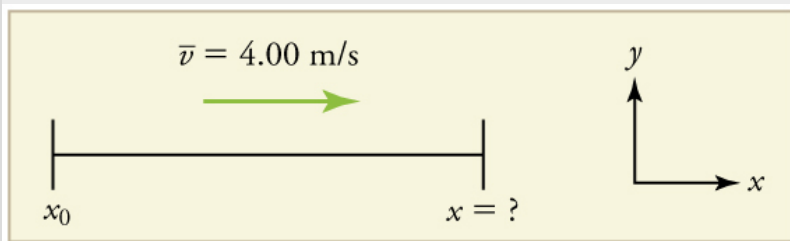
**Example:**

**Calculating Displacement: How Far does the Jogger Run?**

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

**Strategy**

Draw a sketch.



The final position  $x$  is given by the equation

**Equation:**

$$x = x_0 + \bar{v}t.$$

To find  $x$ , we identify the values of  $x_0$ ,  $\bar{v}$ , and  $t$  from the statement of the problem and substitute them into the equation.

**Solution**

1. Identify the knowns.  $\bar{v} = 4.00 \text{ m/s}$ ,  $\Delta t = 2.00 \text{ min}$ , and  $x_0 = 0 \text{ m}$ .
2. Enter the known values into the equation.

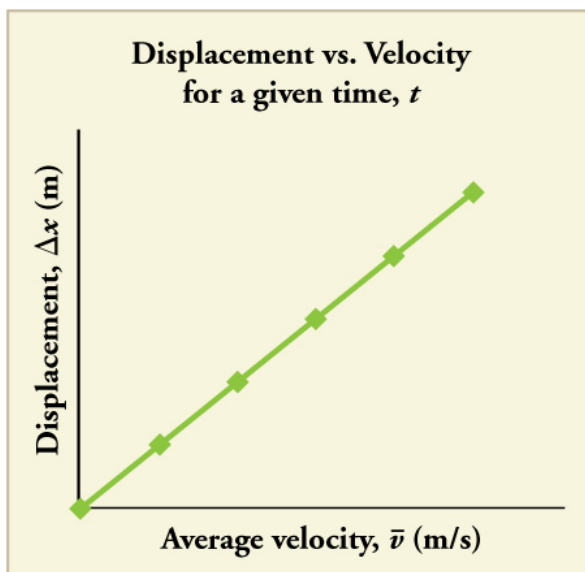
**Equation:**

$$x = x_0 + \bar{v}t = 0 + (4.00 \text{ m/s})(120 \text{ s}) = 480 \text{ m}$$

### Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation  $x = x_0 + \bar{v}t$  gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on  $\bar{v}$  rather than on  $\bar{v}$  raised to some other power, such as  $\bar{v}^2$ . When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.



There is a linear relationship between displacement and average velocity. For a given time  $t$ , an object moving twice as fast as another object will

move twice as far as the other object.

**Note:**

**Solving for Final Velocity**

We can derive another useful equation by manipulating the definition of acceleration.

**Equation:**

$$a = \frac{\Delta v}{\Delta t}$$

Substituting the simplified notation for  $\Delta v$  and  $\Delta t$  gives us

**Equation:**

$$a = \frac{v - v_0}{t} \text{ (constant } a\text{)}.$$

Solving for  $v$  yields

**Equation:**

$$v = v_0 + at \text{ (constant } a\text{)}.$$

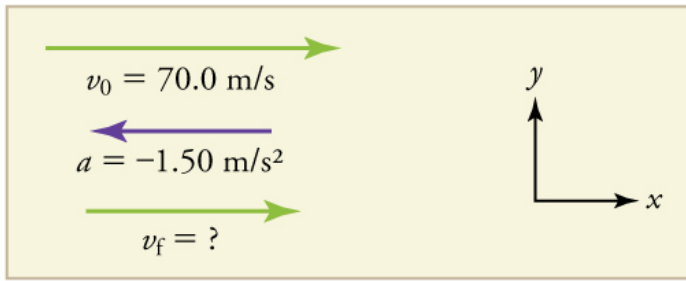
**Example:**

**Calculating Final Velocity: An Airplane Slowing Down after Landing**

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at 1.50 m/s<sup>2</sup> for 40.0 s. What is its final velocity?

**Strategy**

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.



### Solution

1. Identify the knowns.  $v_0 = 70.0 \text{ m/s}$ ,  $a = -1.50 \text{ m/s}^2$ ,  $t = 40.0 \text{ s}$ .
2. Identify the unknown. In this case, it is final velocity,  $v_f$ .
3. Determine which equation to use. We can calculate the final velocity using the equation  $v = v_0 + at$ .
4. Plug in the known values and solve.

### Equation:

$$v = v_0 + at = 70.0 \text{ m/s} + (-1.50 \text{ m/s}^2)(40.0 \text{ s}) = 10.0 \text{ m/s}$$

### Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.



The airplane lands with an initial velocity of  $70.0 \text{ m/s}$  and slows to a final velocity of  $10.0 \text{ m/s}$  before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation  $v = v_0 + at$  gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity ( $v = v_0$ ), as expected (i.e., velocity is constant)
- if  $a$  is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

**Note:**

Making Connections: Real-World Connection



The Space Shuttle *Endeavor*  
blasts off from the Kennedy  
Space Center in February 2010.  
(credit: Matthew Simantov,  
Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the

first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

**Note:**

Solving for Final Position When Velocity is Not Constant ( $a \neq 0$ )

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

**Equation:**

$$v = v_0 + at.$$

Adding  $v_0$  to each side of this equation and dividing by 2 gives

**Equation:**

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}at.$$

Since  $\frac{v_0 + v}{2} = \bar{v}$  for constant acceleration, then

**Equation:**

$$\bar{v} = v_0 + \frac{1}{2}at.$$

Now we substitute this expression for  $\bar{v}$  into the equation for displacement,

$x = x_0 + \bar{v}t$ , yielding

**Equation:**

$$x = x_0 + v_0t + \frac{1}{2}at^2 \text{ (constant } a\text{)}.$$

**Example:****Calculating Displacement of an Accelerating Object: Dragsters**

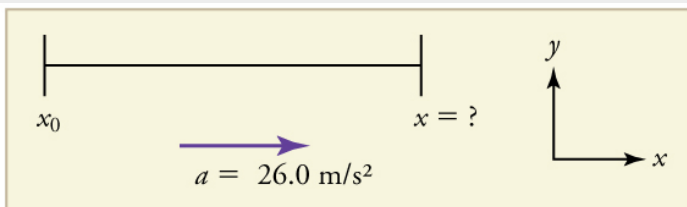
Dragsters can achieve average accelerations of  $26.0 \text{ m/s}^2$ . Suppose such a dragster accelerates from rest at this rate for  $5.56 \text{ s}$ . How far does it travel in this time?



U.S. Army Top Fuel pilot  
Tony “The Sarge”  
Schumacher begins a race  
with a controlled burnout.  
(credit: Lt. Col. William  
Thurmond. Photo  
Courtesy of U.S. Army.)

**Strategy**

Draw a sketch.



We are asked to find displacement, which is  $x$  if we take  $x_0$  to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation  $x = x_0 + v_0 t + \frac{1}{2} a t^2$  once we identify  $v_0$ ,  $a$ , and  $t$  from the statement of the problem.

**Solution**

1. Identify the knowns. Starting from rest means that  $v_0 = 0$ ,  $a$  is given as  $26.0 \text{ m/s}^2$  and  $t$  is given as  $5.56 \text{ s}$ .
2. Plug the known values into the equation to solve for the unknown  $x$ :

**Equation:**

$$x = x_0 + v_0 t + \frac{1}{2} a t^2.$$

Since the initial position and velocity are both zero, this simplifies to

**Equation:**

$$x = \frac{1}{2} a t^2.$$

Substituting the identified values of  $a$  and  $t$  gives

**Equation:**

$$x = \frac{1}{2} (26.0 \text{ m/s}^2) (5.56 \text{ s})^2,$$

yielding

**Equation:**

$$x = 402 \text{ m}.$$

### Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation  $x = x_0 + v_0 t + \frac{1}{2} a t^2$ ?  
We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In [\[link\]](#), the dragster covers only one fourth of the total distance in the first half of the elapsed time

- if acceleration is zero, then the initial velocity equals average velocity ( $v_0 = \bar{v}$ ) and  $x = x_0 + v_0 t + \frac{1}{2}at^2$  becomes  $x = x_0 + v_0 t$

**Note:**

Solving for Final Velocity when Velocity Is Not Constant ( $a \neq 0$ )

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve  $v = v_0 + at$  for  $t$ , we get

**Equation:**

$$t = \frac{v - v_0}{a}.$$

Substituting this and  $\bar{v} = \frac{v_0 + v}{2}$  into  $x = x_0 + \bar{v}t$ , we get

**Equation:**

$$v^2 = v_0^2 + 2a(x - x_0) \text{ (constant } a\text{)}.$$

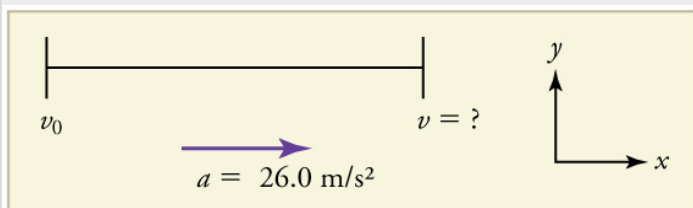
**Example:**

**Calculating Final Velocity: Dragsters**

Calculate the final velocity of the dragster in [\[link\]](#) without using information about time.

**Strategy**

Draw a sketch.



The equation  $v^2 = v_0^2 + 2a(x - x_0)$  is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

**Solution**

1. Identify the known values. We know that  $v_0 = 0$ , since the dragster starts from rest. Then we note that  $x - x_0 = 402$  m (this was the answer in [\[link\]](#)). Finally, the average acceleration was given to be  $a = 26.0$  m/s<sup>2</sup>.
2. Plug the knowns into the equation  $v^2 = v_0^2 + 2a(x - x_0)$  and solve for  $v$ .

**Equation:**

$$v^2 = 0 + 2(26.0 \text{ m/s}^2)(402 \text{ m}).$$

Thus

**Equation:**

$$v^2 = 2.09 \times 10^4 \text{ m}^2/\text{s}^2.$$

To get  $v$ , we take the square root:

**Equation:**

$$v = \sqrt{2.09 \times 10^4 \text{ m}^2/\text{s}^2} = 145 \text{ m/s}.$$

**Discussion**

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation  $v^2 = v_0^2 + 2a(x - x_0)$  can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed deceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to stop. (This is why

we have reduced speed zones near schools.)

## Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

**Note:**

Summary of Kinematic Equations (constant  $a$ )

**Equation:**

$$x = x_0 + \bar{v}t$$

**Equation:**

$$\bar{v} = \frac{v_0 + v}{2}$$

**Equation:**

$$v = v_0 + at$$

**Equation:**

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

**Equation:**

$$v^2 = v_0^2 + 2a(x - x_0)$$

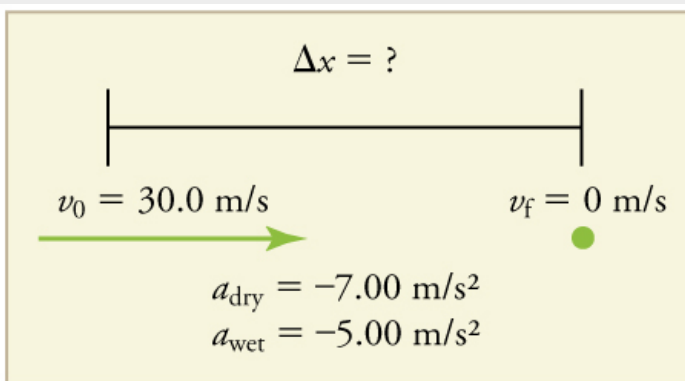
**Example:**

## Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of  $7.00 \text{ m/s}^2$ , whereas on wet concrete it can decelerate at only  $5.00 \text{ m/s}^2$ . Find the distances necessary to stop a car moving at  $30.0 \text{ m/s}$  (about  $110 \text{ km/h}$ ) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of  $0.500 \text{ s}$  to get his foot on the brake.

### Strategy

Draw a sketch.



In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

### Solution for (a)

1. Identify the knowns and what we want to solve for. We know that  $v_0 = 30.0 \text{ m/s}$ ;  $v = 0$ ;  $a = -7.00 \text{ m/s}^2$  ( $a$  is negative because it is in a direction opposite to velocity). We take  $x_0$  to be 0. We are looking for displacement  $\Delta x$ , or  $x - x_0$ .

2. Identify the equation that will help up solve the problem. The best equation to use is

### Equation:

$$v^2 = v_0^2 + 2a(x - x_0).$$

This equation is best because it includes only one unknown,  $x$ . We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for  $x$ , but they require us to know

the stopping time,  $t$ , which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for  $x$ .

**Equation:**

$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

4. Enter known values.

**Equation:**

$$x - 0 = \frac{0^2 - (30.0 \text{ m/s})^2}{2(-7.00 \text{ m/s}^2)}$$

Thus,

**Equation:**

$$x = 64.3 \text{ m on dry concrete.}$$

### **Solution for (b)**

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is  $-5.00 \text{ m/s}^2$ . The result is

**Equation:**

$$x_{\text{wet}} = 90.0 \text{ m on wet concrete.}$$

### **Solution for (c)**

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that

$\bar{v} = 30.0 \text{ m/s}$ ;  $t_{\text{reaction}} = 0.500 \text{ s}$ ;  $a_{\text{reaction}} = 0$ . We take  $x_{0-\text{reaction}}$  to be 0. We are looking for  $x_{\text{reaction}}$ .

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$  works well because the only unknown value is  $x$ , which is what we want to solve for.

3. Plug in the knowns to solve the equation.

**Equation:**

$$x = 0 + (30.0 \text{ m/s})(0.500 \text{ s}) = 15.0 \text{ m}.$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

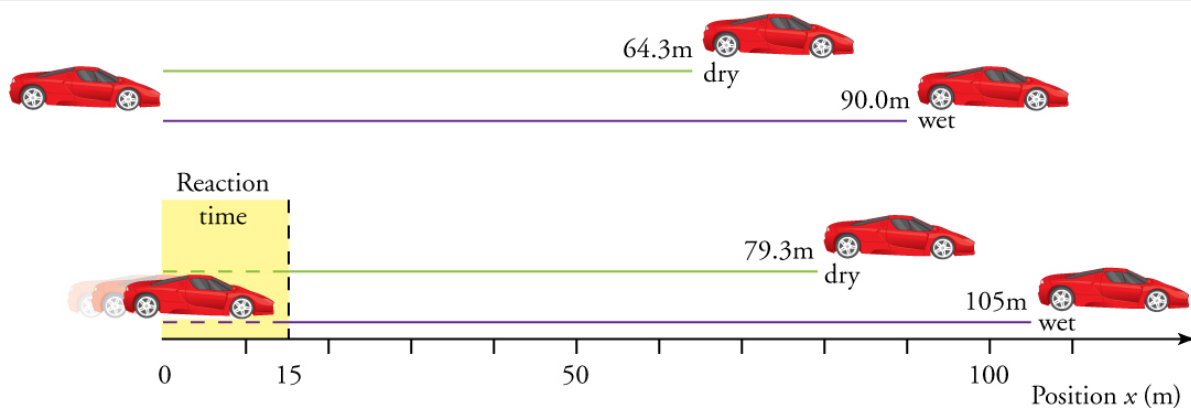
4. Add the displacement during the reaction time to the displacement when braking.

**Equation:**

$$x_{\text{braking}} + x_{\text{reaction}} = x_{\text{total}}$$

a.  $64.3 \text{ m} + 15.0 \text{ m} = 79.3 \text{ m}$  when dry

b.  $90.0 \text{ m} + 15.0 \text{ m} = 105 \text{ m}$  when wet



The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

## Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

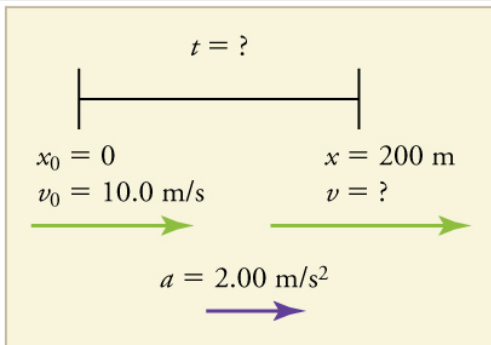
## Example:

### Calculating Time: A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at  $2.00 \text{ m/s}^2$ , how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

### Strategy

Draw a sketch.



We are asked to solve for the time  $t$ . As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown,  $t$ ).

### Solution

1. Identify the knowns and what we want to solve for. We know that  $v_0 = 10 \text{ m/s}$ ;  $a = 2.00 \text{ m/s}^2$ ; and  $x = 200 \text{ m}$ .
2. We need to solve for  $t$ . Choose the best equation.  $x = x_0 + v_0 t + \frac{1}{2} a t^2$  works best because the only unknown in the equation is the variable  $t$  for which we need to solve.

3. We will need to rearrange the equation to solve for  $t$ . In this case, it will be easier to plug in the knowns first.

**Equation:**

$$200 \text{ m} = 0 \text{ m} + (10.0 \text{ m/s})t + \frac{1}{2} (2.00 \text{ m/s}^2) t^2$$

4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking  $t = t \text{ s}$ , where  $t$  is the magnitude of time and s is the unit. Doing so leaves

**Equation:**

$$200 = 10t + t^2.$$

5. Use the quadratic formula to solve for  $t$ .

(a) Rearrange the equation to get 0 on one side of the equation.

**Equation:**

$$t^2 + 10t - 200 = 0$$

This is a quadratic equation of the form

**Equation:**

$$at^2 + bt + c = 0,$$

where the constants are  $a = 1.00$ ,  $b = 10.0$ , and  $c = -200$ .

(b) Its solutions are given by the quadratic formula:

**Equation:**

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This yields two solutions for  $t$ , which are

**Equation:**

$$t = 10.0 \text{ and } -20.0.$$

In this case, then, the time is  $t = t$  in seconds, or

**Equation:**

$$t = 10.0 \text{ s and } -20.0 \text{ s.}$$

A negative value for time is unreasonable, since it would mean that the event happened 20 s before the motion began. We can discard that solution. Thus,

**Equation:**

$$t = 10.0 \text{ s.}$$

### Discussion

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. [Problem-Solving Basics](#) discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

### Note:

#### Making Connections: Take-Home Experiment—Breaking News

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration,  $\bar{a} = \Delta v / \Delta t$ . While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

**Exercise:**  
**Check Your Understanding**

**Problem:**

A manned rocket accelerates at a rate of  $20 \text{ m/s}^2$  during launch. How long does it take the rocket to reach a velocity of  $400 \text{ m/s}$ ?

---

**Solution:**

To answer this, choose an equation that allows you to solve for time  $t$ , given only  $a$ ,  $v_0$ , and  $v$ .

**Equation:**

$$v = v_0 + at$$

Rearrange to solve for  $t$ .

**Equation:**

$$t = \frac{v - v_0}{a} = \frac{400 \text{ m/s} - 0 \text{ m/s}}{20 \text{ m/s}^2} = 20 \text{ s}$$

**Section Summary**

- To simplify calculations we take acceleration to be constant, so that  $\bar{a} = a$  at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,

**Equation:**

$$\Delta t = t$$

$$\Delta x = x - x_0$$

$$\Delta v = v - v_0$$

- The following kinematic equations for motion with constant  $a$  are useful:

**Equation:**

$$x = x_0 + \bar{v}t$$

**Equation:**

$$\bar{v} = \frac{v_0 + v}{2}$$

**Equation:**

$$v = v_0 + at$$

**Equation:**

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

**Equation:**

$$v^2 = v_0^2 + 2a(x - x_0)$$

- In vertical motion,  $y$  is substituted for  $x$ .

## Problems & Exercises

**Exercise:**

**Problem:**

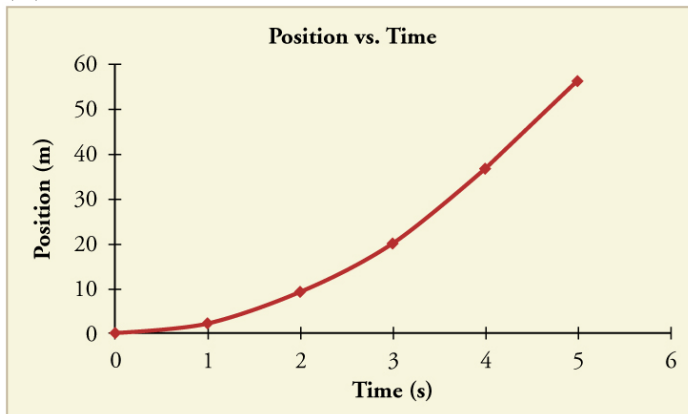
An Olympic-class sprinter starts a race with an acceleration of  $4.50 \text{ m/s}^2$ . (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.

---

**Solution:**

(a)  $10.8 \text{ m/s}$

(b)



**Exercise:**

**Problem:**

A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is  $2.10 \times 10^4 \text{ m/s}^2$ , and 1.85 ms ( $1 \text{ ms} = 10^{-3} \text{ s}$ ) elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

---

**Solution:**

38.9 m/s (about 87 miles per hour)

**Exercise:**

**Problem:**

A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of  $6.20 \times 10^5 \text{ m/s}^2$  for  $8.10 \times 10^{-4} \text{ s}$ . What is its muzzle velocity (that is, its final velocity)?

**Exercise:**

**Problem:**

(a) A light-rail commuter train accelerates at a rate of  $1.35 \text{ m/s}^2$ . How long does it take to reach its top speed of  $80.0 \text{ km/h}$ , starting from rest? (b) The same train ordinarily decelerates at a rate of  $1.65 \text{ m/s}^2$ . How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from  $80.0 \text{ km/h}$  in  $8.30 \text{ s}$ . What is its emergency deceleration in  $\text{m/s}^2$ ?

---

**Solution:**

(a)  $16.5 \text{ s}$

(b)  $13.5 \text{ s}$

(c)  $-2.68 \text{ m/s}^2$

**Exercise:****Problem:**

While entering a freeway, a car accelerates from rest at a rate of  $2.40 \text{ m/s}^2$  for  $12.0 \text{ s}$ . (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those  $12.0 \text{ s}$ ? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

**Exercise:****Problem:**

At the end of a race, a runner decelerates from a velocity of  $9.00 \text{ m/s}$  at a rate of  $2.00 \text{ m/s}^2$ . (a) How far does she travel in the next  $5.00 \text{ s}$ ? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?

---

**Solution:**

(a) 20.0 m

(b)  $-1.00 \text{ m/s}$

(c) This result does not really make sense. If the runner starts at  $9.00 \text{ m/s}$  and decelerates at  $2.00 \text{ m/s}^2$ , then she will have stopped after  $4.50 \text{ s}$ . If she continues to decelerate, she will be running backwards.

**Exercise:****Problem: Professional Application:**

Blood is accelerated from rest to  $30.0 \text{ cm/s}$  in a distance of  $1.80 \text{ cm}$  by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

**Exercise:****Problem:**

In a slap shot, a hockey player accelerates the puck from a velocity of  $8.00 \text{ m/s}$  to  $40.0 \text{ m/s}$  in the same direction. If this shot takes  $3.33 \times 10^{-2} \text{ s}$ , calculate the distance over which the puck accelerates.

---

**Solution:**

$0.799 \text{ m}$

**Exercise:**

**Problem:**

A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

**Exercise:****Problem:**

Freight trains can produce only relatively small accelerations and decelerations. (a) What is the final velocity of a freight train that accelerates at a rate of  $0.0500 \text{ m/s}^2$  for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of  $0.550 \text{ m/s}^2$ , how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

---

**Solution:**

(a) 28.0 m/s

(b) 50.9 s

(c) 7.68 km to accelerate and 713 m to decelerate

**Exercise:****Problem:**

A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.

**Exercise:**

**Problem:**

A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of  $0.350 \text{ m/s}^2$ , how far will it travel before becoming airborne? (b) How long does this take?

---

**Solution:**

(a) 51.4 m

(b) 17.1 s

**Exercise:****Problem: Professional Application:**

A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in  $\text{m/s}^2$  and in multiples of  $g$  ( $g = 9.80 \text{ m/s}^2$ ). (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of  $g$ ?

**Exercise:****Problem:**

An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?

---

**Solution:**

(a)  $-80.4 \text{ m/s}^2$

(b)  $9.33 \times 10^{-2} \text{ s}$

**Exercise:**

**Problem:**

In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

**Exercise:**

**Problem:**

Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m. (b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

---

**Solution:**

(a)  $7.7 \text{ m/s}$

(b)  $-15 \times 10^2 \text{ m/s}^2$ . This is about 3 times the deceleration of the pilots, who were falling from thousands of meters high!

**Exercise:**

**Problem:**

An express train passes through a station. It enters with an initial velocity of  $22.0 \text{ m/s}$  and decelerates at a rate of  $0.150 \text{ m/s}^2$  as it goes through. The station is  $210 \text{ m}$  long. (a) How long is the nose of the train in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is  $130 \text{ m}$  long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?

**Exercise:****Problem:**

Dragsters can actually reach a top speed of  $145 \text{ m/s}$  in only  $4.45 \text{ s}$ —considerably less time than given in [\[link\]](#) and [\[link\]](#). (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for  $402 \text{ m}$  (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? *Hint:* Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

---

**Solution:**

(a)  $32.6 \text{ m/s}^2$

(b)  $162 \text{ m/s}$

(c)  $v > v_{\text{max}}$ , because the assumption of constant acceleration is not valid for a dragster. A dragster changes gears, and would have a greater acceleration in first gear than second gear than third gear, etc. The acceleration would be greatest at the beginning, so it would not be accelerating at  $32.6 \text{ m/s}^2$  during the last few meters, but substantially less, and the final velocity would be less than  $162 \text{ m/s}$ .

**Exercise:****Problem:**

A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of  $11.5 \text{ m/s}$  and accelerates at the rate of  $0.500 \text{ m/s}^2$  for  $7.00 \text{ s}$ . (a) What is his final velocity? (b) The racer continues at this velocity to the finish line. If he was  $300 \text{ m}$  from the finish line when he started to accelerate, how much time did he save? (c) One other racer was  $5.00 \text{ m}$  ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at  $11.8 \text{ m/s}$  until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

**Exercise:****Problem:**

In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of  $183.58 \text{ mi/h}$ . The one-way course was  $5.00 \text{ mi}$  long. Acceleration rates are often described by the time it takes to reach  $60.0 \text{ mi/h}$  from rest. If this time was  $4.00 \text{ s}$ , and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

---

**Solution:**

$104 \text{ s}$

**Exercise:**

**Problem:**

(a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

---

**Solution:**

(a)  $v = 12.2 \text{ m/s}$ ;  $a = 4.07 \text{ m/s}^2$

(b)  $v = 11.2 \text{ m/s}$

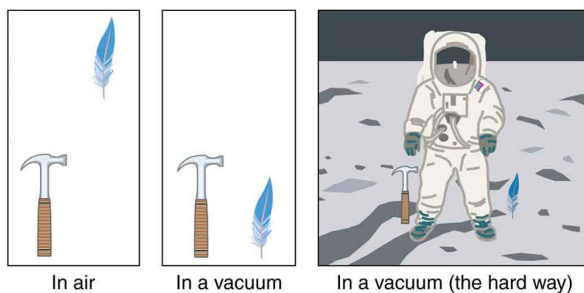
## Falling Objects

- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

## Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration, independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.



A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the

acceleration due to gravity is  
only  $1.67 \text{ m/s}^2$ .

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free-fall**.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the **acceleration due to gravity**. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol,  $g$ . It is constant at any given location on Earth and has the average value

**Equation:**

$$g = 9.80 \text{ m/s}^2.$$

Although  $g$  varies from  $9.78 \text{ m/s}^2$  to  $9.83 \text{ m/s}^2$ , depending on latitude, altitude, underlying geological formations, and local topography, the average value of  $9.80 \text{ m/s}^2$  will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward (towards the center of Earth)*. In fact, its direction *defines* what we call vertical. Note that whether the acceleration  $a$  in the kinematic equations has the value  $+g$  or  $-g$  depends on how we define our coordinate system. If we define the upward direction as positive, then  $a = -g = -9.80 \text{ m/s}^2$ , and if we define the downward direction as positive, then  $a = g = 9.80 \text{ m/s}^2$ .

## One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude  $g$ . We will also represent vertical displacement with the symbol  $y$  and use  $x$  for horizontal displacement.

### **Note:**

Kinematic Equations for Objects in Free-Fall where Acceleration =  $-g$

### **Equation:**

$$v = v_0 - gt$$

### **Equation:**

$$y = y_0 + v_0t - \frac{1}{2}gt^2$$

### **Equation:**

$$v^2 = v_0^2 - 2g(y - y_0)$$

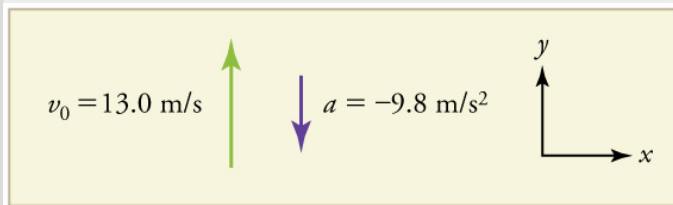
### **Example:**

#### **Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward**

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

### Strategy

Draw a sketch.



We are asked to determine the position  $y$  at various times. It is reasonable to take the initial position  $y_0$  to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so  $a$  is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs.

Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as  $y_1$  and  $v_1$ ;  $y_2$  and  $v_2$ ; and  $y_3$  and  $v_3$ .

#### Solution for Position $y_1$

1. Identify the knowns. We know that  $y_0 = 0$ ;  $v_0 = 13.0 \text{ m/s}$ ;

$a = -g = -9.80 \text{ m/s}^2$ ; and  $t = 1.00 \text{ s}$ .

2. Identify the best equation to use. We will use  $y = y_0 + v_0 t + \frac{1}{2} a t^2$  because it includes only one unknown,  $y$  (or  $y_1$ , here), which is the value we want to find.

3. Plug in the known values and solve for  $y_1$ .

#### Equation:

$$y_1 = 0 + (13.0 \text{ m/s})(1.00 \text{ s}) + \frac{1}{2} (-9.80 \text{ m/s}^2)(1.00 \text{ s})^2 = 8.10 \text{ m}$$

#### Discussion

The rock is 8.10 m above its starting point at  $t = 1.00 \text{ s}$ , since  $y_1 > y_0$ . It could be *moving* up or down; the only way to tell is to calculate  $v_1$  and find out if it is positive or negative.

#### Solution for Velocity $v_1$

1. Identify the knowns. We know that  $y_0 = 0$ ;  $v_0 = 13.0 \text{ m/s}$ ;

$a = -g = -9.80 \text{ m/s}^2$ ; and  $t = 1.00 \text{ s}$ . We also know from the solution above that  $y_1 = 8.10 \text{ m}$ .

2. Identify the best equation to use. The most straightforward is  $v = v_0 - gt$  (from  $v = v_0 + at$ , where  $a = \text{gravitational acceleration} = -g$ ).
3. Plug in the knowns and solve.

**Equation:**

$$v_1 = v_0 - gt = 13.0 \text{ m/s} - (9.80 \text{ m/s}^2)(1.00 \text{ s}) = 3.20 \text{ m/s}$$

### Discussion

The positive value for  $v_1$  means that the rock is still heading upward at  $t = 1.00 \text{ s}$ . However, it has slowed from its original  $13.0 \text{ m/s}$ , as expected.

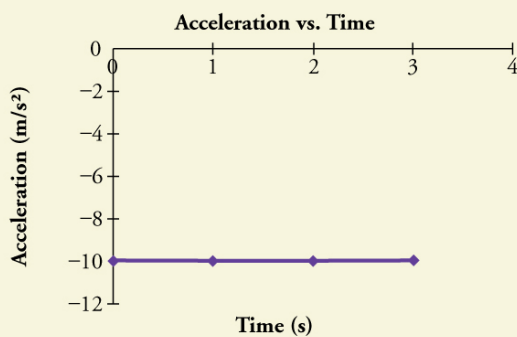
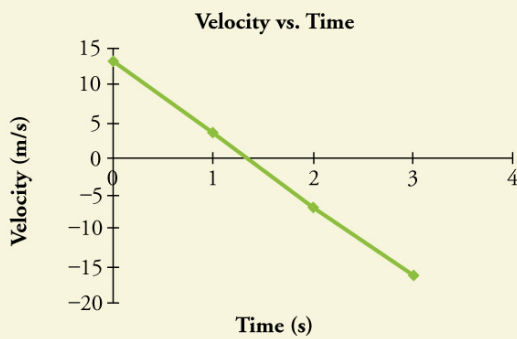
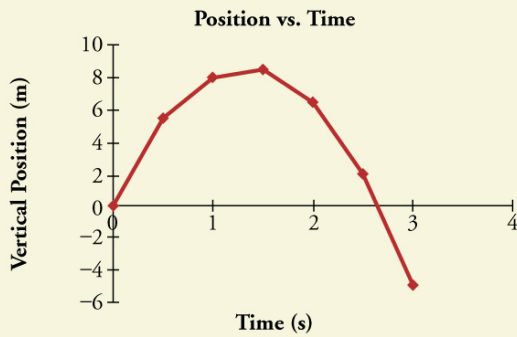
### Solution for Remaining Times

The procedures for calculating the position and velocity at  $t = 2.00 \text{ s}$  and  $3.00 \text{ s}$  are the same as those above. The results are summarized in [\[link\]](#) and illustrated in [\[link\]](#).

Time, $t$	Position, $y$	Velocity, $v$	Acceleration, $a$
1.00 s	8.10 m	3.20 m/s	$-9.80 \text{ m/s}^2$
2.00 s	6.40 m	$-6.60 \text{ m/s}$	$-9.80 \text{ m/s}^2$
3.00 s	$-5.10 \text{ m}$	$-16.4 \text{ m/s}$	$-9.80 \text{ m/s}^2$

### Results

Graphing the data helps us understand it more clearly.



Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant.

*Misconception Alert!* Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some

horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is *time*, not space. The actual path of the rock in space is straight up, and straight down.

### Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since  $y_1$  and  $v_1$  are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both  $y_3$  and  $v_3$  are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still  $-9.80 \text{ m/s}^2$ . Its acceleration is  $-9.80 \text{ m/s}^2$  for the whole trip—while it is moving up and while it is moving down. Note that the values for  $y$  are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

### Note:

#### Making Connections: Take-Home Experiment—Reaction Time

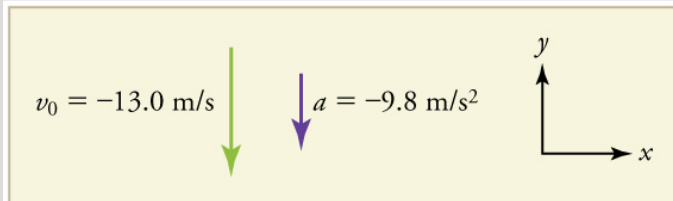
A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

**Example:****Calculating Velocity of a Falling Object: A Rock Thrown Down**

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

**Strategy**

Draw a sketch.



Since up is positive, the final position of the rock will be negative because it finishes below the starting point at  $y_0 = 0$ . Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

**Solution**

1. Identify the knowns.  $y_0 = 0$ ;  $y_1 = -5.10$  m;  $v_0 = -13.0$  m/s;  $a = -g = -9.80$  m/s<sup>2</sup>.
2. Choose the kinematic equation that makes it easiest to solve the problem. The equation  $v^2 = v_0^2 + 2a(y - y_0)$  works well because the only unknown in it is  $v$ . (We will plug  $y_1$  in for  $y$ .)
3. Enter the known values

**Equation:**

$$v^2 = (-13.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-5.10 \text{ m} - 0 \text{ m}) = 268.96 \text{ m}^2/\text{s}^2,$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

**Equation:**

$$v = \pm 16.4 \text{ m/s}.$$

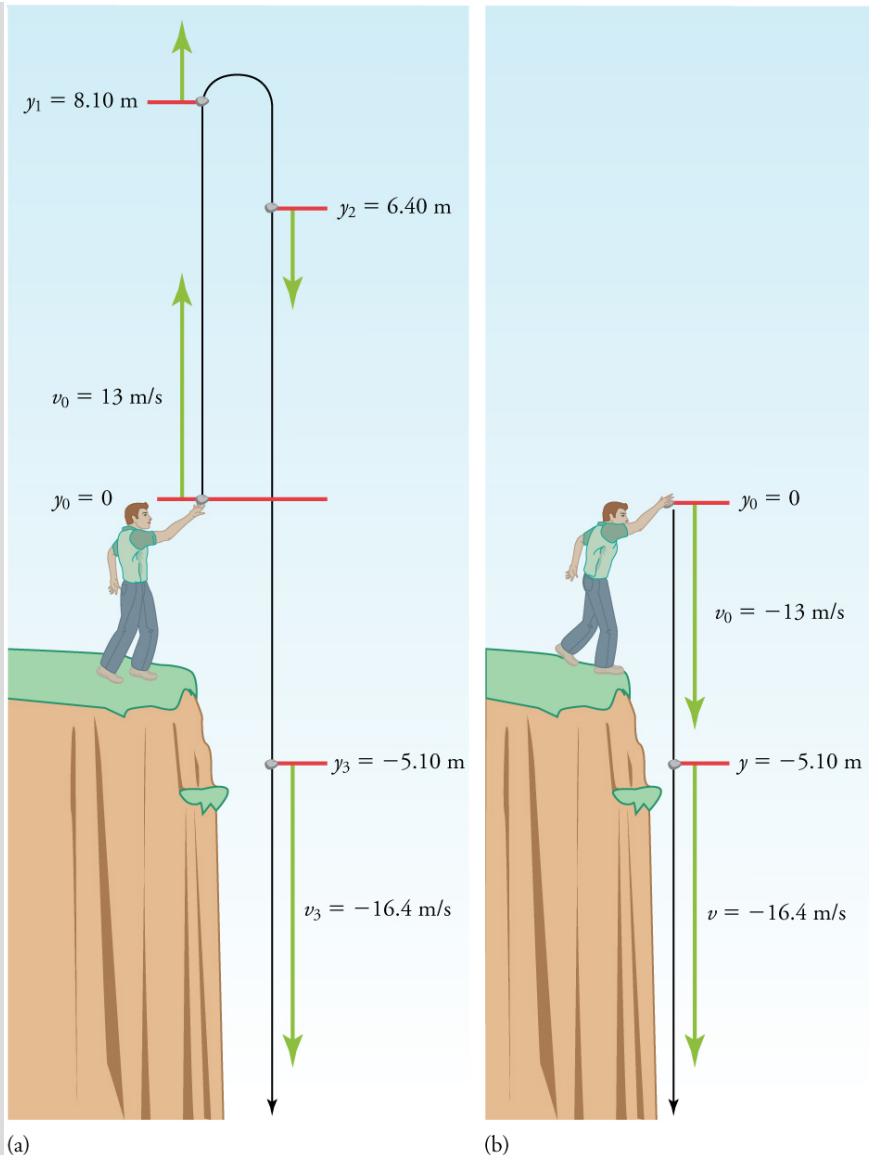
The negative root is chosen to indicate that the rock is still heading down. Thus,

**Equation:**

$$v = -16.4 \text{ m/s.}$$

### Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed.* (See [\[link\]](#) and [\[link\]](#)(a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from [\[link\]](#)) when the initial velocity is 13.0 m/s straight up, a result of  $\pm 3.20 \text{ m/s}$  is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.



(a) A person throws a rock straight up, as explored in [\[link\]](#). The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in [\[link\]](#). Note that at the same distance below the point of release, the rock has the same velocity in both cases.

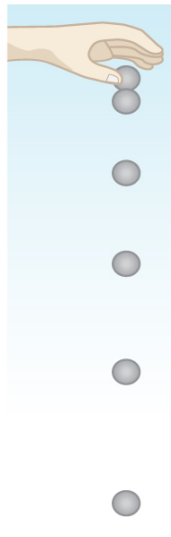
Another way to look at it is this: In [\[link\]](#), the rock is thrown up with an initial velocity of  $13.0 \text{ m/s}$ . It rises and then falls back down. When its

position is  $y = 0$  on its way back down, its velocity is  $-13.0 \text{ m/s}$ . That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of  $y = -5.10 \text{ m}$  to be the same whether we have thrown it upwards at  $+13.0 \text{ m/s}$  or thrown it downwards at  $-13.0 \text{ m/s}$ . The velocity of the rock on its way down from  $y = 0$  is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

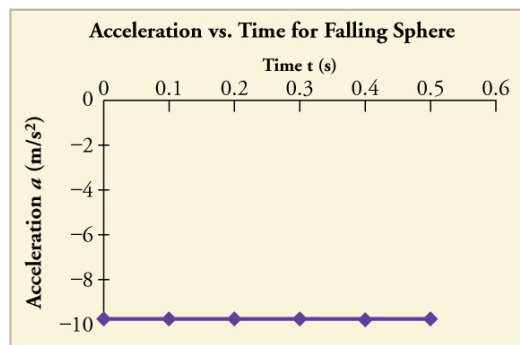
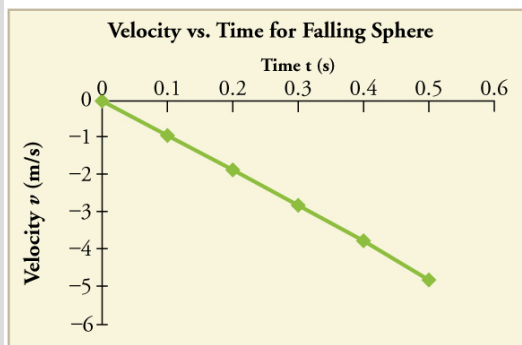
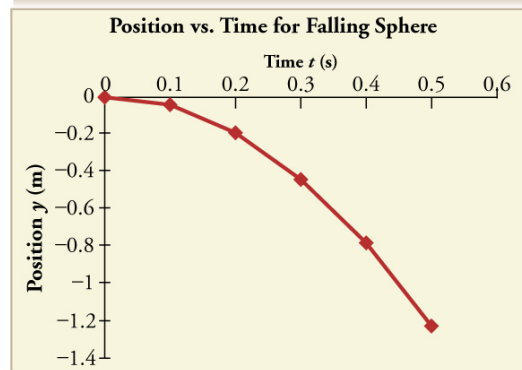
**Example:****Find  $g$  from Data on a Falling Object**

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course.

An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, [\[link\]](#). Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.



$y$ (m)	$v$ (m/s)	$t$ (s)
0	0	0
-0.049	-0.98	0.1
-0.196	-1.96	0.2
-0.441	-2.94	0.3
-0.784	-3.92	0.4
-1.225	-4.90	0.5



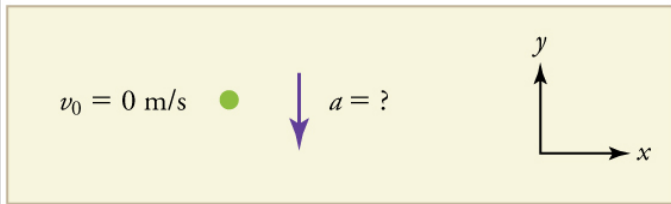
Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared.

Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

**Strategy**

Draw a sketch.



We need to solve for acceleration  $a$ . Note that in this case, displacement is downward and therefore negative, as is acceleration.

**Solution**

1. Identify the knowns.  $y_0 = 0$ ;  $y = -1.0000$  m;  $t = 0.45173$ ;  $v_0 = 0$ .
2. Choose the equation that allows you to solve for  $a$  using the known values.

**Equation:**

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

3. Substitute 0 for  $v_0$  and rearrange the equation to solve for  $a$ . Substituting 0 for  $v_0$  yields

**Equation:**

$$y = y_0 + \frac{1}{2} a t^2.$$

Solving for  $a$  gives

**Equation:**

$$a = \frac{2(y - y_0)}{t^2}.$$

4. Substitute known values yields

**Equation:**

$$a = \frac{2(-1.0000 \text{ m} - 0)}{(0.45173 \text{ s})^2} = -9.8010 \text{ m/s}^2,$$

so, because  $a = -g$  with the directions we have chosen,

**Equation:**

$$g = 9.8010 \text{ m/s}^2.$$

### Discussion

The negative value for  $a$  indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of  $9.80 \text{ m/s}^2$ , so  $9.8010 \text{ m/s}^2$  makes sense. Since the data going into the calculation are relatively precise, this value for  $g$  is more precise than the average value of  $9.80 \text{ m/s}^2$ ; it represents the local value for the acceleration due to gravity.

### Exercise:

#### Check Your Understanding

##### Problem:

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

---

##### Solution:

We know that initial position  $y_0 = 0$ , final position  $y = -30.0 \text{ m}$ , and  $a = -g = -9.80 \text{ m/s}^2$ . We can then use the equation  $y = y_0 + v_0t + \frac{1}{2}at^2$  to solve for  $t$ . Inserting  $a = -g$ , we obtain

**Equation:**

$$y = 0 + 0 - \frac{1}{2}gt^2$$

$$t^2 = \frac{2y}{-g}$$

$$t = \pm \sqrt{\frac{2y}{-g}} = \pm \sqrt{\frac{2(-30.0 \text{ m})}{-9.80 \text{ m/s}^2}} = \pm \sqrt{6.12 \text{ s}^2} = 2.47 \text{ s} \approx 2.5 \text{ s}$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

**Note:**

**PhET Explorations: Equation Grapher**

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g.  $y = bx$ ) to see how they add to generate the polynomial curve.

[https://phet.colorado.edu/sims/equation-grapher/equation-grapher\\_en.html](https://phet.colorado.edu/sims/equation-grapher/equation-grapher_en.html)

## Section Summary

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity  $g$ , which averages

**Equation:**

$$g = 9.80 \text{ m/s}^2.$$

- Whether the acceleration  $a$  should be taken as  $+g$  or  $-g$  is determined by your choice of coordinate system. If you choose the upward direction as positive,  $a = -g = -9.80 \text{ m/s}^2$  is negative. In the opposite case,  $a = +g = 9.80 \text{ m/s}^2$  is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate  $+g$  or  $-g$  substituted for  $a$ .
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

## Conceptual Questions

**Exercise:**

**Problem:**

What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

**Exercise:****Problem:**

An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

**Exercise:****Problem:**

Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

**Exercise:****Problem:**

If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

**Exercise:****Problem:**

The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about  $1/6$  that of the Earth)?

**Exercise:****Problem:**

How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about  $1/6$  of  $g$  on Earth)?

**Problems & Exercises**

Assume air resistance is negligible unless otherwise stated.

**Exercise:****Problem:**

Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be  $y_0 = 0$ .

---

**Solution:**

(a)  $y_1 = 6.28 \text{ m}$ ;  $v_1 = 10.1 \text{ m/s}$

(b)  $y_2 = 10.1 \text{ m}$ ;  $v_2 = 5.20 \text{ m/s}$

(c)  $y_3 = 11.5 \text{ m}$ ;  $v_3 = 0.300 \text{ m/s}$

(d)  $y_4 = 10.4 \text{ m}$ ;  $v_4 = -4.60 \text{ m/s}$

**Exercise:****Problem:**

Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

**Exercise:**

**Problem:**

A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

---

**Solution:**

$$v_0 = 4.95 \text{ m/s}$$

**Exercise:****Problem:**

A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

**Exercise:****Problem:**

A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

---

**Solution:**

$$(a) a = -9.80 \text{ m/s}^2; v_0 = 13.0 \text{ m/s}; y_0 = 0 \text{ m}$$

(b)  $v = 0\text{ m/s}$ . Unknown is distance  $y$  to top of trajectory, where velocity is zero. Use equation  $v^2 = v_0^2 + 2a(y - y_0)$  because it contains all known values except for  $y$ , so we can solve for  $y$ . Solving for  $y$  gives

**Equation:**

$$v^2 - v_0^2 = 2a(y - y_0)$$

$$\frac{v^2 - v_0^2}{2a} = y - y_0$$

$$y = y_0 + \frac{v^2 - v_0^2}{2a} = 0\text{ m} + \frac{(0\text{ m/s})^2 - (13.0\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 8.62\text{ m}$$

Dolphins measure about 2 meters long and can jump several times their length out of the water, so this is a reasonable result.

(c) 2.65 s

**Exercise:**

**Problem:**

A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool. (a) How long are her feet in the air? (b) What is her highest point above the board? (c) What is her velocity when her feet hit the water?

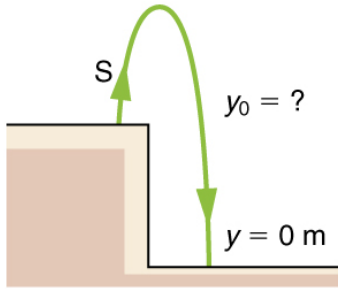
**Exercise:**

**Problem:**

(a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?

---

**Solution:**



(a) 8.26 m

(b) 0.717 s

**Exercise:**

**Problem:**

A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

**Exercise:**

**Problem:**

You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

---

**Solution:**

1.91 s

**Exercise:**

**Problem:**

A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?

**Exercise:**

**Problem:**

Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can't see the rock right away but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?

---

**Solution:**

(a) 94.0 m

(b) 3.13 s

**Exercise:****Problem:**

An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

**Exercise:****Problem:**

There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

---

**Solution:**

(a) -70.0 m/s (downward)

(b) 6.10 s

**Exercise:****Problem:**

A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 0.312 s to go past the window. What was the ball's initial velocity? Hint: First consider only the distance along the window, and solve for the ball's velocity at the bottom of the window. Next, consider only the distance from the ground to the bottom of the window, and solve for the initial velocity using the velocity at the bottom of the window as the final velocity.

**Exercise:****Problem:**

Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

---

**Solution:**

(a) 19.6 m

(b) 18.5 m

**Exercise:****Problem:**

A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms ( $8.00 \times 10^{-5}$  s). (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

**Exercise:**

**Problem:**

A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

---

**Solution:**

(a) 305 m

(b) 262 m, -29.2 m/s

(c) 8.91 s

**Exercise:****Problem:**

A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms ( $3.50 \times 10^{-3}$  s). (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

**Glossary**

free-fall

the state of movement that results from gravitational force only

acceleration due to gravity

acceleration of an object as a result of gravity

## Introduction to Two-Dimensional Kinematics

class="introduction"

Everyday motion  
that we experience  
is, thankfully,  
rarely as tortuous  
as a rollercoaster  
ride like this—the  
Dragon Khan in  
Spain's Universal  
Port Aventura  
Amusement Park.

However, most  
motion is in  
curved, rather than  
straight-line, paths.

Motion along a  
curved path is two-  
or three-  
dimensional  
motion, and can be  
described in a  
similar fashion to  
one-dimensional  
motion. (credit:  
Boris23/Wikimedi  
a Commons)



The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Motion not confined to a plane, such as a car following a winding mountain road, is described by three-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight-line motion in the previous chapter. This simple extension will allow us to apply physics to many more situations, and it will also yield unexpected insights about nature.

## Kinematics in Two Dimensions: An Introduction

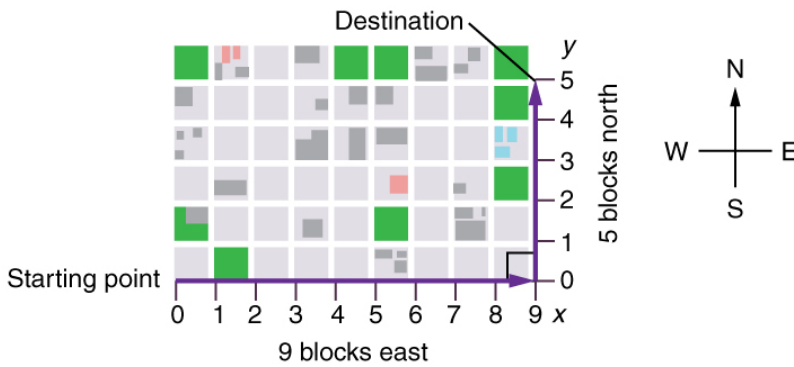
- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.



Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths.  
(credit: Margaret W. Carruthers)

## Two-Dimensional Motion: Walking in a City

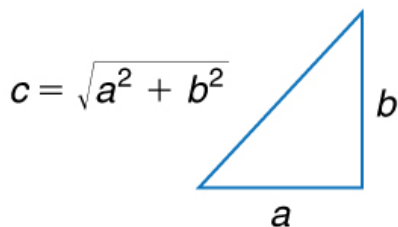
Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in [\[link\]](#).



A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

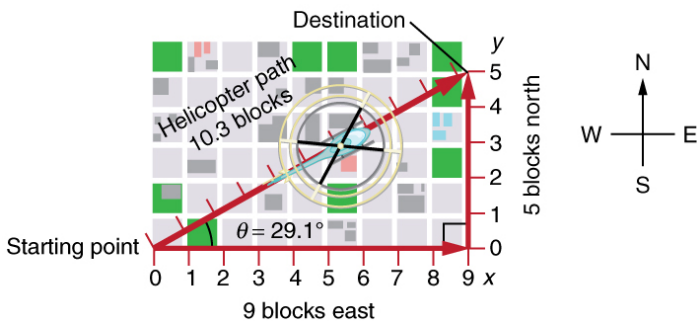
An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem,  $a^2 + b^2 = c^2$ , can be used to find the straight-line distance.



The Pythagorean theorem relates the length of the legs of a right triangle,

labeled  $a$  and  $b$ ,  
 with the  
 hypotenuse, labeled  
 $c$ . The relationship  
 is given by:  
 $a^2 + b^2 = c^2$ . This  
 can be rewritten,  
 solving for  $c$  :  
 $c = \sqrt{a^2 + b^2}$ .

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is  
 $\sqrt{(9 \text{ blocks})^2 + (5 \text{ blocks})^2} = 10.3 \text{ blocks}$ , considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that “9” and “5” have only one significant digit, they are discrete numbers. In this case “9 blocks” is the same as “9.0 or 9.00 blocks.” We have decided to use three significant figures in the answer in order to show the result more precisely.)



The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

The fact that the straight-line distance (10.3 blocks) in [\[link\]](#) is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that **vectors** are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector's magnitude. The arrow's length is indicated by hash marks in [\[link\]](#) and [\[link\]](#). The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in [\[link\]](#). The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in [Vector Addition and Subtraction: Graphical Methods](#) and [Vector Addition and Subtraction: Analytical Methods](#).)

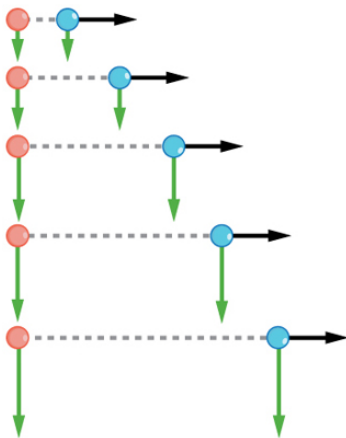
## The Independence of Perpendicular Motions

The person taking the path shown in [\[link\]](#) walks east and then north (two perpendicular directions). How far he or she walks east is only affected by his or her motion eastward. Similarly, how far he or she walks north is only affected by his or her motion northward.

**Note:****Independence of Motion**

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let's compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.



This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent

position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the

ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in [Vector Addition and Subtraction: Graphical Methods](#) and [Vector Addition and Subtraction: Analytical Methods](#). We will find such techniques to be useful in many areas of physics.

**Note:****PhET Explorations: Ladybug Motion 2D**

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

<https://archive.cnx.org/specials/317a2b1e-2fbd-11e5-99b5-e38ffb545fe6/ladybug-motion/#sim-ladybug-motion>

## Summary

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion

in the vertical direction, and vice versa.

## **Glossary**

### **vector**

a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

## Projectile Motion

- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

**Projectile motion** is the **motion** of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a **projectile**, and its path is called its **trajectory**. The motion of falling objects, as covered in [Problem-Solving Basics for One-Dimensional Kinematics](#), is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which **air resistance is negligible**.

The most important fact to remember here is that *motions along perpendicular axes are independent* and thus can be analyzed separately. This fact was discussed in [Kinematics in Two Dimensions: An Introduction](#), where vertical and horizontal motions were seen to be independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the  $x$ -axis and the vertical axis the  $y$ -axis. [\[link\]](#) illustrates the notation for displacement, where  $\mathbf{s}$  is defined to be the total displacement and  $\mathbf{x}$  and  $\mathbf{y}$  are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are  $s$ ,  $x$ , and  $y$ . (Note that in the last section we used the notation  $\mathbf{A}$  to represent a vector with components  $\mathbf{A}_x$  and  $\mathbf{A}_y$ . If we continued this format, we would call displacement  $\mathbf{s}$  with components  $\mathbf{s}_x$  and  $\mathbf{s}_y$ . However, to simplify the notation, we will simply represent the component vectors as  $\mathbf{x}$  and  $\mathbf{y}$ .)

Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the  $x$ - and  $y$ -axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple:  $a_y = -g = -9.80 \text{ m/s}^2$ . (Note that this definition assumes that the upwards direction is defined as the positive direction. If you arrange the coordinate system instead such that the downwards direction is positive, then acceleration due to gravity takes a positive value.) Because gravity is vertical,  $a_x = 0$ . Both accelerations are constant, so the kinematic equations can be used.

### Note:

Review of Kinematic Equations (constant  $a$ )

#### Equation:

$$x = x_0 + \bar{v}t$$

#### Equation:

$$\bar{v} = \frac{v_0 + v}{2}$$

#### Equation:

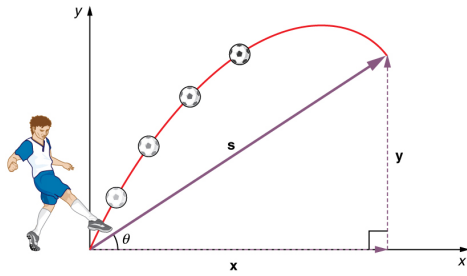
$$v = v_0 + at$$

#### Equation:

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

#### Equation:

$$v^2 = v_0^2 + 2a(x - x_0).$$



The total displacement  $\mathbf{s}$  of a soccer ball at a point along its path. The vector  $\mathbf{s}$  has components  $\mathbf{x}$  and  $\mathbf{y}$  along the horizontal and vertical axes. Its magnitude is  $s$ , and it makes an angle  $\theta$  with the horizontal.

Given these assumptions, the following steps are then used to analyze projectile motion:

**Step 1.** Resolve or break the motion into horizontal and vertical components along the  $x$ - and  $y$ -axes. These axes are perpendicular, so  $A_x = A \cos \theta$  and  $A_y = A \sin \theta$  are used. The magnitude of the components of displacement  $\mathbf{s}$  along these axes are  $x$  and  $y$ . The magnitudes of the components of the velocity  $\mathbf{v}$  are  $v_x = v \cos \theta$  and  $v_y = v \sin \theta$ , where  $v$  is the magnitude of the velocity and  $\theta$  is its direction, as shown in [\[link\]](#). Initial values are denoted with a subscript 0, as usual.

**Step 2.** Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical. The kinematic equations for horizontal and vertical motion take the following forms:

**Equation:**

$$\text{Horizontal Motion}(a_x = 0)$$

**Equation:**

$$x = x_0 + v_x t$$

**Equation:**

$$v_x = v_{0x} = v_x = \text{velocity is a constant.}$$

**Equation:**

$$\text{Vertical Motion(assuming positive is up } a_y = -g = -9.80\text{m/s}^2)$$

**Equation:**

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$

**Equation:**

$$v_y = v_{0y} - gt$$

**Equation:**

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

**Equation:**

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

**Step 3.** Solve for the unknowns in the two separate motions—one horizontal and one vertical. Note that the only common variable between the motions is time  $t$ . The problem solving procedures here are the same as for one-dimensional **kinematics** and are illustrated in the solved examples below.

**Step 4.** Recombine the two motions to find the total displacement  $\mathbf{s}$  and velocity  $\mathbf{v}$ . Because the  $x$  - and  $y$  -motions are perpendicular, we determine these vectors by using the techniques outlined in the [Vector Addition and Subtraction: Analytical Methods](#) and employing  $A = \sqrt{A_x^2 + A_y^2}$  and  $\theta = \tan^{-1}(A_y/A_x)$  in the following form, where  $\theta$  is the direction of the displacement  $\mathbf{s}$  and  $\theta_v$  is the direction of the velocity  $\mathbf{v}$ :

**Total displacement and velocity**

**Equation:**

$$s = \sqrt{x^2 + y^2}$$

**Equation:**

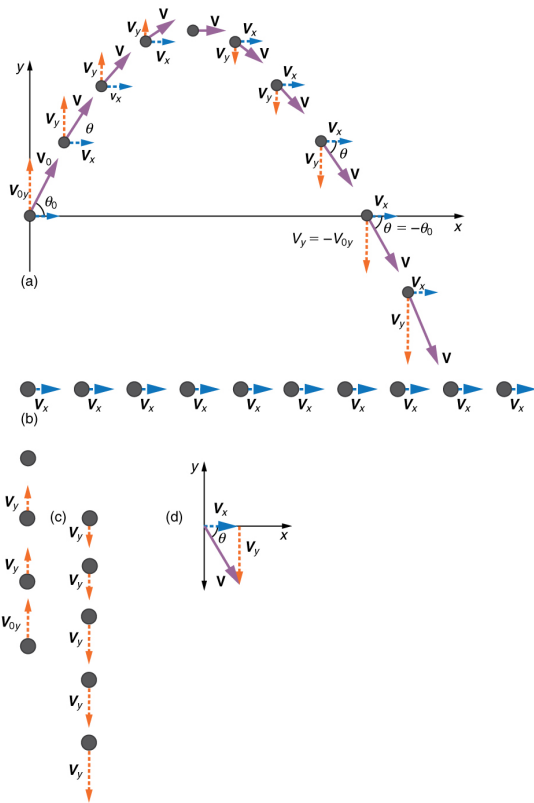
$$\theta = \tan^{-1}(y/x)$$

**Equation:**

$$v = \sqrt{v_x^2 + v_y^2}$$

**Equation:**

$$\theta_v = \tan^{-1}(v_y/v_x).$$



(a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because  $a_x = 0$  and  $v_x$  is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The  $x$  - and  $y$  -motions are recombined to give the total velocity at any given point on the trajectory.

### Example:

#### A Fireworks Projectile Explodes High and Away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of  $75.0^\circ$  above the horizontal, as illustrated in [\[link\]](#). The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?

#### Strategy

Because air resistance is negligible for the unexploded shell, the analysis method outlined above can be used. The motion can be broken into horizontal and vertical motions in which  $a_x = 0$  and  $a_y = -g$ . We can then define  $x_0$

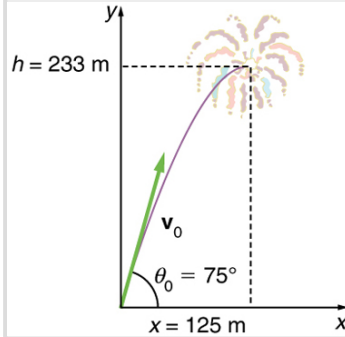
and  $y_0$  to be zero and solve for the desired quantities.

**Solution for (a)**

By “height” we mean the altitude or vertical position  $y$  above the starting point. The highest point in any trajectory, called the apex, is reached when  $v_y = 0$ . Since we know the initial and final velocities as well as the initial position, we use the following equation to find  $y$ :

**Equation:**

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$



The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.

Because  $y_0$  and  $v_y$  are both zero, the equation simplifies to

**Equation:**

$$0 = v_{0y}^2 - 2gy.$$

Solving for  $y$  gives

**Equation:**

$$y = \frac{v_{0y}^2}{2g}.$$

Now we must find  $v_{0y}$ , the component of the initial velocity in the  $y$ -direction. It is given by  $v_{0y} = v_0 \sin \theta$ , where  $v_0$  is the initial velocity of 70.0 m/s, and  $\theta = 75.0^\circ$  is the initial angle. Thus,

**Equation:**

$$v_{0y} = v_0 \sin \theta_0 = (70.0 \text{ m/s})(\sin 75^\circ) = 67.6 \text{ m/s}.$$

and  $y$  is

**Equation:**

$$y = \frac{(67.6 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)},$$

so that

**Equation:**

$$y = 233\text{m}.$$

**Discussion for (a)**

Note that because up is positive, the initial velocity is positive, as is the maximum height, but the acceleration due to gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6 m/s initial vertical component of velocity will reach a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, and so the initial velocity would have to be somewhat larger than that given to reach the same height.

**Solution for (b)**

As in many physics problems, there is more than one way to solve for the time to the highest point. In this case, the easiest method is to use  $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$ . Because  $y_0$  is zero, this equation reduces to simply

**Equation:**

$$y = \frac{1}{2}(v_{0y} + v_y)t.$$

Note that the final vertical velocity,  $v_y$ , at the highest point is zero. Thus,

**Equation:**

$$\begin{aligned} t &= \frac{2y}{(v_{0y} + v_y)} = \frac{2(233 \text{ m})}{(67.6 \text{ m/s})} \\ &= 6.90 \text{ s}. \end{aligned}$$

**Discussion for (b)**

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. (Another way of finding the time is by using  $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ , and solving the quadratic equation for  $t$ .)

**Solution for (c)**

Because air resistance is negligible,  $a_x = 0$  and the horizontal velocity is constant, as discussed above. The horizontal displacement is horizontal velocity multiplied by time as given by  $x = x_0 + v_x t$ , where  $x_0$  is equal to zero:

**Equation:**

$$x = v_x t,$$

where  $v_x$  is the x-component of the velocity, which is given by  $v_x = v_0 \cos \theta_0$ . Now,

**Equation:**

$$v_x = v_0 \cos \theta_0 = (70.0 \text{ m/s})(\cos 75.0^\circ) = 18.1 \text{ m/s}.$$

The time  $t$  for both motions is the same, and so  $x$  is

**Equation:**

$$x = (18.1 \text{ m/s})(6.90 \text{ s}) = 125 \text{ m}.$$

**Discussion for (c)**

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below.

In solving part (a) of the preceding example, the expression we found for  $y$  is valid for any projectile motion where air resistance is negligible. Call the maximum height  $y = h$ ; then,

**Equation:**

$$h = \frac{v_{0y}^2}{2g}.$$

This equation defines the *maximum height of a projectile* and depends only on the vertical component of the initial velocity.

**Note:**

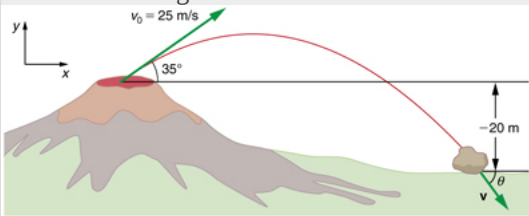
**Defining a Coordinate System**

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the  $x$  and  $y$  positions. Often, it is convenient to choose the initial position of the object as the origin such that  $x_0 = 0$  and  $y_0 = 0$ . It is also important to define the positive and negative directions in the  $x$  and  $y$  directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration,  $g$ , takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case,  $g$  takes a positive value.

**Example:**

**Calculating Projectile Motion: Hot Rock Projectile**

Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle  $35.0^\circ$  above the horizontal, as shown in [link](#). The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact?



The trajectory of a rock ejected from the Kilauea volcano.

**Strategy**

Again, resolving this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the desired quantities. The time a projectile is in the air is governed by its vertical motion alone. We will solve for  $t$  first. While the rock is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, the vertical and horizontal results will be recombined to obtain  $v$  and  $\theta_v$  at the final time  $t$  determined in the first part of the example.

**Solution for (a)**

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

**Equation:**

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2.$$

If we take the initial position  $y_0$  to be zero, then the final position is  $y = -20.0$  m. Now the initial vertical velocity is the vertical component of the initial velocity, found from  $v_{0y} = v_0 \sin \theta_0 = (25.0 \text{ m/s})(\sin 35.0^\circ) = 14.3 \text{ m/s}$ . Substituting known values yields

**Equation:**

$$-20.0 \text{ m} = (14.3 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2.$$

Rearranging terms gives a quadratic equation in  $t$ :

**Equation:**

$$(4.90 \text{ m/s}^2)t^2 - (14.3 \text{ m/s})t - (20.0 \text{ m}) = 0.$$

This expression is a quadratic equation of the form  $at^2 + bt + c = 0$ , where the constants are  $a = 4.90$ ,  $b = -14.3$ , and  $c = -20.0$ . Its solutions are given by the quadratic formula:

**Equation:**

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This equation yields two solutions:  $t = 3.96$  and  $t = -1.03$ . (It is left as an exercise for the reader to verify these solutions.) The time is  $t = 3.96$  s or  $-1.03$  s. The negative value of time implies an event before the start of motion, and so we discard it. Thus,

**Equation:**

$$t = 3.96 \text{ s}.$$

#### Discussion for (a)

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3 m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

#### Solution for (b)

From the information now in hand, we can find the final horizontal and vertical velocities  $v_x$  and  $v_y$  and combine them to find the total velocity  $v$  and the angle  $\theta_0$  it makes with the horizontal. Of course,  $v_x$  is constant so we can solve for it at any horizontal location. In this case, we chose the starting point since we know both the initial velocity and initial angle. Therefore:

**Equation:**

$$v_x = v_0 \cos \theta_0 = (25.0 \text{ m/s})(\cos 35^\circ) = 20.5 \text{ m/s}.$$

The final vertical velocity is given by the following equation:

**Equation:**

$$v_y = v_{0y} - gt,$$

where  $v_{0y}$  was found in part (a) to be 14.3 m/s. Thus,

**Equation:**

$$v_y = 14.3 \text{ m/s} - (9.80 \text{ m/s}^2)(3.96 \text{ s})$$

so that

**Equation:**

$$v_y = -24.5 \text{ m/s}.$$

To find the magnitude of the final velocity  $v$  we combine its perpendicular components, using the following equation:

**Equation:**

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20.5 \text{ m/s})^2 + (-24.5 \text{ m/s})^2},$$

which gives

**Equation:**

$$v = 31.9 \text{ m/s}.$$

The direction  $\theta_v$  is found from the equation:

**Equation:**

$$\theta_v = \tan^{-1}(v_y/v_x)$$

so that

**Equation:**

$$\theta_v = \tan^{-1}(-24.5/20.5) = \tan^{-1}(-1.19).$$

Thus,

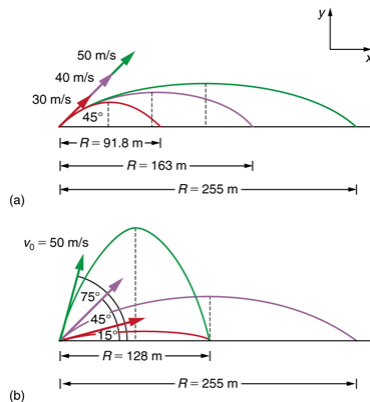
**Equation:**

$$\theta_v = -50.1^\circ.$$

#### Discussion for (b)

The negative angle means that the velocity is  $50.1^\circ$  below the horizontal. This result is consistent with the fact that the final vertical velocity is negative and hence downward—as you would expect because the final altitude is 20.0 m lower than the initial altitude. (See [\[link\]](#).)

One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. Galileo was the first person to fully comprehend this characteristic. He used it to predict the range of a projectile. On level ground, we define **range** to be the horizontal distance  $R$  traveled by a projectile. Galileo and many others were interested in the range of projectiles primarily for military purposes—such as aiming cannons. However, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.



Trajectories of projectiles on level ground. (a) The greater the initial speed  $v_0$ , the greater the range for a given initial angle. (b) The effect of initial angle  $\theta_0$  on the range of a projectile with a

given initial speed. Note that the range is the same for  $15^\circ$  and  $75^\circ$ , although the maximum heights of those paths are different.

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed  $v_0$ , the greater the range, as shown in [\[link\]](#)(a). The initial angle  $\theta_0$  also has a dramatic effect on the range, as illustrated in [\[link\]](#)(b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with  $\theta_0 = 45^\circ$ . This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately  $38^\circ$ . Interestingly, for every initial angle except  $45^\circ$ , there are two angles that give the same range—the sum of those angles is  $90^\circ$ . The range also depends on the value of the acceleration of gravity  $g$ . The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range  $R$  of a projectile on *level ground* for which air resistance is negligible is given by

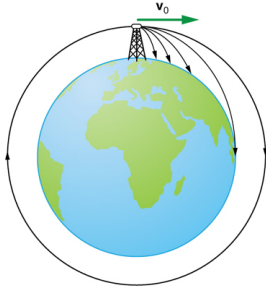
**Equation:**

$$R = \frac{v_0^2 \sin 2\theta_0}{g},$$

where  $v_0$  is the initial speed and  $\theta_0$  is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that  $R$  is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See [\[link\]](#).) If the initial speed is great enough, the projectile goes into orbit. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.

Once again we see that thinking about one topic, such as the range of a projectile, can lead us to others, such as the Earth orbits. In [Addition of Velocities](#), we will examine the addition of velocities, which is another important aspect of two-dimensional kinematics and will also yield insights beyond the immediate topic.



Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the

range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.

**Note:**

**PhET Explorations: Projectile Motion**

Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target.

[https://phet.colorado.edu/sims/projectile-motion/projectile-motion\\_en.html](https://phet.colorado.edu/sims/projectile-motion/projectile-motion_en.html)

## Summary

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:

Determine a coordinate system. Then, resolve the position and/or velocity of the object in the horizontal and vertical components.

$s$  are given  $x$  and  $y$ , and the  $\mathbf{v}$  are  $v_x = v \cos \theta$  and  $v_y = v \sin \theta$ , where  $v$  is the magnitude of the velocity and  $\theta$  is its direction.

The components of position

Analyze the motion of the projectile in the horizontal direction using the following equations:

**Equation:**

Horizontal motion ( $a_x = 0$ )

**Equation:**

$$x = x_0 + v_x t$$

**Equation:**

$v_x = v_{0x} = \mathbf{v}_x = \text{velocity is a constant.}$

Analyze the motion of

**Equation:**

**Equation:**

Vertical motion (Assuming positive direction is up;  $a_y = -g = -9.80 \text{ m/s}^2$ )

$$y = y_0 + \frac{1}{2}(v_{0y} t + a_y t^2)$$

the projectile in the vertical

direction  
using the  
following  
equations:

Recombine the  
horizontal and  
vertical components  
of location and/or  
velocity using the  
following equations:

**Equation:**

$$s = \sqrt{x^2 + y^2}$$

**Equation:**

$$\theta = \tan^{-1}(y/x)$$

**Equation:**

$$v = \sqrt{v_x^2 + v_y^2}$$

**Equation:**

$$\theta_v = \tan^{-1}(v_y/v_x).$$

- The maximum height  $h$  of a projectile launched with initial vertical velocity  $v_{0y}$  is given by

**Equation:**

$$h = \frac{v_{0y}^2}{2g}.$$

- The maximum horizontal distance traveled by a projectile is called the **range**. The range  $R$  of a projectile on level ground launched at an angle  $\theta_0$  above the horizontal with initial speed  $v_0$  is given by

**Equation:**

$$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

## Conceptual Questions

**Exercise:**

**Problem:**

Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither  $0^\circ$  nor  $90^\circ$ ): (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than at  $t = 0$ ? (d) Can the speed ever be the same as the initial speed at a time other than at  $t = 0$ ?

**Exercise:**

**Problem:**

Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither  $0^\circ$  nor  $90^\circ$ ): (a) Is the acceleration ever zero? (b) Is the acceleration ever in the same direction as a component of velocity? (c) Is the acceleration ever opposite in direction to a component of velocity?

**Exercise:**

**Problem:**

For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?

**Exercise:**

**Problem:**

During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

**Problems & Exercises****Exercise:****Problem:**

A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of 30.0° above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the  $x$  and  $y$  distances from where the projectile was launched to where it lands?

---

**Solution:**

$$x = 1.30 \text{ m} \times 10^2$$

$$y = 30.9 \text{ m}.$$

**Exercise:****Problem:**

A ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction. (a) At what speed does the ball hit the ground? (b) For how long does the ball remain in the air? (c) What maximum height is attained by the ball?

**Exercise:****Problem:**

A ball is thrown horizontally from the top of a 60.0-m building and lands 100.0 m from the base of the building. Ignore air resistance. (a) How long is the ball in the air? (b) What must have been the initial horizontal component of the velocity? (c) What is the vertical component of the velocity just before the ball hits the ground? (d) What is the velocity (including both the horizontal and vertical components) of the ball just before it hits the ground?

---

**Solution:**

(a) 3.50 s

(b) 28.6 m/s (c) 34.3 m/s

(d) 44.7 m/s, 50.2° below horizontal

**Exercise:****Problem:**

(a) A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up a 32° ramp at a speed of 40.0 m/s (144 km/h). How many buses can he clear if the top of the takeoff ramp is at the same height as the bus tops and the buses are 20.0 m long? (b) Discuss what your answer implies about the margin of error in this act—that is, consider how much greater the range is than the horizontal distance he must travel to miss the end of the last bus. (Neglect air resistance.)

**Exercise:**

**Problem:**

An archer shoots an arrow at a 75.0 m distant target; the bull's-eye of the target is at same height as the release height of the arrow. (a) At what angle must the arrow be released to hit the bull's-eye if its initial speed is 35.0 m/s? In this part of the problem, explicitly show how you follow the steps involved in solving projectile motion problems. (b) There is a large tree halfway between the archer and the target with an overhanging horizontal branch 3.50 m above the release height of the arrow. Will the arrow go over or under the branch?

---

**Solution:**

(a)  $18.4^\circ$

(b) The arrow will go over the branch.

**Exercise:****Problem:**

A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand. (a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used? (b) What other angle gives the same range, and why would it not be used? (c) How long did this pass take?

**Exercise:**

**Problem:** Verify the ranges for the projectiles in [\[link\]](#)(a) for  $\theta = 45^\circ$  and the given initial velocities.

---

**Solution:**

$$R = \frac{v_0^2}{\sin 2\theta_0 g}$$

$$\text{For } \theta = 45^\circ, \quad R = \frac{v_0^2}{g}$$

$$R = 91.8 \text{ m for } v_0 = 30 \text{ m/s}; \quad R = 163 \text{ m for } v_0 = 40 \text{ m/s}; \quad R = 255 \text{ m for } v_0 = 50 \text{ m/s}.$$

**Exercise:****Problem:**

Verify the ranges shown for the projectiles in [\[link\]](#)(b) for an initial velocity of 50 m/s at the given initial angles.

**Exercise:****Problem:**

The cannon on a battleship can fire a shell a maximum distance of 32.0 km. (a) Calculate the initial velocity of the shell. (b) What maximum height does it reach? (At its highest, the shell is above 60% of the atmosphere—but air resistance is not really negligible as assumed to make this problem easier.) (c) The ocean is not flat, because the Earth is curved. Assume that the radius of the Earth is  $6.37 \times 10^3$  km. How many meters lower will its surface be 32.0 km from the ship along a horizontal line parallel to the surface at the ship? Does your answer imply that error introduced by the assumption of a flat Earth in projectile motion is significant here?

---

**Solution:**

(a) 560 m/s

(b)  $8.00 \times 10^3$  m

(c) 80.0 m. This error is not significant because it is only 1% of the answer in part (b).

**Exercise:**

**Problem:**

An arrow is shot from a height of 1.5 m toward a cliff of height  $H$ . It is shot with a velocity of 30 m/s at an angle of  $60^\circ$  above the horizontal. It lands on the top edge of the cliff 4.0 s later. (a) What is the height of the cliff? (b) What is the maximum height reached by the arrow along its trajectory? (c) What is the arrow's impact speed just before hitting the cliff?

**Exercise:**

**Problem:**

In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.600 m and the acceleration achieved from this position is 1.25 times the acceleration due to gravity,  $g$ . How far can they jump? State your assumptions. (Increased range can be achieved by swinging the arms in the direction of the jump.)

---

**Solution:**

1.50 m, assuming launch angle of  $45^\circ$

**Exercise:**

**Problem:**

The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.

**Exercise:**

**Problem:**

Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle  $\theta$  below the horizontal. The base line is 11.9 m from the net, which is 0.91 m high. What is the angle  $\theta$  such that the ball just crosses the net? Will the ball land in the service box, whose service line is 6.40 m from the net?

---

**Solution:**

$$\theta = 6.1^\circ$$

yes, the ball lands at 5.3 m from the net

**Exercise:**

**Problem:**

A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. (a) If the ball is thrown at an angle of  $25^\circ$  relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground? (b) How long does it take to get to the receiver? (c) What is its maximum height above its point of release?

**Exercise:**

**Problem:**

Gun sights are adjusted to aim high to compensate for the effect of gravity, effectively making the gun accurate only for a specific range. (a) If a gun is sighted to hit targets that are at the same height as the gun and 100.0 m away, how low will the bullet hit if aimed directly at a target 150.0 m away? The muzzle velocity of the bullet is 275 m/s. (b) Discuss qualitatively how a larger muzzle velocity would affect this problem and what would be the effect of air resistance.

---

**Solution:**

(a)  $-0.486\text{ m}$

(b) The larger the muzzle velocity, the smaller the deviation in the vertical direction, because the time of flight would be smaller. Air resistance would have the effect of decreasing the time of flight, therefore increasing the vertical deviation.

**Exercise:****Problem:**

An eagle is flying horizontally at a speed of  $3.00\text{ m/s}$  when the fish in her talons wiggles loose and falls into the lake  $5.00\text{ m}$  below. Calculate the velocity of the fish relative to the water when it hits the water.

**Exercise:****Problem:**

An owl is carrying a mouse to the chicks in its nest. Its position at that time is  $4.00\text{ m}$  west and  $12.0\text{ m}$  above the center of the  $30.0\text{ cm}$  diameter nest. The owl is flying east at  $3.50\text{ m/s}$  at an angle  $30.0^\circ$  below the horizontal when it accidentally drops the mouse. Is the owl lucky enough to have the mouse hit the nest? To answer this question, calculate the horizontal position of the mouse when it has fallen  $12.0\text{ m}$ .

---

**Solution:**

$4.23\text{ m}$ . No, the owl is not lucky; he misses the nest.

**Exercise:****Problem:**

Suppose a soccer player kicks the ball from a distance  $30\text{ m}$  toward the goal. Find the initial speed of the ball if it just passes over the goal,  $2.4\text{ m}$  above the ground, given the initial direction to be  $40^\circ$  above the horizontal.

**Exercise:****Problem:**

Can a goalkeeper at her/ his goal kick a soccer ball into the opponent's goal without the ball touching the ground? The distance will be about  $95\text{ m}$ . A goalkeeper can give the ball a speed of  $30\text{ m/s}$ .

---

**Solution:**

No, the maximum range (neglecting air resistance) is about  $92\text{ m}$ .

**Exercise:****Problem:**

The free throw line in basketball is  $4.57\text{ m}$  ( $15\text{ ft}$ ) from the basket, which is  $3.05\text{ m}$  ( $10\text{ ft}$ ) above the floor. A player standing on the free throw line throws the ball with an initial speed of  $8.15\text{ m/s}$ , releasing it at a height of  $2.44\text{ m}$  ( $8\text{ ft}$ ) above the floor. At what angle above the horizontal must the ball be thrown to exactly hit the basket? Note that most players will use a large initial angle rather than a flat shot because it allows for a larger margin of error. Explicitly show how you follow the steps involved in solving projectile motion problems.

**Exercise:**

**Problem:**

In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw it at an angle of  $38.0^\circ$  above the horizontal? (Although the maximum distance for a projectile on level ground is achieved at  $45^\circ$  when air resistance is neglected, the actual angle to achieve maximum range is smaller; thus,  $38^\circ$  will give a longer range than  $45^\circ$  in the shot put.)

**Solution:**

15.0 m/s

**Exercise:****Problem:**

A basketball player is running at 5.00 m/s directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise 0.750 m above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?

**Exercise:****Problem:**

A football player punts the ball at a  $45.0^\circ$  angle. Without an effect from the wind, the ball would travel 60.0 m horizontally. (a) What is the initial speed of the ball? (b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?

**Solution:**

(a) 24.2 m/s

(b) The ball travels a total of 57.4 m with the brief gust of wind.

**Exercise:****Problem:**

Prove that the trajectory of a projectile is parabolic, having the form  $y = ax + bx^2$ . To obtain this expression, solve the equation  $x = v_{0x}t$  for  $t$  and substitute it into the expression for  $y = v_{0y}t - (1/2)gt^2$  (These equations describe the  $x$  and  $y$  positions of a projectile that starts at the origin.) You should obtain an equation of the form  $y = ax + bx^2$  where  $a$  and  $b$  are constants.

**Exercise:****Problem:**

Derive  $R = \frac{v_0^2 \sin 2\theta_0}{g}$  for the range of a projectile on level ground by finding the time  $t$  at which  $y$  becomes zero and substituting this value of  $t$  into the expression for  $x - x_0$ , noting that  $R = x - x_0$

**Solution:**

$$y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin \theta)t - \frac{1}{2}gt^2,$$

$$\text{so that } t = \frac{2(v_0 \sin \theta)}{g}$$

$x - x_0 = v_{0x}t = (v_0 \cos \theta)t = R$ , and substituting for  $t$  gives:

$$R = v_0 \cos \theta \left( \frac{2v_0 \sin \theta}{g} \right) = \frac{2v_0^2 \sin \theta \cos \theta}{g}$$

since  $2 \sin \theta \cos \theta = \sin 2\theta$ , the range is:

$$R = \frac{v_0^2 \sin 2\theta}{g}.$$

### Exercise:

#### Problem:

**Unreasonable Results** (a) Find the maximum range of a super cannon that has a muzzle velocity of 4.0 km/s. (b) What is unreasonable about the range you found? (c) Is the premise unreasonable or is the available equation inapplicable? Explain your answer. (d) If such a muzzle velocity could be obtained, discuss the effects of air resistance, thinning air with altitude, and the curvature of the Earth on the range of the super cannon.

### Exercise:

#### Problem:

**Construct Your Own Problem** Consider a ball tossed over a fence. Construct a problem in which you calculate the ball's needed initial velocity to just clear the fence. Among the things to determine are; the height of the fence, the distance to the fence from the point of release of the ball, and the height at which the ball is released. You should also consider whether it is possible to choose the initial speed for the ball and just calculate the angle at which it is thrown. Also examine the possibility of multiple solutions given the distances and heights you have chosen.

## Glossary

air resistance

a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

kinematics

the study of motion without regard to mass or force

motion

displacement of an object as a function of time

projectile

an object that travels through the air and experiences only acceleration due to gravity

projectile motion

the motion of an object that is subject only to the acceleration of gravity

range

the maximum horizontal distance that a projectile travels

trajectory

the path of a projectile through the air

## Introduction to Linear Momentum and Collisions

class="introduction"

"Each  
rugby  
player has  
great  
momentum  
, which will  
affect the  
outcome of  
their  
collisions  
with each  
other and  
the ground.  
(credit:  
vjpaul,  
Flickr)"



We use the term momentum in various ways in everyday language, and most of these ways are consistent with its precise scientific definition. We speak of sports teams or politicians gaining and maintaining the momentum to win. We also recognize that momentum has something to do with collisions. For example, looking at the rugby players in the photograph colliding and falling to the ground, we expect their momenta to have great effects in the resulting collisions. Generally, momentum implies a tendency to continue on course—to move in the same direction—and is associated with great mass and speed.

Momentum, like energy, is important because it is conserved. Only a few physical quantities are conserved in nature, and studying them yields fundamental insight into how nature works, as we shall see in our study of momentum.

## Linear Momentum and Force

- Define linear momentum.
- Explain the relationship between momentum and force.
- State Newton's second law of motion in terms of momentum.
- Calculate momentum given mass and velocity.

### Linear Momentum

The scientific definition of linear momentum is consistent with most people's intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. **Linear momentum** is defined as the product of a system's mass multiplied by its velocity. In symbols, linear momentum is expressed as

**Equation:**

$$\mathbf{p} = m\mathbf{v}.$$

Momentum is directly proportional to the object's mass and also its velocity. Thus the greater an object's mass or the greater its velocity, the greater its momentum. Momentum  $\mathbf{p}$  is a vector having the same direction as the velocity  $\mathbf{v}$ . The SI unit for momentum is  $\text{kg} \cdot \text{m/s}$ .

**Note:**

Linear Momentum

Linear momentum is defined as the product of a system's mass multiplied by its velocity:

**Equation:**

$$\mathbf{p} = m\mathbf{v}.$$

**Example:**

### Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

#### Strategy

No information is given regarding direction, and so we can calculate only the magnitude of the momentum,  $p$ . (As usual, a symbol that is in italics is a magnitude, whereas one that is italicized, boldfaced, and has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum given in the equation, which becomes

#### Equation:

$$p = mv$$

when only magnitudes are considered.

#### Solution for (a)

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation.

#### Equation:

$$p_{\text{player}} = (110 \text{ kg})(8.00 \text{ m/s}) = 880 \text{ kg} \cdot \text{m/s}$$

#### Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

#### Equation:

$$p_{\text{ball}} = (0.410 \text{ kg})(25.0 \text{ m/s}) = 10.3 \text{ kg} \cdot \text{m/s}$$

The ratio of the player's momentum to that of the ball is

#### Equation:

$$\frac{p_{\text{player}}}{p_{\text{ball}}} = \frac{880}{10.3} = 85.9.$$

#### Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

## Momentum and Newton's Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the “quantity of motion.” Newton actually stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

**Equation:**

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t},$$

where  $\mathbf{F}_{\text{net}}$  is the net external force,  $\Delta \mathbf{p}$  is the change in momentum, and  $\Delta t$  is the change in time.

**Note:**

Newton's Second Law of Motion in Terms of Momentum

The net external force equals the change in momentum of a system divided by the time over which it changes.

**Equation:**

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$$

**Note:****Making Connections: Force and Momentum**

Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in its most broadly applicable form in terms of momentum. Momentum continues to be a key concept in the study of atomic and subatomic particles in quantum mechanics.

This statement of Newton's second law of motion includes the more familiar  $\mathbf{F}_{\text{net}} = m\mathbf{a}$  as a special case. We can derive this form as follows. First, note that the change in momentum  $\Delta\mathbf{p}$  is given by

**Equation:**

$$\Delta\mathbf{p} = \Delta(m\mathbf{v}).$$

If the mass of the system is constant, then

**Equation:**

$$\Delta(m\mathbf{v}) = m\Delta\mathbf{v}.$$

So that for constant mass, Newton's second law of motion becomes

**Equation:**

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t} = \frac{m\Delta\mathbf{v}}{\Delta t}.$$

Because  $\frac{\Delta\mathbf{v}}{\Delta t} = \mathbf{a}$ , we get the familiar equation

**Equation:**

$$\mathbf{F}_{\text{net}} = m\mathbf{a}$$

*when the mass of the system is constant.*

Newton's second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass. We will consider systems with varying mass in some detail; however, the relationship between momentum and force remains useful when mass is constant, such as in the following example.

**Example:**

**Calculating Force: Venus Williams' Racquet**

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

**Strategy**

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as

**Equation:**

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}.$$

As noted above, when mass is constant, the change in momentum is given by

**Equation:**

$$\Delta p = m\Delta v = m(v_f - v_i).$$

In this example, the velocity just after impact and the change in time are given; thus, once  $\Delta p$  is calculated,  $F_{\text{net}} = \frac{\Delta p}{\Delta t}$  can be used to find the force.

**Solution**

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

**Equation:**

$$\begin{aligned}\Delta p &= m(v_f - v_i) \\ &= (0.057 \text{ kg})(58 \text{ m/s} - 0 \text{ m/s}) \\ &= 3.306 \text{ kg} \cdot \text{m/s} \approx 3.3 \text{ kg} \cdot \text{m/s}\end{aligned}$$

Now the magnitude of the net external force can be determined by using

$$F_{\text{net}} = \frac{\Delta p}{\Delta t}:$$

**Equation:**

$$\begin{aligned}F_{\text{net}} &= \frac{\Delta p}{\Delta t} = \frac{3.306 \text{ kg} \cdot \text{m/s}}{5.0 \times 10^{-3} \text{ s}} \\ &= 661 \text{ N} \approx 660 \text{ N},\end{aligned}$$

where we have retained only two significant figures in the final step.

### Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using  $F_{\text{net}} = ma$ , but one additional step would be required compared with the strategy used in this example.

## Section Summary

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum **p** is defined to be

**Equation:**

$$\mathbf{p} = m\mathbf{v},$$

where  $m$  is the mass of the system and  $\mathbf{v}$  is its velocity.

- The SI unit for momentum is  $\text{kg} \cdot \text{m/s}$ .

- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be

**Equation:**

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t},$$

$\mathbf{F}_{\text{net}}$  is the net external force,  $\Delta \mathbf{p}$  is the change in momentum, and  $\Delta t$  is the change time.

## Conceptual Questions

**Exercise:**

**Problem:**

An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

**Exercise:**

**Problem:**

An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

**Exercise:**

**Problem: Professional Application**

Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

**Exercise:**

**Problem:**

How can a small force impart the same momentum to an object as a large force?

**Problems & Exercises****Exercise:****Problem:**

(a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of 7.50 m/s. (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of 600 m/s. (c) What is the momentum of the 90.0-kg hunter running at 7.40 m/s after missing the elephant?

---

**Solution:**

(a)  $1.50 \times 10^4 \text{ kg} \cdot \text{m/s}$

(b) 625 to 1

(c)  $6.66 \times 10^2 \text{ kg} \cdot \text{m/s}$

**Exercise:****Problem:**

(a) What is the mass of a large ship that has a momentum of  $1.60 \times 10^9 \text{ kg} \cdot \text{m/s}$ , when the ship is moving at a speed of 48.0 km/h? (b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of 1200 m/s.

**Exercise:**

**Problem:**

(a) At what speed would a  $2.00 \times 10^4$ -kg airplane have to fly to have a momentum of  $1.60 \times 10^9 \text{ kg} \cdot \text{m/s}$  (the same as the ship's momentum in the problem above)? (b) What is the plane's momentum when it is taking off at a speed of  $60.0 \text{ m/s}$ ? (c) If the ship is an aircraft carrier that launches these airplanes with a catapult, discuss the implications of your answer to (b) as it relates to recoil effects of the catapult on the ship.

---

**Solution:**

(a)  $8.00 \times 10^4 \text{ m/s}$

(b)  $1.20 \times 10^6 \text{ kg} \cdot \text{m/s}$

(c) Because the momentum of the airplane is 3 orders of magnitude smaller than of the ship, the ship will not recoil very much. The recoil would be  $-0.0100 \text{ m/s}$ , which is probably not noticeable.

**Exercise:****Problem:**

(a) What is the momentum of a garbage truck that is  $1.20 \times 10^4 \text{ kg}$  and is moving at  $10.0 \text{ m/s}$ ? (b) At what speed would an  $8.00$ -kg trash can have the same momentum as the truck?

**Exercise:****Problem:**

A runaway train car that has a mass of  $15,000 \text{ kg}$  travels at a speed of  $5.4 \text{ m/s}$  down a track. Compute the time required for a force of  $1500 \text{ N}$  to bring the car to rest.

---

**Solution:**

$54 \text{ s}$

**Exercise:****Problem:**

The mass of Earth is  $5.972 \times 10^{24}$  kg and its orbital radius is an average of  $1.496 \times 10^{11}$  m. Calculate its linear momentum.

**Glossary**

linear momentum

the product of mass and velocity

second law of motion

physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

## Impulse

- Define impulse.
- Describe effects of impulses in everyday life.
- Determine the average effective force using graphical representation.
- Calculate average force and impulse given mass, velocity, and time.

The effect of a force on an object depends on how long it acts, as well as how great the force is. In [\[link\]](#), a very large force acting for a short time had a great effect on the momentum of the tennis ball. A small force could cause the same **change in momentum**, but it would have to act for a much longer time. For example, if the ball were thrown upward, the gravitational force (which is much smaller than the tennis racquet's force) would eventually reverse the momentum of the ball. Quantitatively, the effect we are talking about is the change in momentum  $\Delta \mathbf{p}$ .

By rearranging the equation  $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$  to be

**Equation:**

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t,$$

we can see how the change in momentum equals the average net external force multiplied by the time this force acts. The quantity  $\mathbf{F}_{\text{net}} \Delta t$  is given the name **impulse**. Impulse is the same as the change in momentum.

### **Note:**

**Impulse: Change in Momentum**

Change in momentum equals the average net external force multiplied by the time this force acts.

**Equation:**

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t$$

The quantity  $\mathbf{F}_{\text{net}} \Delta t$  is given the name impulse.

There are many ways in which an understanding of impulse can save lives, or at least limbs. The dashboard padding in a car, and certainly the airbags, allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant, whether an air bag is deployed or not, but the force (to bring the occupant to a stop) will be much less if it acts over a larger time. Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

Bones in a body will fracture if the force on them is too large. If you jump onto the floor from a table, the force on your legs can be immense if you land stiff-legged on a hard surface. Rolling on the ground after jumping from the table, or landing with a parachute, extends the time over which the force (on you from the ground) acts.

### **Example:**

#### **Calculating Magnitudes of Impulses: Two Billiard Balls Striking a Rigid Wall**

Two identical billiard balls strike a rigid wall with the same speed, and are reflected without any change of speed. The first ball strikes perpendicular to the wall. The second ball strikes the wall at an angle of  $30^\circ$  from the perpendicular, and bounces off at an angle of  $30^\circ$  from perpendicular to the wall.

- (a) Determine the direction of the force on the wall due to each ball.
- (b) Calculate the ratio of the magnitudes of impulses on the two balls by the wall.

#### **Strategy for (a)**

In order to determine the force on the wall, consider the force on the ball due to the wall using Newton's second law and then apply Newton's third law to determine the direction. Assume the  $x$ -axis to be normal to the wall and to be positive in the initial direction of motion. Choose the  $y$ -axis to be

along the wall in the plane of the second ball's motion. The momentum direction and the velocity direction are the same.

**Solution for (a)**

The first ball bounces directly into the wall and exerts a force on it in the  $+x$  direction. Therefore the wall exerts a force on the ball in the  $-x$  direction. The second ball continues with the same momentum component in the  $y$  direction, but reverses its  $x$ -component of momentum, as seen by sketching a diagram of the angles involved and keeping in mind the proportionality between velocity and momentum.

These changes mean the change in momentum for both balls is in the  $-x$  direction, so the force of the wall on each ball is along the  $-x$  direction.

**Strategy for (b)**

Calculate the change in momentum for each ball, which is equal to the impulse imparted to the ball.

**Solution for (b)**

Let  $u$  be the speed of each ball before and after collision with the wall, and  $m$  the mass of each ball. Choose the  $x$ -axis and  $y$ -axis as previously described, and consider the change in momentum of the first ball which strikes perpendicular to the wall.

**Equation:**

$$p_{xi} = mu; p_{yi} = 0$$

**Equation:**

$$p_{xf} = -mu; p_{yf} = 0$$

Impulse is the change in momentum vector. Therefore the  $x$ -component of impulse is equal to  $-2mu$  and the  $y$ -component of impulse is equal to zero.

Now consider the change in momentum of the second ball.

**Equation:**

$$p_{xi} = mu \cos 30^\circ; p_{yi} = -mu \sin 30^\circ$$

**Equation:**

$$p_{xf} = -mu \cos 30^\circ; p_{yf} = -mu \sin 30^\circ$$

It should be noted here that while  $p_x$  changes sign after the collision,  $p_y$  does not. Therefore the  $x$ -component of impulse is equal to  $-2mu \cos 30^\circ$  and the  $y$ -component of impulse is equal to zero.

The ratio of the magnitudes of the impulse imparted to the balls is

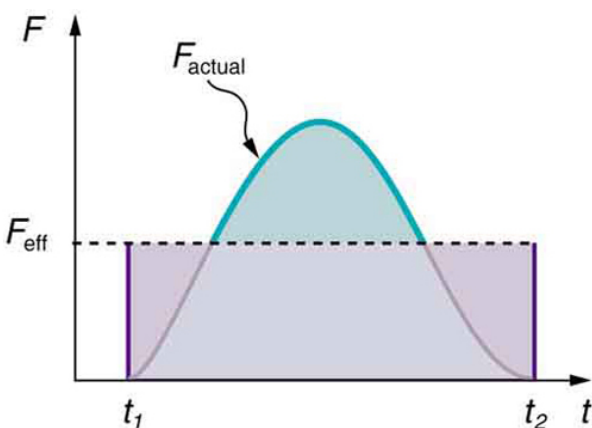
**Equation:**

$$\frac{2mu}{2mu \cos 30^\circ} = \frac{2}{\sqrt{3}} = 1.155.$$

### Discussion

The direction of impulse and force is the same as in the case of (a); it is normal to the wall and along the negative  $x$ -direction. Making use of Newton's third law, the force on the wall due to each ball is normal to the wall along the positive  $x$ -direction.

Our definition of impulse includes an assumption that the force is constant over the time interval  $\Delta t$ . *Forces are usually not constant.* Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force  $F_{\text{eff}}$  that produces the same result as the corresponding time-varying force. [\[link\]](#) shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times  $t_1$  and  $t_2$ . That area is equal to the area inside the rectangle bounded by  $F_{\text{eff}}$ ,  $t_1$ , and  $t_2$ . Thus the impulses and their effects are the same for both the actual and effective forces.



A graph of force versus time with time along the  $x$ -axis and force along the  $y$ -axis for an actual force and an equivalent effective force. The areas under the two curves are equal.

**Note:**

**Making Connections: Take-Home Investigation—Hand Movement and Impulse**

Try catching a ball while “giving” with the ball, pulling your hands toward your body. Then, try catching a ball while keeping your hands still. Hit water in a tub with your full palm. After the water has settled, hit the water again by diving your hand with your fingers first into the water. (Your full palm represents a swimmer doing a belly flop and your diving hand represents a swimmer doing a dive.) Explain what happens in each case and why. Which orientations would you advise people to avoid and why?

**Note:**

**Making Connections: Constant Force and Constant Acceleration**

The assumption of a constant force in the definition of impulse is analogous to the assumption of a constant acceleration in kinematics. In both cases, nature is adequately described without the use of calculus.

## Section Summary

- Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts:

**Equation:**

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t.$$

- Forces are usually not constant over a period of time.

## Conceptual Questions

### Exercise:

#### **Problem: Professional Application**

Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.

### Exercise:

#### **Problem:**

While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?

### Exercise:

#### **Problem: Professional Application**

Tennis racquets have “sweet spots.” If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

## Problems & Exercises

### Exercise:

**Problem:**

A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder. What is the average force exerted on a 0.0300-kg bullet to accelerate it to a speed of 600 m/s in a time of 2.00 ms (milliseconds)?

---

**Solution:**

$$9.00 \times 10^3 \text{ N}$$

**Exercise:****Problem: Professional Application**

A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seat belt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.

**Exercise:****Problem:**

A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s. (a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg? (b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.

---

**Solution:**

a)  $2.40 \times 10^3 \text{ N}$  toward the leg

b) The force on each hand would have the same magnitude as that found in part (a) (but in opposite directions by Newton's third law) because the change in momentum and the time interval are the same.

**Exercise:**

**Problem: Professional Application**

A professional boxer hits his opponent with a 1000-N horizontal blow that lasts for 0.150 s. (a) Calculate the impulse imparted by this blow. (b) What is the opponent's final velocity, if his mass is 105 kg and he is motionless in midair when struck near his center of mass? (c) Calculate the recoil velocity of the opponent's 10.0-kg head if hit in this manner, assuming the head does not initially transfer significant momentum to the boxer's body. (d) Discuss the implications of your answers for parts (b) and (c).

**Exercise:****Problem: Professional Application**

Suppose a child drives a bumper car head on into the side rail, which exerts a force of 4000 N on the car for 0.200 s. (a) What impulse is imparted by this force? (b) Find the final velocity of the bumper car if its initial velocity was 2.80 m/s and the car plus driver have a mass of 200 kg. You may neglect friction between the car and floor.

---

**Solution:**

- a) 800 kg · m/s away from the wall
- b) 1.20 m/s away from the wall

**Exercise:****Problem: Professional Application**

One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of  $4.00 \times 10^3$  m/s, given the collision lasts  $6.00 \times 10^{-8}$  s.

**Exercise:****Problem: Professional Application**

A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment. (a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm. (b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.

---

**Solution:**

(a)  $1.50 \times 10^6$  N away from the dashboard

(b)  $1.00 \times 10^5$  N away from the dashboard

**Exercise:****Problem: Professional Application**

Military rifles have a mechanism for reducing the recoil forces of the gun on the person firing it. An internal part recoils over a relatively large distance and is stopped by damping mechanisms in the gun. The larger distance reduces the average force needed to stop the internal part. (a) Calculate the recoil velocity of a 1.00-kg plunger that directly interacts with a 0.0200-kg bullet fired at 600 m/s from the gun. (b) If this part is stopped over a distance of 20.0 cm, what average force is exerted upon it by the gun? (c) Compare this to the force exerted on the gun if the bullet is accelerated to its velocity in 10.0 ms (milliseconds).

**Exercise:**

**Problem:**

A cruise ship with a mass of  $1.00 \times 10^7$  kg strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)

---

**Solution:**

$4.69 \times 10^5$  N in the boat's original direction of motion

**Exercise:****Problem:**

Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of  $1.76 \times 10^4$  N for  $5.50 \times 10^{-2}$  s.

**Exercise:****Problem:**

Water from a fire hose is directed horizontally against a wall at a rate of 50.0 kg/s and a speed of 42.0 m/s. Calculate the magnitude of the force exerted on the wall, assuming the water's horizontal momentum is reduced to zero.

---

**Solution:**

$2.10 \times 10^3$  N away from the wall

**Exercise:****Problem:**

A 0.450-kg hammer is moving horizontally at 7.00 m/s when it strikes a nail and comes to rest after driving the nail 1.00 cm into a board. (a) Calculate the duration of the impact. (b) What was the average force exerted on the nail?

**Exercise:****Problem:**

Starting with the definitions of momentum and kinetic energy, derive an equation for the kinetic energy of a particle expressed as a function of its momentum.

---

**Solution:****Equation:**

$$\begin{aligned}\mathbf{p} &= m\mathbf{v} \Rightarrow p^2 = m^2v^2 \Rightarrow \frac{p^2}{m} = mv^2 \\ \Rightarrow \frac{p^2}{2m} &= \frac{1}{2}mv^2 = \text{KE} \\ KE &= \frac{p^2}{2m}\end{aligned}$$

**Exercise:****Problem:**

A ball with an initial velocity of 10 m/s moves at an angle  $60^\circ$  above the  $+x$ -direction. The ball hits a vertical wall and bounces off so that it is moving  $60^\circ$  above the  $-x$ -direction with the same speed. What is the impulse delivered by the wall?

**Exercise:****Problem:**

When serving a tennis ball, a player hits the ball when its velocity is zero (at the highest point of a vertical toss). The racquet exerts a force of 540 N on the ball for 5.00 ms, giving it a final velocity of 45.0 m/s. Using these data, find the mass of the ball.

---

**Solution:**

60.0 g

**Exercise:**

**Problem:**

A punter drops a ball from rest vertically 1 meter down onto his foot. The ball leaves the foot with a speed of 18 m/s at an angle  $55^\circ$  above the horizontal. What is the impulse delivered by the foot (magnitude and direction)?

**Glossary**

change in momentum

the difference between the final and initial momentum; the mass times the change in velocity

impulse

the average net external force times the time it acts; equal to the change in momentum

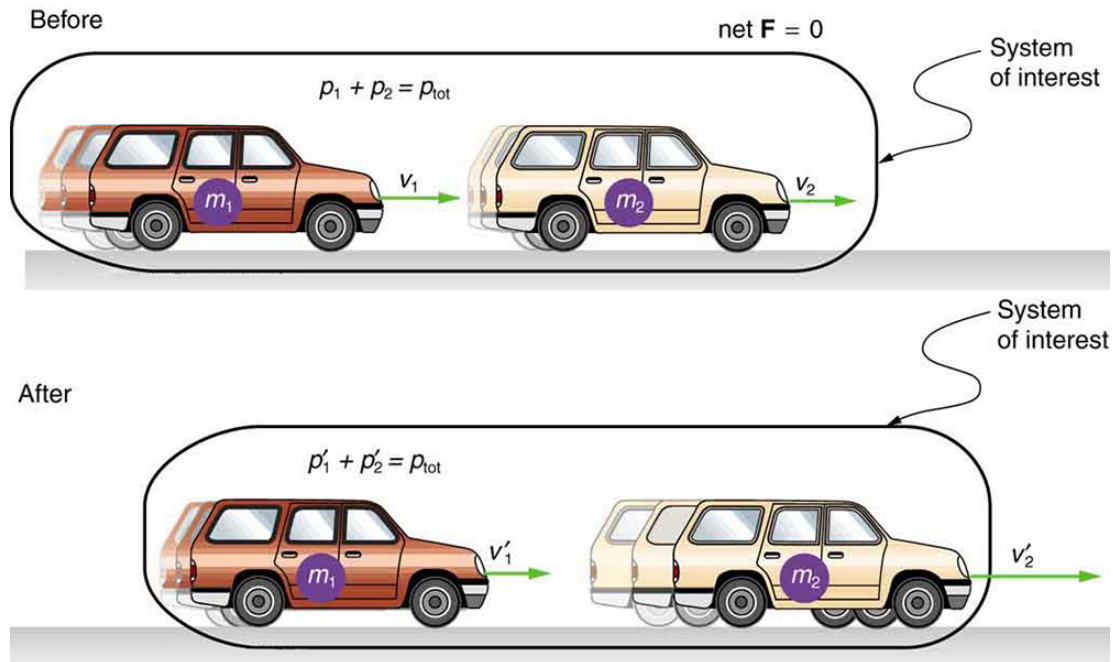
## Conservation of Momentum

- Describe the principle of conservation of momentum.
- Derive an expression for the conservation of momentum.
- Explain conservation of momentum with examples.
- Explain the principle of conservation of momentum as it relates to atomic and subatomic particles.

Momentum is an important quantity because it is conserved. Yet it was not conserved in the examples in [Impulse](#) and [Linear Momentum and Force](#), where large changes in momentum were produced by forces acting on the system of interest. Under what circumstances is momentum conserved?

The answer to this question entails considering a sufficiently large system. It is always possible to find a larger system in which total momentum is constant, even if momentum changes for components of the system. If a football player runs into the goalpost in the end zone, there will be a force on him that causes him to bounce backward. However, the Earth also recoils—conserving momentum—because of the force applied to it through the goalpost. Because Earth is many orders of magnitude more massive than the player, its recoil is immeasurably small and can be neglected in any practical sense, but it is real nevertheless.

Consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—for example, one car bumping into another, as shown in [\[link\]](#). Both cars are coasting in the same direction when the lead car (labeled  $m_2$ ) is bumped by the trailing car (labeled  $m_1$ ). The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.



A car of mass  $m_1$  moving with a velocity of  $v_1$  bumps into another car of mass  $m_2$  and velocity  $v_2$  that it is following. As a result, the first car slows down to a velocity of  $v'_1$  and the second speeds up to a velocity of  $v'_2$ . The momentum of each car is changed, but the total momentum  $p_{\text{tot}}$  of the two cars is the same before and after the collision (if you assume friction is negligible).

Using the definition of impulse, the change in momentum of car 1 is given by

**Equation:**

$$\Delta p_1 = F_1 \Delta t,$$

where  $F_1$  is the force on car 1 due to car 2, and  $\Delta t$  is the time the force acts (the duration of the collision). Intuitively, it seems obvious that the collision time is the same for both cars, but it is only true for objects traveling at ordinary speeds. This assumption must be modified for objects travelling

near the speed of light, without affecting the result that momentum is conserved.

Similarly, the change in momentum of car 2 is

**Equation:**

$$\Delta p_2 = F_2 \Delta t,$$

where  $F_2$  is the force on car 2 due to car 1, and we assume the duration of the collision  $\Delta t$  is the same for both cars. We know from Newton's third law that  $F_2 = -F_1$ , and so

**Equation:**

$$\Delta p_2 = -F_1 \Delta t = -\Delta p_1.$$

Thus, the changes in momentum are equal and opposite, and

**Equation:**

$$\Delta p_1 + \Delta p_2 = 0.$$

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

**Equation:**

$$p_1 + p_2 = \text{constant},$$

**Equation:**

$$p_1 + p_2 = p'_1 + p'_2,$$

where  $p'_1$  and  $p'_2$  are the momenta of cars 1 and 2 after the collision. (We often use primes to denote the final state.)

This result—that momentum is conserved—has validity far beyond the preceding one-dimensional case. It can be similarly shown that total momentum is conserved for any isolated system, with any number of

objects in it. In equation form, the **conservation of momentum principle** for an isolated system is written

**Equation:**

$$\mathbf{p}_{\text{tot}} = \text{constant},$$

or

**Equation:**

$$\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}},$$

where  $\mathbf{p}_{\text{tot}}$  is the total momentum (the sum of the momenta of the individual objects in the system) and  $\mathbf{p}'_{\text{tot}}$  is the total momentum some time later. (The total momentum can be shown to be the momentum of the center of mass of the system.) An **isolated system** is defined to be one for which the net external force is zero ( $\mathbf{F}_{\text{net}} = 0$ ).

**Note:**

Conservation of Momentum Principle

**Equation:**

$$\begin{aligned}\mathbf{p}_{\text{tot}} &= \text{constant} \\ \mathbf{p}_{\text{tot}} &= \mathbf{p}'_{\text{tot}} \text{ (isolated system)}\end{aligned}$$

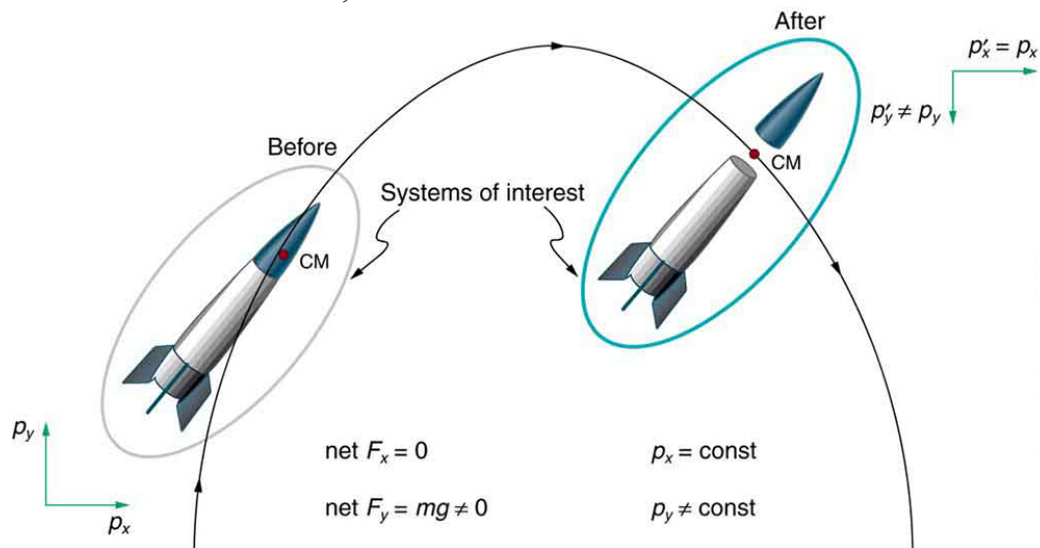
**Note:**

Isolated System

An isolated system is defined to be one for which the net external force is zero ( $\mathbf{F}_{\text{net}} = 0$ ).

Perhaps an easier way to see that momentum is conserved for an isolated system is to consider Newton's second law in terms of momentum,  $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}_{\text{tot}}}{\Delta t}$ . For an isolated system, ( $\mathbf{F}_{\text{net}} = 0$ ); thus,  $\Delta \mathbf{p}_{\text{tot}} = 0$ , and  $\mathbf{p}_{\text{tot}}$  is constant.

We have noted that the three length dimensions in nature— $x$ ,  $y$ , and  $z$ —are independent, and it is interesting to note that momentum can be conserved in different ways along each dimension. For example, during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero and momentum is unchanged. But along the vertical direction, the net vertical force is not zero and the momentum of the projectile is not conserved. (See [\[link\]](#).) However, if the momentum of the projectile-Earth system is considered in the vertical direction, we find that the total momentum is conserved.



The horizontal component of a projectile's momentum is conserved if air resistance is negligible, even in this case where a space probe separates. The forces causing the separation are internal to the system, so that the net external horizontal force  $F_{x-\text{net}}$  is still zero. The vertical component of the momentum is not conserved, because the net vertical force  $F_{y-\text{net}}$  is not zero. In the vertical direction, the space probe-Earth system needs to be considered and we find that the total momentum is conserved. The center of mass of the

space probe takes the same path it would if the separation did not occur.

The conservation of momentum principle can be applied to systems as different as a comet striking Earth and a gas containing huge numbers of atoms and molecules. Conservation of momentum is violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

**Note:**

**Making Connections: Take-Home Investigation—Drop of Tennis Ball and a Basketball**

Hold a tennis ball side by side and in contact with a basketball. Drop the balls together. (Be careful!) What happens? Explain your observations. Now hold the tennis ball above and in contact with the basketball. What happened? Explain your observations. What do you think will happen if the basketball ball is held above and in contact with the tennis ball?

**Note:**

**Making Connections: Take-Home Investigation—Two Tennis Balls in a Ballistic Trajectory**

Tie two tennis balls together with a string about a foot long. Hold one ball and let the other hang down and throw it in a ballistic trajectory. Explain your observations. Now mark the center of the string with bright ink or attach a brightly colored sticker to it and throw again. What happened? Explain your observations.

Some aquatic animals such as jellyfish move around based on the principles of conservation of momentum. A jellyfish fills its umbrella section with water and then pushes the water out resulting in motion in the opposite direction to that of the jet of water. Squids propel themselves in a

similar manner but, in contrast with jellyfish, are able to control the direction in which they move by aiming their nozzle forward or backward. Typical squids can move at speeds of 8 to 12 km/h.

The ballistocardiograph (BCG) was a diagnostic tool used in the second half of the 20th century to study the strength of the heart. About once a second, your heart beats, forcing blood into the aorta. A force in the opposite direction is exerted on the rest of your body (recall Newton's third law). A ballistocardiograph is a device that can measure this reaction force. This measurement is done by using a sensor (resting on the person) or by using a moving table suspended from the ceiling. This technique can gather information on the strength of the heart beat and the volume of blood passing from the heart. However, the electrocardiogram (ECG or EKG) and the echocardiogram (cardiac ECHO or ECHO; a technique that uses ultrasound to see an image of the heart) are more widely used in the practice of cardiology.

**Note:**

**Making Connections: Conservation of Momentum and Collision**

Conservation of momentum is quite useful in describing collisions. Momentum is crucial to our understanding of atomic and subatomic particles because much of what we know about these particles comes from collision experiments.

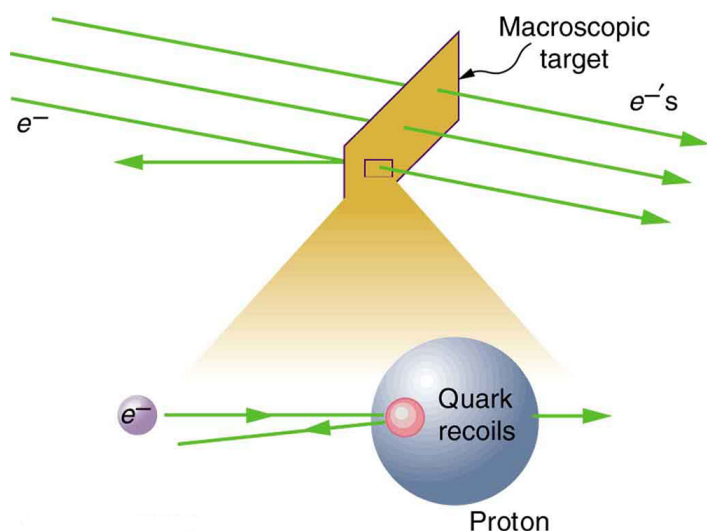
## **Subatomic Collisions and Momentum**

The conservation of momentum principle not only applies to the macroscopic objects, it is also essential to our explorations of atomic and subatomic particles. Giant machines hurl subatomic particles at one another, and researchers evaluate the results by assuming conservation of momentum (among other things).

On the small scale, we find that particles and their properties are invisible to the naked eye but can be measured with our instruments, and models of these subatomic particles can be constructed to describe the results.

Momentum is found to be a property of all subatomic particles including massless particles such as photons that compose light. Momentum being a property of particles hints that momentum may have an identity beyond the description of an object's mass multiplied by the object's velocity. Indeed, momentum relates to wave properties and plays a fundamental role in what measurements are taken and how we take these measurements.

Furthermore, we find that the conservation of momentum principle is valid when considering systems of particles. We use this principle to analyze the masses and other properties of previously undetected particles, such as the nucleus of an atom and the existence of quarks that make up particles of nuclei. [\[link\]](#) below illustrates how a particle scattering backward from another implies that its target is massive and dense. Experiments seeking evidence that **quarks** make up protons (one type of particle that makes up nuclei) scattered high-energy electrons off of protons (nuclei of hydrogen atoms). Electrons occasionally scattered straight backward in a manner that implied a very small and very dense particle makes up the proton—this observation is considered nearly direct evidence of quarks. The analysis was based partly on the same conservation of momentum principle that works so well on the large scale.



A subatomic particle scatters straight backward from a target particle. In experiments seeking evidence for

quarks, electrons were observed to occasionally scatter straight backward from a proton.

## Section Summary

- The conservation of momentum principle is written  
**Equation:**

$$\mathbf{p}_{\text{tot}} = \text{constant}$$

or

**Equation:**

$$\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}} \text{ (isolated system),}$$

$\mathbf{p}_{\text{tot}}$  is the initial total momentum and  $\mathbf{p}'_{\text{tot}}$  is the total momentum some time later.

- An isolated system is defined to be one for which the net external force is zero ( $\mathbf{F}_{\text{net}} = 0$ ).
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero.
- Conservation of momentum applies only when the net external force is zero.
- The conservation of momentum principle is valid when considering systems of particles.

## Conceptual Questions

**Exercise:**

**Problem: Professional Application**

If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.

**Exercise:**

**Problem:** Under what circumstances is momentum conserved?

**Exercise:**

**Problem:**

Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?

**Exercise:**

**Problem:**

Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

**Exercise:**

**Problem: Professional Application**

Explain in terms of momentum and Newton's laws how a car's air resistance is due in part to the fact that it pushes air in its direction of motion.

**Exercise:**

**Problem:**

Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

**Exercise:**

**Problem:**

Must the total energy of a system be conserved whenever its momentum is conserved? Explain why or why not.

**Problems & Exercises****Exercise:****Problem: Professional Application**

Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of  $-0.120$  m/s. (The minus indicates direction of motion.) What is their final velocity?

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**Solution:**

0.122 m/s

**Exercise:****Problem:**

Suppose a clay model of a koala bear has a mass of 0.200 kg and slides on ice at a speed of 0.750 m/s. It runs into another clay model, which is initially motionless and has a mass of 0.350 kg. Both being soft clay, they naturally stick together. What is their final velocity?

**Exercise:****Problem: Professional Application**

Consider the following question: *A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seatbelt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.* Would the answer to this question be different if the car with

the 70-kg passenger had collided with a car that has a mass equal to and is traveling in the opposite direction and at the same speed? Explain your answer.

---

**Solution:**

In a collision with an identical car, momentum is conserved. Afterwards  $v_f = 0$  for both cars. The change in momentum will be the same as in the crash with the tree. However, the force on the body is not determined since the time is not known. A padded stop will reduce injurious force on body.

**Exercise:**

**Problem:**

What is the velocity of a 900-kg car initially moving at 30.0 m/s, just after it hits a 150-kg deer initially running at 12.0 m/s in the same direction? Assume the deer remains on the car.

**Exercise:**

**Problem:**

A 1.80-kg falcon catches a 0.650-kg dove from behind in midair. What is their velocity after impact if the falcon's velocity is initially 28.0 m/s and the dove's velocity is 7.00 m/s in the same direction?

---

**Solution:**

22.4 m/s in the same direction as the original motion

## Glossary

conservation of momentum principle

when the net external force is zero, the total momentum of the system is conserved or constant

isolated system

a system in which the net external force is zero

quark

fundamental constituent of matter and an elementary particle

## Inelastic Collisions in One Dimension

- Define inelastic collision.
- Explain perfectly inelastic collision.
- Apply an understanding of collisions to sports.
- Determine recoil velocity and loss in kinetic energy given mass and initial velocity.

We have seen that in an elastic collision, internal kinetic energy is conserved. An **inelastic collision** is one in which the internal kinetic energy changes (it is not conserved). This lack of conservation means that the forces between colliding objects may remove or add internal kinetic energy. Work done by internal forces may change the forms of energy within a system. For inelastic collisions, such as when colliding objects stick together, this internal work may transform some internal kinetic energy into heat transfer. Or it may convert stored energy into internal kinetic energy, such as when exploding bolts separate a satellite from its launch vehicle.

### Note:

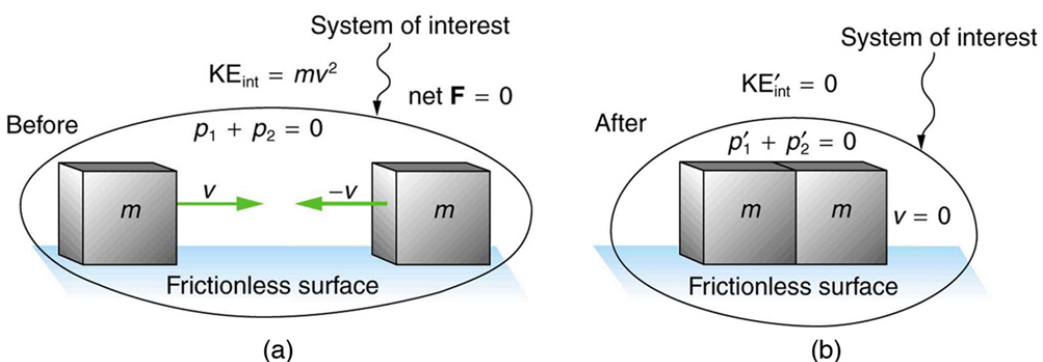
#### Inelastic Collision

An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).

[\[link\]](#) shows an example of an inelastic collision. Two objects that have equal masses head toward one another at equal speeds and then stick together. Their total internal kinetic energy is initially  $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$ . The two objects come to rest after sticking together, conserving momentum. But the internal kinetic energy is zero after the collision. A collision in which the objects stick together is sometimes called a **perfectly inelastic collision** because it reduces internal kinetic energy more than does any other type of inelastic collision. In fact, such a collision reduces internal kinetic energy to the minimum it can have while still conserving momentum.

**Note:****Perfectly Inelastic Collision**

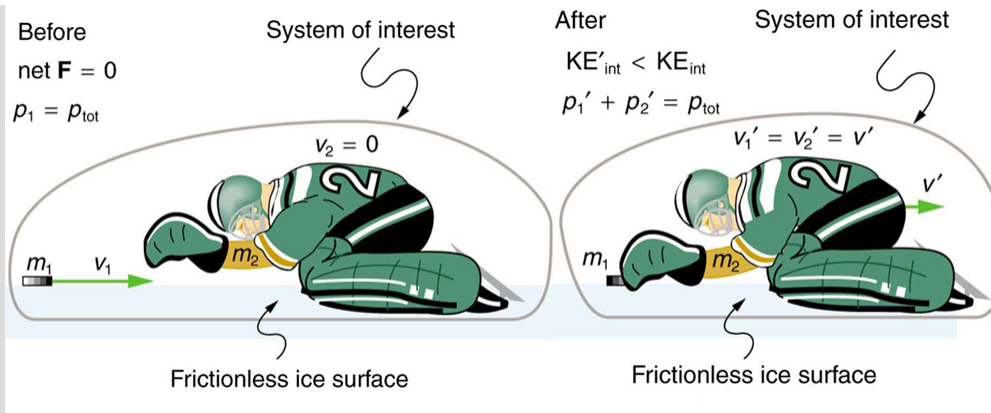
A collision in which the objects stick together is sometimes called “perfectly inelastic.”



An inelastic one-dimensional two-object collision. Momentum is conserved, but internal kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward one another at the same speed. (b) The objects stick together (a perfectly inelastic collision), and so their final velocity is zero. The internal kinetic energy of the system changes in any inelastic collision and is reduced to zero in this example.

**Example:****Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie**

(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See [link](#) )



An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.

### Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

### Solution for (a)

Momentum is conserved because the net external force on the puck-goalie system is zero.

Conservation of momentum is

**Equation:**

$$p_1 + p_2 = p'_1 + p'_2$$

or

**Equation:**

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2.$$

Because the goalie is initially at rest, we know  $v_2 = 0$ . Because the goalie catches the puck, the final velocities are equal, or  $v'_1 = v'_2 = v'$ . Thus, the

conservation of momentum equation simplifies to

**Equation:**

$$m_1 v_1 = (m_1 + m_2) v_f.$$

Solving for  $v_f$  yields

**Equation:**

$$v_f = \frac{m_1}{m_1 + m_2} v_1.$$

Entering known values in this equation, we get

**Equation:**

$$v_f = \left( \frac{0.150 \text{ kg}}{0.150 \text{ kg} + 70.0 \text{ kg}} \right) (35.0 \text{ m/s}) = 7.48 \times 10^{-2} \text{ m/s}.$$

### **Discussion for (a)**

This recoil velocity is small and in the same direction as the puck's original velocity, as we might expect.

### **Solution for (b)**

Before the collision, the internal kinetic energy  $\text{KE}_{\text{int}}$  of the system is that of the hockey puck, because the goalie is initially at rest. Therefore,  $\text{KE}_{\text{int}}$  is initially

**Equation:**

$$\begin{aligned} \text{KE}_{\text{int}} &= \frac{1}{2} m v^2 = \frac{1}{2} (0.150 \text{ kg}) (35.0 \text{ m/s})^2 \\ &= 91.9 \text{ J}. \end{aligned}$$

After the collision, the internal kinetic energy is

**Equation:**

$$\begin{aligned} \text{KE}'_{\text{int}} &= \frac{1}{2} (m + M) v^2 = \frac{1}{2} (70.15 \text{ kg}) (7.48 \times 10^{-2} \text{ m/s})^2 \\ &= 0.196 \text{ J}. \end{aligned}$$

The change in internal kinetic energy is thus

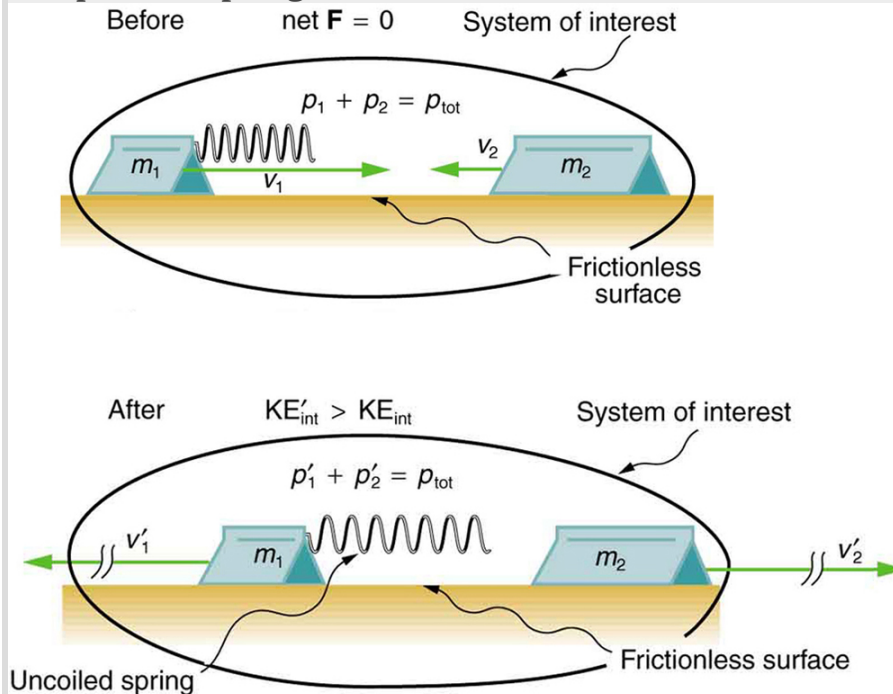
**Equation:**

$$\begin{aligned} KE_{\text{int}} - KE_{\text{int}} &= 0.196 \text{ J} - 91.9 \text{ J} \\ &= -91.7 \text{ J} \end{aligned}$$

where the minus sign indicates that the energy was lost.

### Discussion for (b)

Nearly all of the initial internal kinetic energy is lost in this perfectly inelastic collision.  $KE_{\text{int}}$  is mostly converted to thermal energy and sound. During some collisions, the objects do not stick together and less of the internal kinetic energy is removed—such as happens in most automobile accidents. Alternatively, stored energy may be converted into internal kinetic energy during a collision. [\[link\]](#) shows a one-dimensional example in which two carts on an air track collide, releasing potential energy from a compressed spring. [\[link\]](#) deals with data from such a collision.



An air track is nearly frictionless, so that momentum is conserved. Motion is one-dimensional. In this collision, examined in [\[link\]](#), the potential energy of a compressed spring is released during the collision and is converted to internal kinetic energy.

Collisions are particularly important in sports and the sporting and leisure industry utilizes elastic and inelastic collisions. Let us look briefly at tennis. Recall that in a collision, it is momentum and not force that is important. So, a heavier tennis racquet will have the advantage over a lighter one. This conclusion also holds true for other sports—a lightweight bat (such as a softball bat) cannot hit a hardball very far.

The location of the impact of the tennis ball on the racquet is also important, as is the part of the stroke during which the impact occurs. A smooth motion results in the maximizing of the velocity of the ball after impact and reduces sports injuries such as tennis elbow. A tennis player tries to hit the ball on the “sweet spot” on the racquet, where the vibration and impact are minimized and the ball is able to be given more velocity. Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

**Note:**

**Take-Home Experiment—Bouncing of Tennis Ball**

1. Find a racquet (a tennis, badminton, or other racquet will do). Place the racquet on the floor and stand on the handle. Drop a tennis ball on the strings from a measured height. Measure how high the ball bounces. Now ask a friend to hold the racquet firmly by the handle and drop a tennis ball from the same measured height above the racquet. Measure how high the ball bounces and observe what happens to your friend’s hand during the collision. Explain your observations and measurements.
2. The coefficient of restitution ( $c$ ) is a measure of the elasticity of a collision between a ball and an object, and is defined as the ratio of the speeds after and before the collision. A perfectly elastic collision has a  $c$  of 1. For a ball bouncing off the floor (or a racquet on the floor),  $c$  can be shown to be  $c = (h/H)^{1/2}$  where  $h$  is the height to which the ball bounces and  $H$  is the height from which the ball is dropped. Determine  $c$  for the cases in Part 1 and for the case of a tennis ball bouncing off a concrete or wooden floor ( $c = 0.85$  for new tennis balls used on a tennis court).

**Example:****Calculating Final Velocity and Energy Release: Two Carts Collide**

In the collision pictured in [\[link\]](#), two carts collide inelastically. Cart 1 (denoted  $m_1$ ) carries a spring which is initially compressed. During the collision, the spring releases its potential energy and converts it to internal kinetic energy. The mass of cart 1 and the spring is 0.350 kg, and the cart and the spring together have an initial velocity of 2.00 m/s. Cart 2 (denoted  $m_2$  in [\[link\]](#)) has a mass of 0.500 kg and an initial velocity of  $-0.500$  m/s. After the collision, cart 1 is observed to recoil with a velocity of  $-4.00$  m/s. (a) What is the final velocity of cart 2? (b) How much energy was released by the spring (assuming all of it was converted into internal kinetic energy)?

**Strategy**

We can use conservation of momentum to find the final velocity of cart 2, because  $F_{\text{net}} = 0$  (the track is frictionless and the force of the spring is internal). Once this velocity is determined, we can compare the internal kinetic energy before and after the collision to see how much energy was released by the spring.

**Solution for (a)**

As before, the equation for conservation of momentum in a two-object system is

**Equation:**

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2.$$

The only unknown in this equation is  $v'_2$ . Solving for  $v'_2$  and substituting known values into the previous equation yields

**Equation:**

$$\begin{aligned} v'_2 &= \frac{m_1 v_1 + m_2 v_2 - m_1 v'_1}{m_2} \\ &= \frac{(0.350 \text{ kg})(2.00 \text{ m/s}) + (0.500 \text{ kg})(-0.500 \text{ m/s})}{0.500 \text{ kg}} - \frac{(0.350 \text{ kg})(-4.00 \text{ m/s})}{0.500 \text{ kg}} \\ &= 3.70 \text{ m/s.} \end{aligned}$$

**Solution for (b)**

The internal kinetic energy before the collision is

**Equation:**

$$\begin{aligned}\text{KE}_{\text{int}} &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \\ &= \frac{1}{2}(0.350 \text{ kg})(2.00 \text{ m/s})^2 + \frac{1}{2}(0.500 \text{ kg})(-0.500 \text{ m/s})^2 \\ &= 0.763 \text{ J}.\end{aligned}$$

After the collision, the internal kinetic energy is

**Equation:**

$$\begin{aligned}\text{KE}'_{\text{int}} &= \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \\ &= \frac{1}{2}(0.350 \text{ kg})(-4.00 \text{ m/s})^2 + \frac{1}{2}(0.500 \text{ kg})(3.70 \text{ m/s})^2 \\ &= 6.22 \text{ J}.\end{aligned}$$

The change in internal kinetic energy is thus

**Equation:**

$$\begin{aligned}\text{KE}'_{\text{int}} - \text{KE}_{\text{int}} &= 6.22 \text{ J} - 0.763 \text{ J} \\ &= 5.46 \text{ J}.\end{aligned}$$

**Discussion**

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision. The internal kinetic energy in this collision increases by 5.46 J. That energy was released by the spring.

**Section Summary**

- An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).
- A collision in which the objects stick together is sometimes called perfectly inelastic because it reduces internal kinetic energy more than does any other type of inelastic collision.

- Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

## Conceptual Questions

### Exercise:

#### Problem:

What is an inelastic collision? What is a perfectly inelastic collision?

### Exercise:

#### Problem:

Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?

### Exercise:

#### Problem:

A small pickup truck that has a camper shell slowly coasts toward a red light with negligible friction. Two dogs in the back of the truck are moving and making various inelastic collisions with each other and the walls. What is the effect of the dogs on the motion of the center of mass of the system (truck plus entire load)? What is their effect on the motion of the truck?

## Problems & Exercises

### Exercise:

**Problem:**

A 0.240-kg billiard ball that is moving at 3.00 m/s strikes the bumper of a pool table and bounces straight back at 2.40 m/s (80% of its original speed). The collision lasts 0.0150 s. (a) Calculate the average force exerted on the ball by the bumper. (b) How much kinetic energy in joules is lost during the collision? (c) What percent of the original energy is left?

---

**Solution:**

(a) 86.4 N perpendicularly away from the bumper

(b) 0.389 J

(c) 64.0%

**Exercise:****Problem:**

During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater. (a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s? (b) How much kinetic energy is lost?

**Exercise:****Problem: Professional Application**

Using mass and speed data from [\[link\]](#) and assuming that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate: (a) the final velocity if the ball and player are going in the same direction and (b) the loss of kinetic energy in this case. (c) Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?

---

**Solution:**

(a) 8.06 m/s

(b) -56.0 J

(c)(i) 7.88 m/s; (ii) -223 J

**Exercise:**

**Problem:**

A battleship that is  $6.00 \times 10^7$  kg and is originally at rest fires a 1100-kg artillery shell horizontally with a velocity of 575 m/s. (a) If the shell is fired straight aft (toward the rear of the ship), there will be negligible friction opposing the ship's recoil. Calculate its recoil velocity. (b) Calculate the increase in internal kinetic energy (that is, for the ship and the shell). This energy is less than the energy released by the gun powder—significant heat transfer occurs.

**Exercise:**

**Problem: Professional Application**

Two manned satellites approaching one another, at a relative speed of 0.250 m/s, intending to dock. The first has a mass of  $4.00 \times 10^3$  kg, and the second a mass of  $7.50 \times 10^3$  kg. (a) Calculate the final velocity (after docking) by using the frame of reference in which the first satellite was originally at rest. (b) What is the loss of kinetic energy in this inelastic collision? (c) Repeat both parts by using the frame of reference in which the second satellite was originally at rest. Explain why the change in velocity is different in the two frames, whereas the change in kinetic energy is the same in both.

---

**Solution:**

(a) 0.163 m/s in the direction of motion of the more massive satellite

(b) 81.6 J

(c)  $8.70 \times 10^{-2}$  m/s in the direction of motion of the less massive satellite, 81.5 J. Because there are no external forces, the velocity of the center of mass of the two-satellite system is unchanged by the collision. The two velocities calculated above are the velocity of the center of mass in each of the two different individual reference frames. The loss in KE is the same in both reference frames because the KE lost to internal forces (heat, friction, etc.) is the same regardless of the coordinate system chosen.

**Exercise:**

**Problem: Professional Application**

A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it. (a) What is the final velocity of the loaded freight car? (b) How much kinetic energy is lost?

**Exercise:**

**Problem: Professional Application**

Space probes may be separated from their launchers by exploding bolts. (They bolt away from one another.) Suppose a 4800-kg satellite uses this method to separate from the 1500-kg remains of its launcher, and that 5000 J of kinetic energy is supplied to the two parts. What are their subsequent velocities using the frame of reference in which they were at rest before separation?

---

**Solution:**

0.704 m/s

−2.25 m/s

**Exercise:**

**Problem:**

A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder. (a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder. (b) How much kinetic energy does the rifle gain? (c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg? (d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation. (e) Calculate the momentum of a 110-kg football player running at 8.00 m/s. Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s. Discuss its relationship to this problem.

---

**Solution:**

(a) 4.58 m/s away from the bullet

(b) 31.5 J

(c)  $-0.491$  m/s

(d) 3.38 J

**Exercise:****Problem: Professional Application**

One of the waste products of a nuclear reactor is plutonium-239 ( $^{239}\text{Pu}$ ). This nucleus is radioactive and decays by splitting into a helium-4 nucleus and a uranium-235 nucleus ( $^4\text{He} + ^{235}\text{U}$ ), the latter of which is also radioactive and will itself decay some time later. The energy emitted in the plutonium decay is  $8.40 \times 10^{-13}$  J and is entirely converted to kinetic energy of the helium and uranium nuclei.

The mass of the helium nucleus is  $6.68 \times 10^{-27}$  kg, while that of the uranium is  $3.92 \times 10^{-25}$  kg (note that the ratio of the masses is 4 to 235). (a) Calculate the velocities of the two nuclei, assuming the plutonium nucleus is originally at rest. (b) How much kinetic energy does each nucleus carry away? Note that the data given here are accurate to three digits only.

**Exercise:**

**Problem: Professional Application**

The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of  $5.00 \times 10^{12}$  kg (about a kilometer across) strikes the Moon at a speed of 15.0 km/s. (a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is  $7.36 \times 10^{22}$  kg)? (b) How much kinetic energy is lost in the collision? Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon. (c) In October 2009, NASA crashed a rocket into the Moon, and analyzed the plume produced by the impact. (Significant amounts of water were detected.) Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000 kg and its speed upon impact was 9000 km/h. How does the plume produced alter these results?

---

**Solution:**

(a)  $1.02 \times 10^{-6}$  m/s

(b)  $5.63 \times 10^{20}$  J (almost all KE lost)

(c) Recoil speed is  $6.79 \times 10^{-17}$  m/s, energy lost is  $6.25 \times 10^9$  J. The plume will not affect the momentum result because the plume is still part of the Moon system. The plume may affect the kinetic energy result because a significant part of the initial kinetic energy may be transferred to the kinetic energy of the plume particles.

**Exercise:**

**Problem: Professional Application**

Two football players collide head-on in midair while trying to catch a thrown football. The first player is 95.0 kg and has an initial velocity of 6.00 m/s, while the second player is 115 kg and has an initial velocity of  $-3.50$  m/s. What is their velocity just after impact if they cling together?

**Exercise:****Problem:**

What is the speed of a garbage truck that is  $1.20 \times 10^4$  kg and is initially moving at 25.0 m/s just after it hits and adheres to a trash can that is 80.0 kg and is initially at rest?

---

**Solution:**

24.8 m/s

**Exercise:****Problem:**

During a circus act, an elderly performer thrills the crowd by catching a cannon ball shot at him. The cannon ball has a mass of 10.0 kg and the horizontal component of its velocity is 8.00 m/s when the 65.0-kg performer catches it. If the performer is on nearly frictionless roller skates, what is his recoil velocity?

**Exercise:****Problem:**

(a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a velocity of 10.0 m/s, what is the mass of the barbell? (b) How much kinetic energy is gained by this maneuver? (c) Where does the kinetic energy come from?

---

**Solution:**

(a) 4.00 kg

(b) 210 J

(c) The clown does work to throw the barbell, so the kinetic energy comes from the muscles of the clown. The muscles convert the chemical potential energy of ATP into kinetic energy.

**Glossary**

inelastic collision

a collision in which internal kinetic energy is not conserved

perfectly inelastic collision

a collision in which the colliding objects stick together

## Elastic Collisions in One Dimension

- Describe an elastic collision of two objects in one dimension.
- Define internal kinetic energy.
- Derive an expression for conservation of internal kinetic energy in a one dimensional collision.
- Determine the final velocities in an elastic collision given masses and initial velocities.

Let us consider various types of two-object collisions. These collisions are the easiest to analyze, and they illustrate many of the physical principles involved in collisions. The conservation of momentum principle is very useful here, and it can be used whenever the net external force on a system is zero.

We start with the elastic collision of two objects moving along the same line—a one-dimensional problem. An **elastic collision** is one that also conserves internal kinetic energy. **Internal kinetic energy** is the sum of the kinetic energies of the objects in the system. [\[link\]](#) illustrates an elastic collision in which internal kinetic energy and momentum are conserved.

Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

### **Note:**

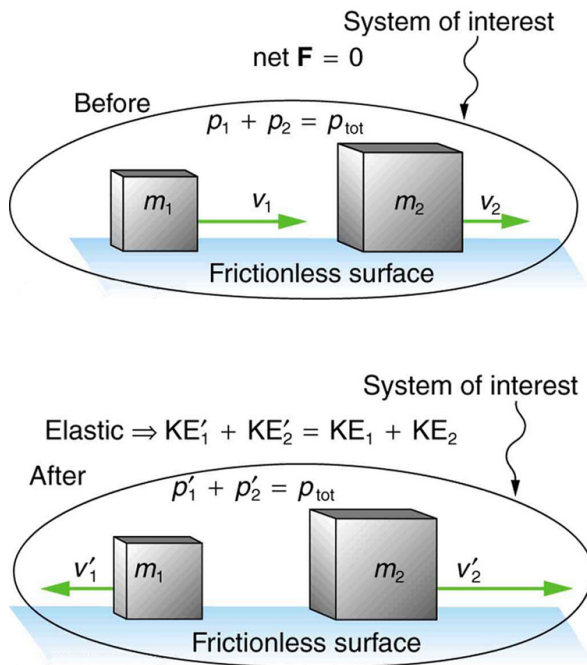
#### Elastic Collision

An **elastic collision** is one that conserves internal kinetic energy.

### **Note:**

#### Internal Kinetic Energy

**Internal kinetic energy** is the sum of the kinetic energies of the objects in the system.



An elastic one-dimensional  
two-object collision.  
Momentum and internal kinetic  
energy are conserved.

Now, to solve problems involving one-dimensional elastic collisions between two objects we can use the equations for conservation of momentum and conservation of internal kinetic energy. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

**Equation:**

$$p_1 + p_2 = p'_1 + p'_2 \quad (F_{\text{net}} = 0)$$

or

**Equation:**

$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2 \quad (F_{\text{net}} = 0),$$

where the primes (') indicate values after the collision. By definition, an elastic collision conserves internal kinetic energy, and so the sum of kinetic energies before the collision equals the sum after the collision. Thus,

**Equation:**

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2 \quad (\text{two-object elastic collision})$$

expresses the equation for conservation of internal kinetic energy in a one-dimensional collision.

**Example:****Calculating Velocities Following an Elastic Collision**

Calculate the velocities of two objects following an elastic collision, given that

**Equation:**

$$m_1 = 0.500 \text{ kg}, \quad m_2 = 3.50 \text{ kg}, \quad v_1 = 4.00 \text{ m/s}, \text{ and } v_2 = 0.$$

**Strategy and Concept**

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation shown in [\[link\]](#) where both objects are initially moving. We are asked to find two unknowns (the final velocities  $v'_1$  and  $v'_2$ ). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus  $v_2 = 0$ . Once we simplify these equations, we combine them algebraically to solve for the unknowns.

**Solution**

For this problem, note that  $v_2 = 0$  and use conservation of momentum. Thus,

**Equation:**

$$p_1 = p'_1 + p'_2$$

or

**Equation:**

$$m_1 v_1 = m_1 v'_1 + m_2 v'_2.$$

Using conservation of internal kinetic energy and that  $v_2 = 0$ ,

**Equation:**

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2.$$

Solving the first equation (momentum equation) for  $v'_2$ , we obtain

**Equation:**

$$v'_2 = \frac{m_1}{m_2} (v_1 - v'_1).$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable  $v'_2$ , leaving only  $v'_1$  as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are

**Equation:**

$$v'_1 = 4.00 \text{ m/s}$$

and

**Equation:**

$$v'_1 = -3.00 \text{ m/s}.$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution ( $v'_1 = -3.00 \text{ m/s}$ ) is negative, meaning that the first object bounces backward. When this negative value of  $v'_1$  is used to find the velocity of the second object after the collision, we get

**Equation:**

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1) = \frac{0.500 \text{ kg}}{3.50 \text{ kg}}[4.00 - (-3.00)] \text{ m/s}$$

or

**Equation:**

$$v'_2 = 1.00 \text{ m/s.}$$

### Discussion

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but with a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy as written above can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.

### Note:

**Making Connections: Take-Home Investigation—Ice Cubes and Elastic Collision**

Find a few ice cubes which are about the same size and a smooth kitchen tabletop or a table with a glass top. Place the ice cubes on the surface several centimeters away from each other. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes. Have you created approximately elastic collisions? Explain the speeds and directions of the ice cubes using momentum.

### Note:

**PhET Explorations: Collision Lab**

Investigate collisions on an air hockey table. Set up your own experiments: vary the number of discs, masses and initial conditions. Is momentum

conserved? Is kinetic energy conserved? Vary the elasticity and see what happens.

[https://phet.colorado.edu/sims/collision-lab/collision-lab\\_en.html](https://phet.colorado.edu/sims/collision-lab/collision-lab_en.html)

## Section Summary

- An elastic collision is one that conserves internal kinetic energy.
- Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions.

## Conceptual Questions

### Exercise:

**Problem:** What is an elastic collision?

## Problems & Exercises

### Exercise:

#### **Problem:**

Two identical objects (such as billiard balls) have a one-dimensional collision in which one is initially motionless. After the collision, the moving object is stationary and the other moves with the same speed as the other originally had. Show that both momentum and kinetic energy are conserved.

### Exercise:

#### **Problem: Professional Application**

Two manned satellites approach one another at a relative speed of 0.250 m/s, intending to dock. The first has a mass of  $4.00 \times 10^3$  kg, and the

second a mass of  $7.50 \times 10^3$  kg. If the two satellites collide elastically rather than dock, what is their final relative velocity?

---

**Solution:**

0.250 m/s

**Exercise:**

**Problem:**

A 70.0-kg ice hockey goalie, originally at rest, catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. Suppose the goalie and the ice puck have an elastic collision and the puck is reflected back in the direction from which it came. What would their final velocities be in this case?

## Glossary

elastic collision

a collision that also conserves internal kinetic energy

internal kinetic energy

the sum of the kinetic energies of the objects in a system

## Newton's Universal Law of Gravitation

- Explain Earth's gravitational force.
- Describe the gravitational effect of the Moon on Earth.
- Discuss weightlessness in space.
- Examine the Cavendish experiment

What do aching feet, a falling apple, and the orbit of the Moon have in common? Each is caused by the gravitational force. Our feet are strained by supporting our weight—the force of Earth's gravity on us. An apple falls from a tree because of the same force acting a few meters above Earth's surface. And the Moon orbits Earth because gravity is able to supply the necessary centripetal force at a distance of hundreds of millions of meters. In fact, the same force causes planets to orbit the Sun, stars to orbit the center of the galaxy, and galaxies to cluster together. Gravity is another example of underlying simplicity in nature. It is the weakest of the four basic forces found in nature, and in some ways the least understood. It is a force that acts at a distance, without physical contact, and is expressed by a formula that is valid everywhere in the universe, for masses and distances that vary from the tiny to the immense.

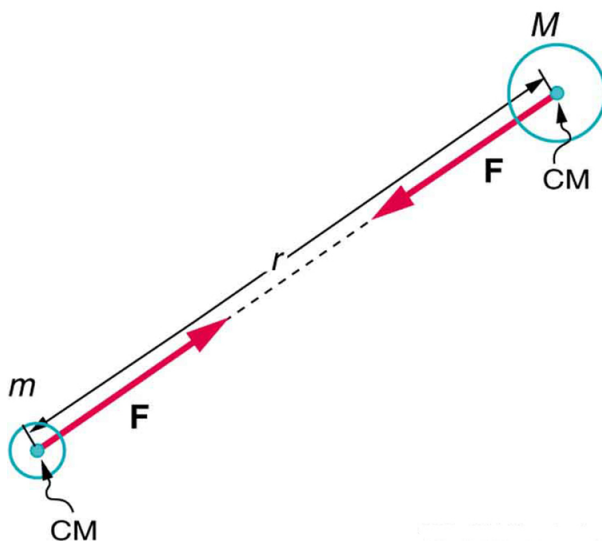
Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See [\[link\]](#). But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner Galileo Galilei had contended that falling bodies and planetary motions had the same cause. Some of Newton's contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph—it had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose a mechanism that caused them to follow these paths and not others.



According to early accounts, Newton was inspired to make the connection between falling bodies and astronomical motions when he saw an apple fall from a tree and realized that if the gravitational force could extend above the ground to a tree, it might also reach the Sun. The inspiration of Newton's apple is a part of worldwide folklore and may even be based in fact. Great importance is attached to it because Newton's

universal law of  
gravitation and his  
laws of motion  
answered very old  
questions about nature  
and gave tremendous  
support to the notion  
of underlying  
simplicity and unity in  
nature. Scientists still  
expect underlying  
simplicity to emerge  
from their ongoing  
inquiries into nature.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Stated in modern language, **Newton's universal law of gravitation** states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton's third law.

**Note:**

**Misconception Alert**

The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton's third law.

The bodies we are dealing with tend to be large. To simplify the situation we assume that the body acts as if its entire mass is concentrated at one specific point called the **center of mass** (CM), which will be further explored in [Linear Momentum and Collisions](#). For two bodies having masses  $m$  and  $M$  with a distance  $r$  between their centers of mass, the equation for Newton's universal law of gravitation is

**Equation:**

$$F = G \frac{mM}{r^2},$$

where  $F$  is the magnitude of the gravitational force and  $G$  is a proportionality factor called the **gravitational constant**.  $G$  is a universal gravitational constant—that is, it is thought to be the same everywhere in the universe. It has been measured experimentally to be

**Equation:**

$$G = 6.674 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

in SI units. Note that the units of  $G$  are such that a force in newtons is obtained from  $F = G \frac{mM}{r^2}$ , when considering masses in kilograms and distance in meters. For example, two 1.000 kg masses separated by 1.000 m will experience a gravitational attraction of  $6.674 \times 10^{-11}$  N. This is an extraordinarily small force. The small magnitude of the gravitational force is consistent with everyday experience. We are unaware that even large objects like mountains exert gravitational forces on us. In fact, our body weight is the force of attraction of the *entire Earth* on us with a mass of  $6 \times 10^{24}$  kg.

Recall that the acceleration due to gravity  $g$  is about  $9.80 \text{ m/s}^2$  on Earth. We can now determine why this is so. The weight of an object  $mg$  is the gravitational force between it and Earth. Substituting  $mg$  for  $F$  in Newton's universal law of gravitation gives

**Equation:**

$$mg = G \frac{mM}{r^2},$$

where  $m$  is the mass of the object,  $M$  is the mass of Earth, and  $r$  is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See [\[link\]](#). The mass  $m$  of the object cancels, leaving an equation for  $g$ :

**Equation:**

$$g = G \frac{M}{r^2}.$$

Substituting known values for Earth's mass and radius (to three significant figures),

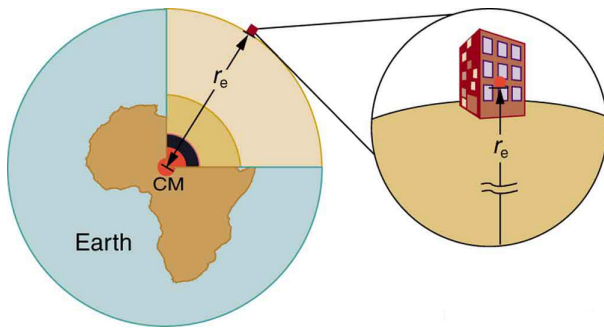
**Equation:**

$$g = \left( 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) \times \frac{5.98 \times 10^{24} \text{ kg}}{(6.38 \times 10^6 \text{ m})^2},$$

and we obtain a value for the acceleration of a falling body:

**Equation:**

$$g = 9.80 \text{ m/s}^2.$$



The distance between the centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object.

This is the expected value *and is independent of the body's mass*. Newton's law of gravitation takes Galileo's observation that all masses fall with the same acceleration a step further, explaining the observation in terms of a force that causes objects to fall—in fact, in terms of a universally existing force of attraction between masses.

**Note:**

**Take-Home Experiment**

Take a marble, a ball, and a spoon and drop them from the same height. Do they hit the floor at the same time? If you drop a piece of paper as well, does it behave like the other objects? Explain your observations.

**Note:****Making Connections**

Attempts are still being made to understand the gravitational force. As we shall see in [Particle Physics](#), modern physics is exploring the connections of gravity to other forces, space, and time. General relativity alters our view of gravitation, leading us to think of gravitation as bending space and time.

In the following example, we make a comparison similar to one made by Newton himself. He noted that if the gravitational force caused the Moon to orbit Earth, then the acceleration due to gravity should equal the centripetal acceleration of the Moon in its orbit. Newton found that the two accelerations agreed “pretty nearly.”

**Example:****Earth’s Gravitational Force Is the Centripetal Force Making the Moon Move in a Curved Path**

- (a) Find the acceleration due to Earth’s gravity at the distance of the Moon.
- (b) Calculate the centripetal acceleration needed to keep the Moon in its orbit (assuming a circular orbit about a fixed Earth), and compare it with the value of the acceleration due to Earth’s gravity that you have just found.

**Strategy for (a)**

This calculation is the same as the one finding the acceleration due to gravity at Earth’s surface, except that  $r$  is the distance from the center of Earth to the center of the Moon. The radius of the Moon’s nearly circular orbit is  $3.84 \times 10^8$  m.

**Solution for (a)**

Substituting known values into the expression for  $g$  found above, remembering that  $M$  is the mass of Earth not the Moon, yields

**Equation:**

$$\begin{aligned}
 g &= G \frac{M}{r^2} = \left( 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) \times \frac{5.98 \times 10^{24} \text{ kg}}{(3.84 \times 10^8 \text{ m})^2} \\
 &= 2.70 \times 10^{-3} \text{ m/s}^2
 \end{aligned}$$

### Strategy for (b)

Centripetal acceleration can be calculated using either form of **Equation:**

$$\left. \begin{aligned} a_c &= \frac{v^2}{r} \\ a_c &= r\omega^2 \end{aligned} \right\}.$$

We choose to use the second form:

**Equation:**

$$a_c = r\omega^2,$$

where  $\omega$  is the angular velocity of the Moon about Earth.

### Solution for (b)

Given that the period (the time it takes to make one complete rotation) of the Moon's orbit is 27.3 days, (d) and using

**Equation:**

$$1 \text{ d} \times 24 \frac{\text{hr}}{\text{d}} \times 60 \frac{\text{min}}{\text{hr}} \times 60 \frac{\text{s}}{\text{min}} = 86,400 \text{ s}$$

we see that

**Equation:**

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{(27.3 \text{ d})(86,400 \text{ s/d})} = 2.66 \times 10^{-6} \frac{\text{rad}}{\text{s}}.$$

The centripetal acceleration is

**Equation:**

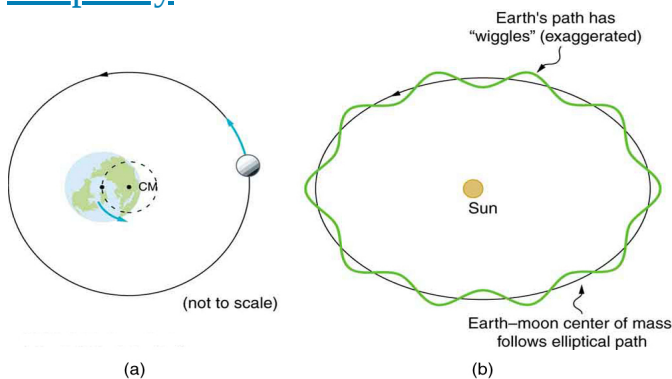
$$\begin{aligned}
 a_c &= r\omega^2 = (3.84 \times 10^8 \text{ m})(2.66 \times 10^{-6} \text{ rad/s})^2 \\
 &= 2.72 \times 10^{-3} \text{ m/s}^2
 \end{aligned}$$

The direction of the acceleration is toward the center of the Earth.

### Discussion

The centripetal acceleration of the Moon found in (b) differs by less than 1% from the acceleration due to Earth's gravity found in (a). This agreement is approximate because the Moon's orbit is slightly elliptical, and Earth is not stationary (rather the Earth-Moon system rotates about its center of mass, which is located some 1700 km below Earth's surface). The clear implication is that Earth's gravitational force causes the Moon to orbit Earth.

Why does Earth not remain stationary as the Moon orbits it? This is because, as expected from Newton's third law, if Earth exerts a force on the Moon, then the Moon should exert an equal and opposite force on Earth (see [\[link\]](#)). We do not sense the Moon's effect on Earth's motion, because the Moon's gravity moves our bodies right along with Earth but there are other signs on Earth that clearly show the effect of the Moon's gravitational force as discussed in [Satellites and Kepler's Laws: An Argument for Simplicity](#).

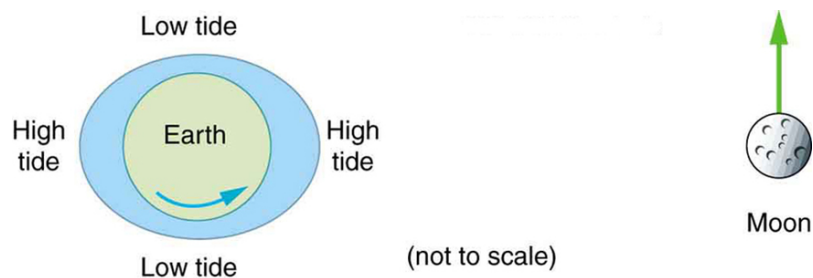


(a) Earth and the Moon rotate approximately once a month around their common center of mass. (b) Their center of mass orbits the Sun in an elliptical orbit, but Earth's path around the Sun has "wiggles" in it. Similar wiggles in the paths of stars have been observed and are

considered direct evidence of planets orbiting those stars. This is important because the planets' reflected light is often too dim to be observed.

## Tides

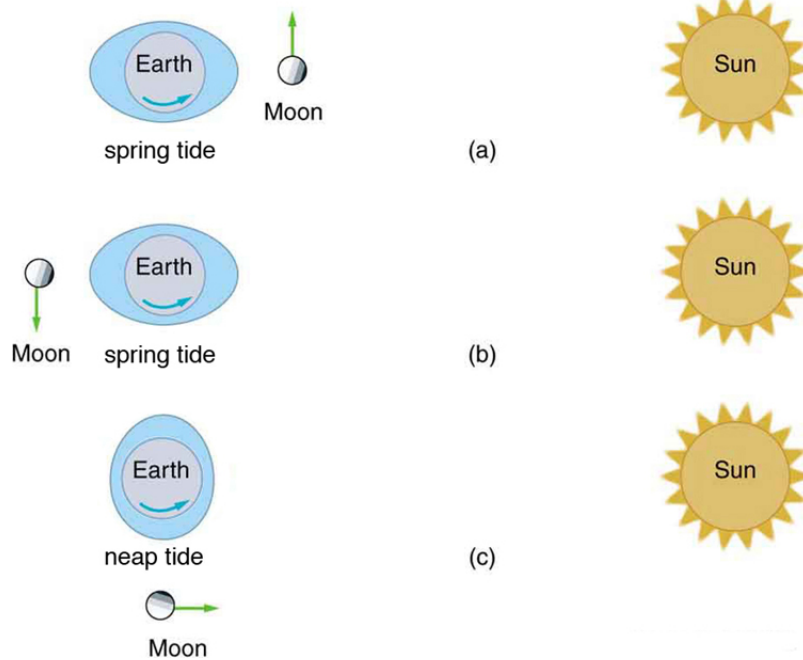
Ocean tides are one very observable result of the Moon's gravity acting on Earth. [\[link\]](#) is a simplified drawing of the Moon's position relative to the tides. Because water easily flows on Earth's surface, a high tide is created on the side of Earth nearest to the Moon, where the Moon's gravitational pull is strongest. Why is there also a high tide on the opposite side of Earth? The answer is that Earth is pulled toward the Moon more than the water on the far side, because Earth is closer to the Moon. So the water on the side of Earth closest to the Moon is pulled away from Earth, and Earth is pulled away from water on the far side. As Earth rotates, the tidal bulge (an effect of the tidal forces between an orbiting natural satellite and the primary planet that it orbits) keeps its orientation with the Moon. Thus there are two tides per day (the actual tidal period is about 12 hours and 25.2 minutes), because the Moon moves in its orbit each day as well).



The Moon causes ocean tides by attracting the water on the near side more than Earth, and by attracting Earth more than the water on the far side. The distances and sizes are

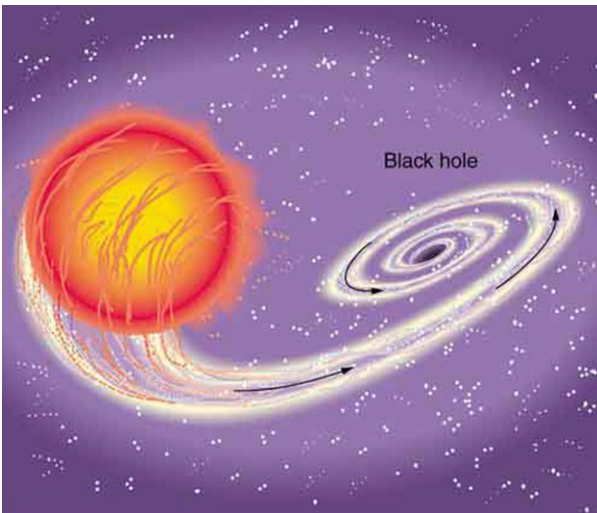
not to scale. For this simplified representation of the Earth-Moon system, there are two high and two low tides per day at any location, because Earth rotates under the tidal bulge.

The Sun also affects tides, although it has about half the effect of the Moon. However, the largest tides, called spring tides, occur when Earth, the Moon, and the Sun are aligned. The smallest tides, called neap tides, occur when the Sun is at a  $90^\circ$  angle to the Earth-Moon alignment.



(a, b) Spring tides: The highest tides occur when Earth, the Moon, and the Sun are aligned. (c) Neap tide: The lowest tides occur when the Sun lies at  $90^\circ$  to the Earth-Moon alignment. Note that this figure is not drawn to scale.

Tides are not unique to Earth but occur in many astronomical systems. The most extreme tides occur where the gravitational force is the strongest and varies most rapidly, such as near black holes (see [\[link\]](#)). A few likely candidates for black holes have been observed in our galaxy. These have masses greater than the Sun but have diameters only a few kilometers across. The tidal forces near them are so great that they can actually tear matter from a companion star.



A black hole is an object with such strong gravity that not even light can escape it. This black hole was created by the supernova of one star in a two-star system. The tidal forces created by the black hole are so great that it tears matter from the companion star. This matter is compressed and heated as it is sucked into the black hole, creating light and X-rays observable from Earth.

## **”Weightlessness” and Microgravity**

In contrast to the tremendous gravitational force near black holes is the apparent gravitational field experienced by astronauts orbiting Earth. What is the effect of “weightlessness” upon an astronaut who is in orbit for months? Or what about the effect of weightlessness upon plant growth? Weightlessness doesn’t mean that an astronaut is not being acted upon by the gravitational force. There is no “zero gravity” in an astronaut’s orbit. The term just means that the astronaut is in free-fall, accelerating with the acceleration due to gravity. If an elevator cable breaks, the passengers inside will be in free fall and will experience weightlessness. You can experience short periods of weightlessness in some rides in amusement parks.



Astronauts experiencing weightlessness on board the International Space Station. (credit: NASA)

**Microgravity** refers to an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface. Many interesting biology and physics topics have been studied over the past three decades in the presence of microgravity. Of immediate concern is the effect on astronauts of extended times in outer space, such as at the International Space Station. Researchers have observed that muscles will atrophy (waste away) in this environment. There is also a corresponding loss of bone mass. Study continues on cardiovascular adaptation to space flight. On Earth, blood pressure is usually higher in the feet than in the head, because the higher column of blood exerts a downward force on it, due to gravity. When standing, 70% of your blood is below the level of the heart, while in a horizontal position, just the opposite occurs. What difference does the absence of this pressure differential have upon the heart?

Some findings in human physiology in space can be clinically important to the management of diseases back on Earth. On a somewhat negative note, spaceflight is known to affect the human immune system, possibly making the crew members more vulnerable to infectious diseases. Experiments flown in space also have shown that some bacteria grow faster in microgravity than they do on Earth. However, on a positive note, studies indicate that microbial antibiotic production can increase by a factor of two in space-grown cultures. One hopes to be able to understand these mechanisms so that similar successes can be achieved on the ground. In another area of physics space research, inorganic crystals and protein crystals have been grown in outer space that have much higher quality than any grown on Earth, so crystallography studies on their structure can yield much better results.

Plants have evolved with the stimulus of gravity and with gravity sensors. Roots grow downward and shoots grow upward. Plants might be able to provide a life support system for long duration space missions by regenerating the atmosphere, purifying water, and producing food. Some studies have indicated that plant growth and development are not affected by gravity, but there is still uncertainty about structural changes in plants grown in a microgravity environment.

## The Cavendish Experiment: Then and Now

As previously noted, the universal gravitational constant  $G$  is determined experimentally. This definition was first done accurately by Henry Cavendish (1731–1810), an English scientist, in 1798, more than 100 years after Newton published his universal law of gravitation. The measurement of  $G$  is very basic and important because it determines the strength of one of the four forces in nature. Cavendish's experiment was very difficult because he measured the tiny gravitational attraction between two ordinary-sized masses (tens of kilograms at most), using apparatus like that in [\[link\]](#). Remarkably, his value for  $G$  differs by less than 1% from the best modern value.

One important consequence of knowing  $G$  was that an accurate value for Earth's mass could finally be obtained. This was done by measuring the acceleration due to gravity as accurately as possible and then calculating the mass of Earth  $M$  from the relationship Newton's universal law of gravitation gives

**Equation:**

$$mg = G \frac{mM}{r^2},$$

where  $m$  is the mass of the object,  $M$  is the mass of Earth, and  $r$  is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See [\[link\]](#). The mass  $m$  of the object cancels, leaving an equation for  $g$ :

**Equation:**

$$g = G \frac{M}{r^2}.$$

Rearranging to solve for  $M$  yields

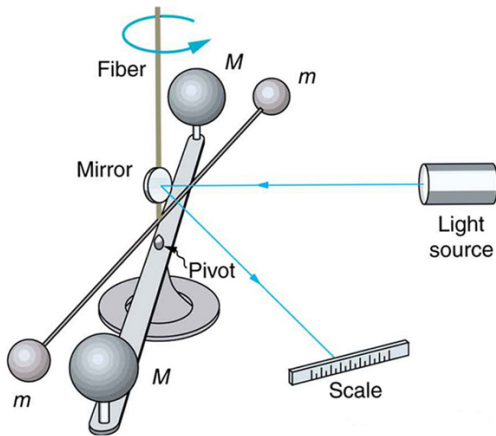
**Equation:**

$$M = \frac{gr^2}{G}.$$

So  $M$  can be calculated because all quantities on the right, including the radius of Earth  $r$ , are known from direct measurements. We shall see in [Satellites and Kepler's Laws: An Argument for Simplicity](#) that knowing  $G$  also allows for the determination of astronomical masses. Interestingly, of all the fundamental constants in physics,  $G$  is by far the least well determined.

The Cavendish experiment is also used to explore other aspects of gravity. One of the most interesting questions is whether the gravitational force depends on substance as well as mass—for example, whether one kilogram of lead exerts the same gravitational pull as one kilogram of water. A Hungarian scientist named Roland von Eötvös pioneered this inquiry early in the 20th century. He found, with an accuracy of five parts per billion, that the gravitational force does not depend on the substance. Such experiments continue today, and have improved upon Eötvös' measurements.

Cavendish-type experiments such as those of Eric Adelberger and others at the University of Washington, have also put severe limits on the possibility of a fifth force and have verified a major prediction of general relativity—that gravitational energy contributes to rest mass. Ongoing measurements there use a torsion balance and a parallel plate (not spheres, as Cavendish used) to examine how Newton's law of gravitation works over sub-millimeter distances. On this small-scale, do gravitational effects depart from the inverse square law? So far, no deviation has been observed.



Cavendish used an apparatus like this to measure the gravitational attraction between the two suspended spheres ( $m$ ) and the two on the stand ( $M$ ) by observing the amount of torsion (twisting) created in the fiber. Distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity.

## Section Summary

- Newton's universal law of gravitation: Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In equation form, this is

**Equation:**

$$F = G \frac{mM}{r^2},$$

where  $F$  is the magnitude of the gravitational force.  $G$  is the gravitational constant, given by  $G = 6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ .

- Newton's law of gravitation applies universally.

**Conceptual Questions****Exercise:****Problem:**

Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

**Exercise:****Problem:**

Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not  $9.80 \text{ m/s}^2$ . Who do you agree with and why?

**Exercise:****Problem:**

Draw a free body diagram for a satellite in an elliptical orbit showing why its speed increases as it approaches its parent body and decreases as it moves away.

**Exercise:**

**Problem:**

Newton's laws of motion and gravity were among the first to convincingly demonstrate the underlying simplicity and unity in nature. Many other examples have since been discovered, and we now expect to find such underlying order in complex situations. Is there proof that such order will always be found in new explorations?

**Problem Exercises****Exercise:****Problem:**

(a) Calculate Earth's mass given the acceleration due to gravity at the North Pole is  $9.830 \text{ m/s}^2$  and the radius of the Earth is 6371 km from center to pole.

(b) Compare this with the accepted value of  $5.979 \times 10^{24} \text{ kg}$ .

---

**Solution:**

a)  $5.979 \times 10^{24} \text{ kg}$

b) This is identical to the best value to three significant figures.

**Exercise:****Problem:**

(a) Calculate the magnitude of the acceleration due to gravity on the surface of Earth due to the Moon.

(b) Calculate the magnitude of the acceleration due to gravity at Earth due to the Sun.

(c) Take the ratio of the Moon's acceleration to the Sun's and comment on why the tides are predominantly due to the Moon in spite of this

number.

**Exercise:**

**Problem:**

- (a) What is the acceleration due to gravity on the surface of the Moon?
  - (b) On the surface of Mars? The mass of Mars is  $6.418 \times 10^{23}$  kg and its radius is  $3.38 \times 10^6$  m.
- 

**Solution:**

- a)  $1.62 \text{ m/s}^2$
- b)  $3.75 \text{ m/s}^2$

**Exercise:**

**Problem:**

- (a) Calculate the acceleration due to gravity on the surface of the Sun.
- (b) By what factor would your weight increase if you could stand on the Sun? (Never mind that you cannot.)

**Exercise:**

**Problem:**

The Moon and Earth rotate about their common center of mass, which is located about 4700 km from the center of Earth. (This is 1690 km below the surface.)

- (a) Calculate the magnitude of the acceleration due to the Moon's gravity at that point.
- (b) Calculate the magnitude of the centripetal acceleration of the center of Earth as it rotates about that point once each lunar month (about 27.3 d) and compare it with the acceleration found in part (a).

Comment on whether or not they are equal and why they should or should not be.

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**Solution:**

a)  $3.42 \times 10^{-5} \text{ m/s}^2$

b)  $3.34 \times 10^{-5} \text{ m/s}^2$

The values are nearly identical. One would expect the gravitational force to be the same as the centripetal force at the core of the system.

**Exercise:**

**Problem:** Solve part (b) of [\[link\]](#) using  $a_c = v^2/r$ .

**Exercise:**

**Problem:**

Astrology, that unlikely and vague pseudoscience, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational.

(a) Calculate the magnitude of the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child).

(b) Calculate the magnitude of the force on the baby due to Jupiter if it is at its closest distance to Earth, some  $6.29 \times 10^{11} \text{ m}$  away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)

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**Solution:**

a)  $7.01 \times 10^{-7} \text{ N}$

b)  $1.35 \times 10^{-6} \text{ N}$ , 0.521

**Exercise:**

**Problem:**

The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune's orbit. Pluto was subsequently discovered near its predicted position. But it now appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune's orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared with the closest planet to Neptune:

(a) Calculate the acceleration due to gravity at Neptune due to Pluto when they are  $4.50 \times 10^{12} \text{ m}$  apart, as they are at present. The mass of Pluto is  $1.4 \times 10^{22} \text{ kg}$ .

(b) Calculate the acceleration due to gravity at Neptune due to Uranus, presently about  $2.50 \times 10^{12} \text{ m}$  apart, and compare it with that due to Pluto. The mass of Uranus is  $8.62 \times 10^{25} \text{ kg}$ .

**Exercise:**

**Problem:**

(a) The Sun orbits the Milky Way galaxy once each  $2.60 \times 10^8 \text{ y}$ , with a roughly circular orbit averaging  $3.00 \times 10^4$  light years in radius. (A light year is the distance traveled by light in 1 y.) Calculate the centripetal acceleration of the Sun in its galactic orbit. Does your result support the contention that a nearly inertial frame of reference can be located at the Sun?

(b) Calculate the average speed of the Sun in its galactic orbit. Does the answer surprise you?

---

**Solution:**

a)  $1.66 \times 10^{-10} \text{ m/s}^2$

b)  $2.17 \times 10^5 \text{ m/s}$

## Exercise:

### Problem: Unreasonable Result

A mountain 10.0 km from a person exerts a gravitational force on him equal to 2.00% of his weight.

- (a) Calculate the mass of the mountain.
  - (b) Compare the mountain's mass with that of Earth.
  - (c) What is unreasonable about these results?
  - (d) Which premises are unreasonable or inconsistent? (Note that accurate gravitational measurements can easily detect the effect of nearby mountains and variations in local geology.)
- 

### Solution:

a)  $2.937 \times 10^{17} \text{ kg}$

b)  $4.91 \times 10^{-8}$

of the Earth's mass.

c) The mass of the mountain and its fraction of the Earth's mass are too great.

d) The gravitational force assumed to be exerted by the mountain is too great.

## Glossary

gravitational constant,  $G$

a proportionality factor used in the equation for Newton's universal law of gravitation; it is a universal constant—that is, it is thought to be the same everywhere in the universe

center of mass

the point where the entire mass of an object can be thought to be concentrated

microgravity

an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface

Newton's universal law of gravitation

every particle in the universe attracts every other particle with a force along a line joining them; the force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

## Further Applications of Newton's Laws of Motion

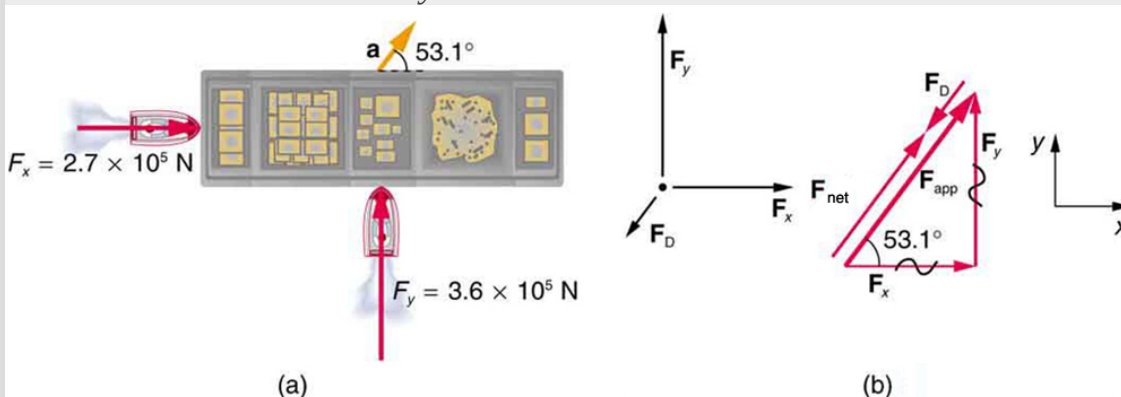
- Apply problem-solving techniques to solve for quantities in more complex systems of forces.
- Integrate concepts from kinematics to solve problems using Newton's laws of motion.

There are many interesting applications of Newton's laws of motion, a few more of which are presented in this section. These serve also to illustrate some further subtleties of physics and to help build problem-solving skills.

### Example:

#### Drag Force on a Barge

Suppose two tugboats push on a barge at different angles, as shown in [\[link\]](#). The first tugboat exerts a force of  $2.7 \times 10^5 \text{ N}$  in the  $x$ -direction, and the second tugboat exerts a force of  $3.6 \times 10^5 \text{ N}$  in the  $y$ -direction.



(a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the  $x$ - and  $y$ -axes are in the same direction as  $\mathbf{F}_x$  and  $\mathbf{F}_y$ . The problem quickly becomes a one-dimensional problem along the direction of  $\mathbf{F}_{app}$ , since friction is in the direction opposite to  $\mathbf{F}_{app}$ .

If the mass of the barge is  $5.0 \times 10^6 \text{ kg}$  and its acceleration is observed to be  $7.5 \times 10^{-2} \text{ m/s}^2$  in the direction shown, what is the drag force of the water on the

barge resisting the motion? (Note: drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object.)

**Strategy**

The directions and magnitudes of acceleration and the applied forces are given in [\[link\]\(a\)](#). We will define the total force of the tugboats on the barge as  $\mathbf{F}_{\text{app}}$  so that:

**Equation:**

$$\mathbf{F}_{\text{app}} = \mathbf{F}_x + \mathbf{F}_y$$

Since the barge is flat bottomed, the drag of the water  $\mathbf{F}_D$  will be in the direction opposite to  $\mathbf{F}_{\text{app}}$ , as shown in the free-body diagram in [\[link\]\(b\)](#). The system of interest here is the barge, since the forces on *it* are given as well as its acceleration. Our strategy is to find the magnitude and direction of the net applied force  $\mathbf{F}_{\text{app}}$ , and then apply Newton's second law to solve for the drag force  $\mathbf{F}_D$ .

**Solution**

Since  $\mathbf{F}_x$  and  $\mathbf{F}_y$  are perpendicular, the magnitude and direction of  $\mathbf{F}_{\text{app}}$  are easily found. First, the resultant magnitude is given by the Pythagorean theorem:

**Equation:**

$$F_{\text{app}} = \sqrt{F_x^2 + F_y^2}$$
$$F_{\text{app}} = \sqrt{(2.7 \times 10^5 \text{ N})^2 + (3.6 \times 10^5 \text{ N})^2} = 4.5 \times 10^5 \text{ N}.$$

The angle is given by

**Equation:**

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$
$$\theta = \tan^{-1}\left(\frac{3.6 \times 10^5 \text{ N}}{2.7 \times 10^5 \text{ N}}\right) = 53^\circ,$$

which we know, because of Newton's first law, is the same direction as the acceleration.  $\mathbf{F}_D$  is in the opposite direction of  $\mathbf{F}_{\text{app}}$ , since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as  $\mathbf{F}_{\text{app}}$ , but its magnitude is slightly less than  $\mathbf{F}_{\text{app}}$ . The problem is now one-dimensional. From [\[link\]\(b\)](#), we can see that

**Equation:**

$$F_{\text{net}} = F_{\text{app}} - F_D.$$

But Newton's second law states that

**Equation:**

$$F_{\text{net}} = ma.$$

Thus,

**Equation:**

$$F_{\text{app}} - F_{\text{D}} = ma.$$

This can be solved for the magnitude of the drag force of the water  $F_{\text{D}}$  in terms of known quantities:

**Equation:**

$$F_{\text{D}} = F_{\text{app}} - ma.$$

Substituting known values gives

**Equation:**

$$F_{\text{D}} = (4.5 \times 10^5 \text{ N}) - (5.0 \times 10^6 \text{ kg})(7.5 \times 10^{-2} \text{ m/s}^2) = 7.5 \times 10^4 \text{ N}.$$

The direction of  $\mathbf{F}_{\text{D}}$  has already been determined to be in the direction opposite to  $\mathbf{F}_{\text{app}}$ , or at an angle of  $53^\circ$  south of west.

### Discussion

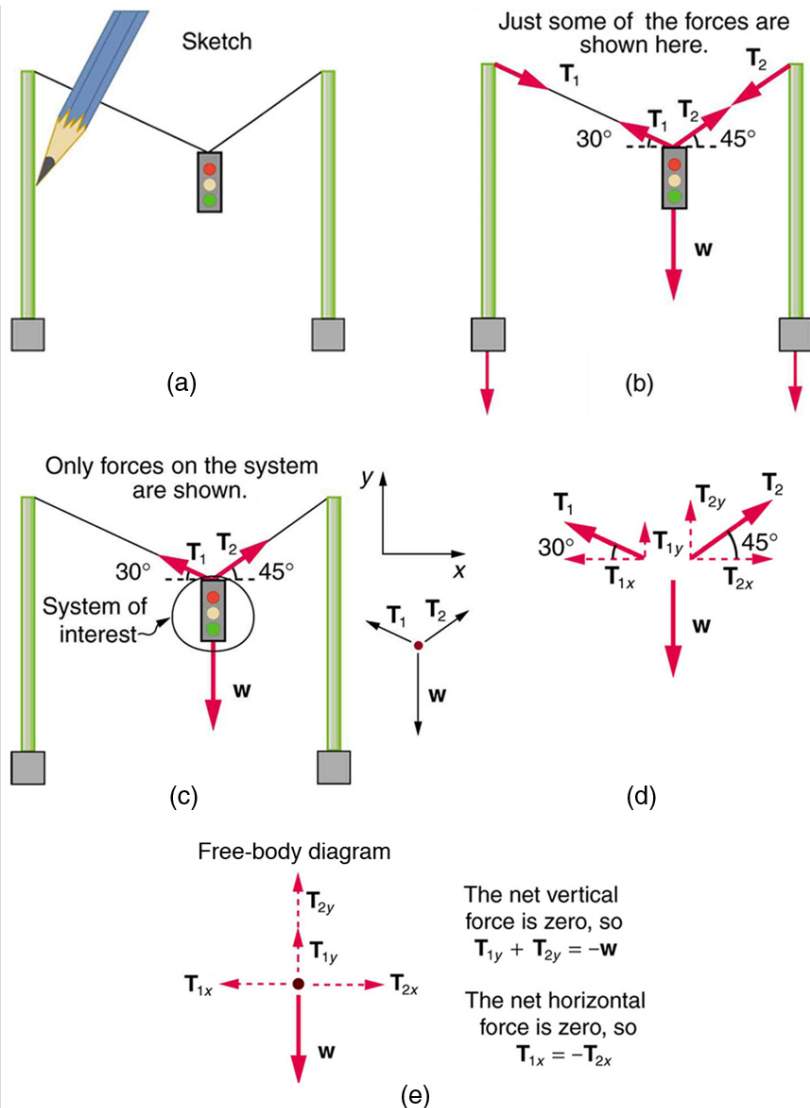
The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where  $F_{\text{D}}$  is less than 1/600th of the weight of the ship.

In the earlier example of a tightrope walker we noted that the tensions in wires supporting a mass were equal only because the angles on either side were equal. Consider the following example, where the angles are not equal; slightly more trigonometry is involved.

### Example:

#### Different Tensions at Different Angles

Consider the traffic light (mass 15.0 kg) suspended from two wires as shown in [\[link\]](#). Find the tension in each wire, neglecting the masses of the wires.



A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical ( $y$ ) and horizontal ( $x$ ) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The free-body diagram shows the vertical and horizontal forces acting on the traffic light.

## Strategy

The system of interest is the traffic light, and its free-body diagram is shown in [\[link\]](#) (c). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in part (d) of the figure. There are two unknowns in this problem ( $T_1$  and  $T_2$ ), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

**Solution**

First consider the horizontal or  $x$ -axis:

**Equation:**

$$F_{\text{net}x} = T_{2x} - T_{1x} = 0.$$

Thus, as you might expect,

**Equation:**

$$T_{1x} = T_{2x}.$$

This gives us the following relationship between  $T_1$  and  $T_2$ :

**Equation:**

$$T_1 \cos(30^\circ) = T_2 \cos(45^\circ).$$

Thus,

**Equation:**

$$T_2 = (1.225)T_1.$$

Note that  $T_1$  and  $T_2$  are not equal in this case, because the angles on either side are not equal. It is reasonable that  $T_2$  ends up being greater than  $T_1$ , because it is exerted more vertically than  $T_1$ .

Now consider the force components along the vertical or  $y$ -axis:

**Equation:**

$$F_{\text{net}y} = T_{1y} + T_{2y} - w = 0.$$

This implies

**Equation:**

$$T_{1y} + T_{2y} = w.$$

Substituting the expressions for the vertical components gives

**Equation:**

$$T_1 \sin (30^\circ) + T_2 \sin (45^\circ) = w.$$

There are two unknowns in this equation, but substituting the expression for  $T_2$  in terms of  $T_1$  reduces this to one equation with one unknown:

**Equation:**

$$T_1(0.500) + (1.225T_1)(0.707) = w = mg,$$

which yields

**Equation:**

$$(1.366)T_1 = (15.0 \text{ kg})(9.80 \text{ m/s}^2).$$

Solving this last equation gives the magnitude of  $T_1$  to be

**Equation:**

$$T_1 = 108 \text{ N}.$$

Finally, the magnitude of  $T_2$  is determined using the relationship between them,  $T_2 = 1.225 T_1$ , found above. Thus we obtain

**Equation:**

$$T_2 = 132 \text{ N}.$$

### Discussion

Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker).

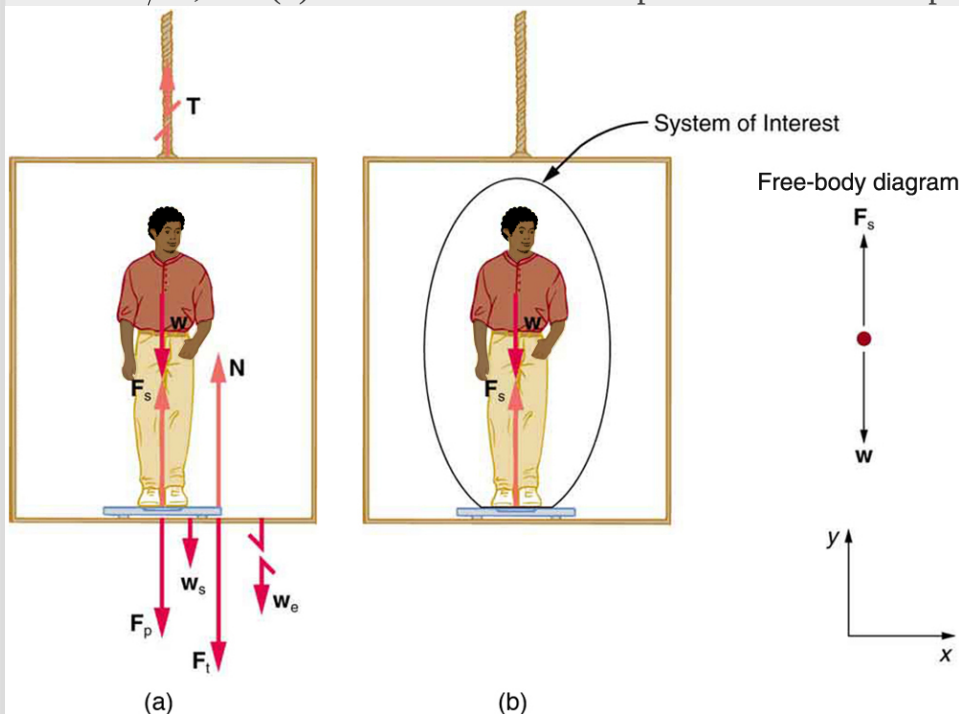
The bathroom scale is an excellent example of a normal force acting on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the dial of a bathroom scale if you stood on it during an elevator ride? Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed: will the scale still read more than your weight at rest? Consider the following example.

### Example:

#### What Does the Bathroom Scale Read in an Elevator?

[\[link\]](#) shows a 75.0-kg man (weight of about 165 lb) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate

of  $1.20 \text{ m/s}^2$ , and (b) if the elevator moves upward at a constant speed of  $1 \text{ m/s}$ .



(a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale.  $\mathbf{T}$  is the tension in the supporting cable,  $\mathbf{w}$  is the weight of the person,  $\mathbf{w}_s$  is the weight of the scale,  $\mathbf{w}_e$  is the weight of the elevator,  $\mathbf{F}_s$  is the force of the scale on the person,  $\mathbf{F}_p$  is the force of the person on the scale,  $\mathbf{F}_t$  is the force of the scale on the floor of the elevator, and  $\mathbf{N}$  is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting on the designated system of interest—the person.

### Strategy

If the scale is accurate, its reading will equal  $F_p$ , the magnitude of the force the person exerts downward on it. [\[link\]](#)(a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn as in [\[link\]](#)(b). Analysis of the free-body diagram using Newton's laws can produce answers to both parts (a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight  $\mathbf{w}$  and the upward force of the scale  $\mathbf{F}_s$ . According to Newton's third law  $\mathbf{F}_p$  and  $\mathbf{F}_s$  are

equal in magnitude and opposite in direction, so that we need to find  $F_s$  in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,

**Equation:**

$$F_{\text{net}} = ma.$$

From the free-body diagram we see that  $F_{\text{net}} = F_s - w$ , so that

**Equation:**

$$F_s - w = ma.$$

Solving for  $F_s$  gives an equation with only one unknown:

**Equation:**

$$F_s = ma + w,$$

or, because  $w = mg$ , simply

**Equation:**

$$F_s = ma + mg.$$

No assumptions were made about the acceleration, and so this solution should be valid for a variety of accelerations in addition to the ones in this exercise.

**Solution for (a)**

In this part of the problem,  $a = 1.20 \text{ m/s}^2$ , so that

**Equation:**

$$F_s = (75.0 \text{ kg})(1.20 \text{ m/s}^2) + (75.0 \text{ kg})(9.80 \text{ m/s}^2),$$

yielding

**Equation:**

$$F_s = 825 \text{ N}.$$

**Discussion for (a)**

This is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

**Equation:**

$$\begin{aligned} F_{\text{net}} &= ma = 0 = F_s - w \\ F_s &= w = mg \\ F_s &= (75.0 \text{ kg})(9.80 \text{ m/s}^2) \\ F_s &= 735 \text{ N}. \end{aligned}$$

So, the scale reading in the elevator is greater than his 735-N (165 lb) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators.

**Solution for (b)**

Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because  $a = \frac{\Delta v}{\Delta t}$ , and  $\Delta v = 0$ .

Thus,

**Equation:**

$$F_s = ma + mg = 0 + mg.$$

Now

**Equation:**

$$F_s = (75.0 \text{ kg})(9.80 \text{ m/s}^2),$$

which gives

**Equation:**

$$F_s = 735 \text{ N}.$$

**Discussion for (b)**

The scale reading is 735 N, which equals the person's weight. This will be the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward,  $a$  is negative, and the scale reading is *less* than the weight of the person, until a constant downward velocity is reached, at which time the scale reading again becomes equal to the person's weight. If the elevator is in free-fall and accelerating downward at  $g$ , then the scale reading will be zero and the person will *appear* to be weightless.

## **Integrating Concepts: Newton's Laws of Motion and Kinematics**

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton's laws of motion can also be integrated with other concepts that have been discussed previously in this text to

solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters. When approaching problems that involve various types of forces, acceleration, velocity, and/or position, use the following steps to approach the problem:

**Problem-Solving Strategy**

- Step 1. *Identify which physical principles are involved.* Listing the givens and the quantities to be calculated will allow you to identify the principles involved.
- Step 2. *Solve the problem using strategies outlined in the text.* If these are available for the specific topic, you should refer to them. You should also refer to the sections of the text that deal with a particular topic. The following worked example illustrates how these strategies are applied to an integrated concept problem.

**Example:**

**What Force Must a Soccer Player Exert to Reach Top Speed?**

A soccer player starts from rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player’s mass is 70.0 kg, and air resistance is negligible.

**Strategy**

To solve an integrated concept problem, we must first identify the physical principles involved and then identify the chapters in which they are found. Part (a) of this example considers acceleration along a straight line. This is a topic of kinematics. Part (b) deals with force, a topic of dynamics found in this chapter.

The following solutions to each part of the example illustrate how the specific

problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so forth.

**Solution for (a)**

We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is  $\Delta v = 8.00 \text{ m/s}$ . We are given the elapsed time, and so  $\Delta t = 2.50 \text{ s}$ . The unknown is acceleration, which can be found from its definition:

**Equation:**

$$a = \frac{\Delta v}{\Delta t}.$$

Substituting the known values yields

**Equation:**

$$\begin{aligned} a &= \frac{8.00 \text{ m/s}}{2.50 \text{ s}} \\ &= 3.20 \text{ m/s}^2. \end{aligned}$$

**Discussion for (a)**

This is an attainable acceleration for an athlete in good condition.

**Solution for (b)**

Here we are asked to find the average force the player exerts backward to achieve this forward acceleration. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes his acceleration. Since we now know the player's acceleration and are given his mass, we can use Newton's second law to find the force exerted. That is,

**Equation:**

$$F_{\text{net}} = ma.$$

Substituting the known values of  $m$  and  $a$  gives

**Equation:**

$$\begin{aligned} F_{\text{net}} &= (70.0 \text{ kg})(3.20 \text{ m/s}^2) \\ &= 224 \text{ N}. \end{aligned}$$

**Discussion for (b)**

This is about 50 pounds, a reasonable average force.

This worked example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These strategies are found throughout the text, and many worked examples show how to use them for single topics. You will find these

techniques for integrated concept problems useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

## Summary

- Newton's laws of motion can be applied in numerous situations to solve problems of motion.
- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether  $F_{\text{net}} = ma$  or  $F_{\text{net}} = 0$ .
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.
- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

## Conceptual Questions

### Exercise:

#### Problem:

To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at  $g$ . Why will they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?

### Exercise:

#### Problem:

A cartoon shows the toupee coming off the head of an elevator passenger when the elevator rapidly stops during an upward ride. Can this really happen without the person being tied to the floor of the elevator? Explain your answer.

## Problem Exercises

### Exercise:

#### Problem:

A flea jumps by exerting a force of  $1.20 \times 10^{-5}$  N straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of  $0.500 \times 10^{-6}$  N on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is  $6.00 \times 10^{-7}$  kg. Do not neglect the gravitational force.

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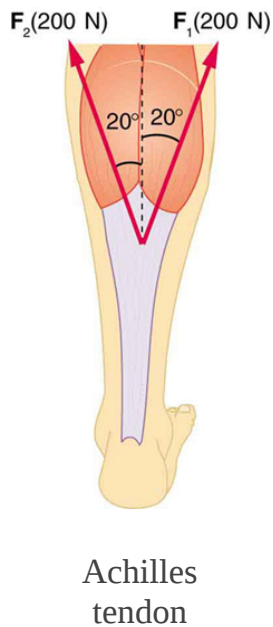
#### Solution:

$10.2 \text{ m/s}^2$ ,  $4.67^\circ$  from vertical

### Exercise:

#### Problem:

Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in [\[link\]](#). (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?

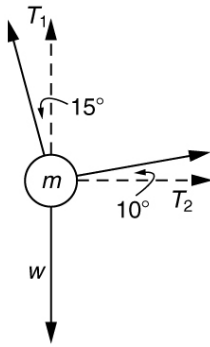


### Exercise:

**Problem:**

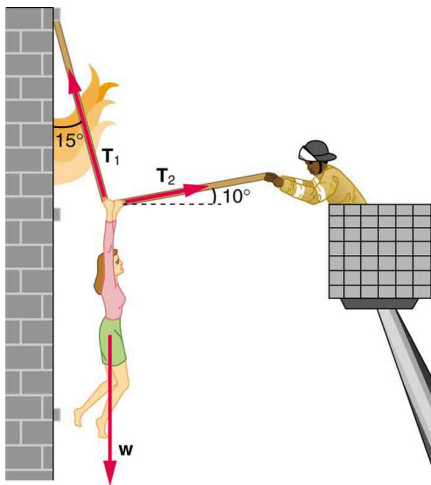
A 76.0-kg person is being pulled away from a burning building as shown in [\[link\]](#). Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.

---

**Solution:**

$$T_1 = 736 \text{ N}$$

$$T_2 = 194 \text{ N}$$



The force  $T_2$  needed to hold steady the person being rescued from the fire is less than her weight and less than the force  $T_1$  in the other rope, since the more

vertical rope supports a greater part of her weight (a vertical force).

**Exercise:**

**Problem:**

**Integrated Concepts** A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow him if he was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)

**Exercise:**

**Problem:**

**Integrated Concepts** When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?

---

**Solution:**

(a) 7.43 m/s

(b) 2.97 m

**Exercise:**

**Problem:**

**Integrated Concepts** A large rocket has a mass of  $2.00 \times 10^6$  kg at takeoff, and its engines produce a thrust of  $3.50 \times 10^7$  N. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust? (c) In reality, the mass of a rocket decreases significantly as its fuel is consumed. Describe qualitatively how this affects the acceleration and time for this motion.

**Exercise:**

**Problem:**

**Integrated Concepts** A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force he exerts on the floor to do this, given that his mass is 110 kg.

---

**Solution:**

(a) 4.20 m/s

(b) 29.4 m/s<sup>2</sup>

(c)  $4.31 \times 10^3$  N

**Exercise:****Problem:**

**Integrated Concepts** A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110 m. (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell's velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.

**Exercise:****Problem:**

**Integrated Concepts** Repeat [\[link\]](#) for a shell fired at an angle 10.0° from the vertical.

---

**Solution:**

(a) 47.1 m/s

(b)  $2.47 \times 10^3$  m/s<sup>2</sup>

(c)  $6.18 \times 10^3$  N . The average force is 252 times the shell's weight.

**Exercise:**

**Problem:**

**Integrated Concepts** An elevator filled with passengers has a mass of 1700 kg. (a) The elevator accelerates upward from rest at a rate of  $1.20 \text{ m/s}^2$  for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of  $0.600 \text{ m/s}^2$  for 3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?

**Exercise:**

**Problem:**

**Unreasonable Results** (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of  $0.400 \text{ m/s}^2$  for 50.0 s? (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

**Exercise:**

**Problem:**

**Unreasonable Results** A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

# Introduction to Fluid Dynamics and Its Biological and Medical Applications

class="introduction"

Many fluids are flowing in this scene.

Water from the hose and smoke from the fire are visible flows.

Less visible are the flow of air and the flow of fluids on the ground and within the people fighting the fire.

Explore all types of flow, such as visible, implied, turbulent, laminar, and so on,

present in  
this scene.

Make a  
list and  
discuss  
the  
relative  
energies  
involved  
in the  
various  
flows,  
including  
the level  
of  
confidence  
in your  
estimates.

(credit:  
Andrew  
Magill,  
Flickr)



We have dealt with many situations in which fluids are static. But by their very definition, fluids flow. Examples come easily—a column of smoke rises from a camp fire, water streams from a fire hose, blood courses through your veins. Why does rising smoke curl and twist? How does a nozzle increase the speed of water emerging from a hose? How does the body regulate blood flow? The physics of fluids in motion—**fluid dynamics**—allows us to answer these and many other questions.

## Glossary

fluid dynamics

the physics of fluids in motion

## Flow Rate and Its Relation to Velocity

- Calculate flow rate.
- Define units of volume.
- Describe incompressible fluids.
- Explain the consequences of the equation of continuity.

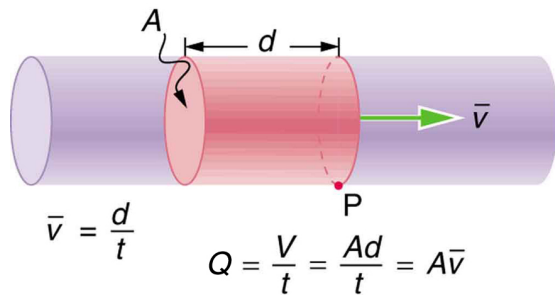
**Flow rate**  $Q$  is defined to be the volume of fluid passing by some location through an area during a period of time, as seen in [\[link\]](#). In symbols, this can be written as

**Equation:**

$$Q = \frac{V}{t},$$

where  $V$  is the volume and  $t$  is the elapsed time.

The SI unit for flow rate is  $\text{m}^3/\text{s}$ , but a number of other units for  $Q$  are in common use. For example, the heart of a resting adult pumps blood at a rate of 5.00 liters per minute (L/min). Note that a **liter** (L) is 1/1000 of a cubic meter or 1000 cubic centimeters ( $10^{-3} \text{ m}^3$  or  $10^3 \text{ cm}^3$ ). In this text we shall use whatever metric units are most convenient for a given situation.



Flow rate is the volume of fluid per unit time flowing past a point through the area  $A$ . Here the shaded cylinder of fluid flows past point P in a uniform pipe in time  $t$ . The volume of the cylinder is  $Ad$

and the average velocity is  
 $v = d/t$  so that the flow rate  
is  $Q = Ad/t = Av$ .

**Example:**

**Calculating Volume from Flow Rate: The Heart Pumps a Lot of Blood in a Lifetime**

How many cubic meters of blood does the heart pump in a 75-year lifetime, assuming the average flow rate is 5.00 L/min?

**Strategy**

Time and flow rate  $Q$  are given, and so the volume  $V$  can be calculated from the definition of flow rate.

**Solution**

Solving  $Q = V/t$  for volume gives

**Equation:**

$$V = Qt.$$

Substituting known values yields

**Equation:**

$$\begin{aligned} V &= \left( \frac{5.00 \text{ L}}{1 \text{ min}} \right) (75 \text{ y}) \left( \frac{1 \text{ m}^3}{10^3 \text{ L}} \right) \left( 5.26 \times 10^5 \frac{\text{min}}{\text{y}} \right) \\ &= 2.0 \times 10^5 \text{ m}^3. \end{aligned}$$

**Discussion**

This amount is about 200,000 tons of blood. For comparison, this value is equivalent to about 200 times the volume of water contained in a 6-lane 50-m lap pool.

Flow rate and velocity are related, but quite different, physical quantities. To make the distinction clear, think about the flow rate of a river. The

greater the velocity of the water, the greater the flow rate of the river. But flow rate also depends on the size of the river. A rapid mountain stream carries far less water than the Amazon River in Brazil, for example. The precise relationship between flow rate  $Q$  and velocity  $v$  is

**Equation:**

$$Q = Av,$$

where  $A$  is the cross-sectional area and  $v$  is the average velocity. This equation seems logical enough. The relationship tells us that flow rate is directly proportional to both the magnitude of the average velocity (hereafter referred to as the speed) and the size of a river, pipe, or other conduit. The larger the conduit, the greater its cross-sectional area. [\[link\]](#) illustrates how this relationship is obtained. The shaded cylinder has a volume

**Equation:**

$$V = Ad,$$

which flows past the point P in a time  $t$ . Dividing both sides of this relationship by  $t$  gives

**Equation:**

$$\frac{V}{t} = \frac{Ad}{t}.$$

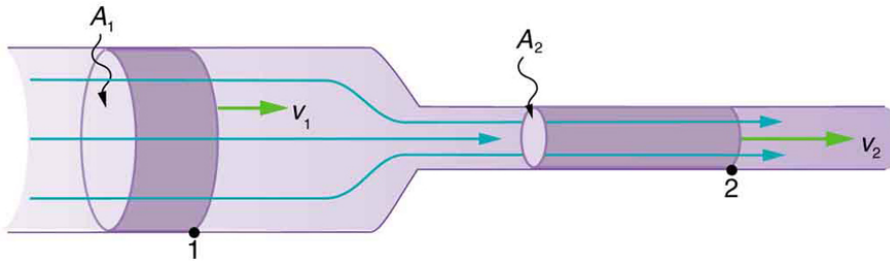
We note that  $Q = V/t$  and the average speed is  $v = d/t$ . Thus the equation becomes  $Q = Av$ .

[\[link\]](#) shows an incompressible fluid flowing along a pipe of decreasing radius. Because the fluid is incompressible, the same amount of fluid must flow past any point in the tube in a given time to ensure continuity of flow. In this case, because the cross-sectional area of the pipe decreases, the velocity must necessarily increase. This logic can be extended to say that the flow rate must be the same at all points along the pipe. In particular, for points 1 and 2,

**Equation:**

$$\left. \begin{array}{l} Q_1 = Q_2 \\ A_1 v_1 = A_2 v_2 \end{array} \right\}.$$

This is called the equation of continuity and is valid for any incompressible fluid. The consequences of the equation of continuity can be observed when water flows from a hose into a narrow spray nozzle: it emerges with a large speed—that is the purpose of the nozzle. Conversely, when a river empties into one end of a reservoir, the water slows considerably, perhaps picking up speed again when it leaves the other end of the reservoir. In other words, speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases.



When a tube narrows, the same volume occupies a greater length. For the same volume to pass points 1 and 2 in a given time, the speed must be greater at point 2. The process is exactly reversible. If the fluid flows in the opposite direction, its speed will decrease when the tube widens. (Note that the relative volumes of the two cylinders and the corresponding velocity vector arrows are not drawn to scale.)

Since liquids are essentially incompressible, the equation of continuity is valid for all liquids. However, gases are compressible, and so the equation must be applied with caution to gases if they are subjected to compression or expansion.

**Example:****Calculating Fluid Speed: Speed Increases When a Tube Narrows**

A nozzle with a radius of 0.250 cm is attached to a garden hose with a radius of 0.900 cm. The flow rate through hose and nozzle is 0.500 L/s. Calculate the speed of the water (a) in the hose and (b) in the nozzle.

**Strategy**

We can use the relationship between flow rate and speed to find both velocities. We will use the subscript 1 for the hose and 2 for the nozzle.

**Solution for (a)**

First, we solve  $Q = Av$  for  $v_1$  and note that the cross-sectional area is  $A = \pi r^2$ , yielding

**Equation:**

$$v_1 = \frac{Q}{A_1} = \frac{Q}{\pi r_1^2}.$$

Substituting known values and making appropriate unit conversions yields

**Equation:**

$$v_1 = \frac{(0.500 \text{ L/s})(10^{-3} \text{ m}^3/\text{L})}{\pi(9.00 \times 10^{-3} \text{ m})^2} = 1.96 \text{ m/s}.$$

**Solution for (b)**

We could repeat this calculation to find the speed in the nozzle  $v_2$ , but we will use the equation of continuity to give a somewhat different insight.

Using the equation which states

**Equation:**

$$A_1 v_1 = A_2 v_2,$$

solving for  $v_2$  and substituting  $\pi r^2$  for the cross-sectional area yields

**Equation:**

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi r_1^2}{\pi r_2^2} v_1 = \frac{r_1^2}{r_2^2} v_1.$$

Substituting known values,

**Equation:**

$$v_2 = \frac{(0.900 \text{ cm})^2}{(0.250 \text{ cm})^2} 1.96 \text{ m/s} = 25.5 \text{ m/s}.$$

**Discussion**

A speed of 1.96 m/s is about right for water emerging from a nozzleless hose. The nozzle produces a considerably faster stream merely by constricting the flow to a narrower tube.

The solution to the last part of the example shows that speed is inversely proportional to the *square* of the radius of the tube, making for large effects when radius varies. We can blow out a candle at quite a distance, for example, by pursing our lips, whereas blowing on a candle with our mouth wide open is quite ineffective.

In many situations, including in the cardiovascular system, branching of the flow occurs. The blood is pumped from the heart into arteries that subdivide into smaller arteries (arterioles) which branch into very fine vessels called capillaries. In this situation, continuity of flow is maintained but it is the *sum* of the flow rates in each of the branches in any portion along the tube that is maintained. The equation of continuity in a more general form becomes

**Equation:**

$$n_1 A_1 v_1 = n_2 A_2 v_2,$$

where  $n_1$  and  $n_2$  are the number of branches in each of the sections along the tube.

**Example:**

**Calculating Flow Speed and Vessel Diameter: Branching in the Cardiovascular System**

The aorta is the principal blood vessel through which blood leaves the heart in order to circulate around the body. (a) Calculate the average speed of the blood in the aorta if the flow rate is 5.0 L/min. The aorta has a radius of 10 mm. (b) Blood also flows through smaller blood vessels known as capillaries. When the rate of blood flow in the aorta is 5.0 L/min, the speed of blood in the capillaries is about 0.33 mm/s. Given that the average diameter of a capillary is 8.0  $\mu\text{m}$ , calculate the number of capillaries in the blood circulatory system.

### Strategy

We can use  $Q = Av$  to calculate the speed of flow in the aorta and then use the general form of the equation of continuity to calculate the number of capillaries as all of the other variables are known.

### Solution for (a)

The flow rate is given by  $Q = Av$  or  $v = \frac{Q}{\pi r^2}$  for a cylindrical vessel. Substituting the known values (converted to units of meters and seconds) gives

### Equation:

$$v = \frac{(5.0 \text{ L/min})(10^{-3} \text{ m}^3/\text{L})(1 \text{ min}/60 \text{ s})}{\pi(0.010 \text{ m})^2} = 0.27 \text{ m/s}.$$

### Solution for (b)

Using  $n_1 A_1 v_1 = n_2 A_2 v_2$ , assigning the subscript 1 to the aorta and 2 to the capillaries, and solving for  $n_2$  (the number of capillaries) gives  $n_2 = \frac{n_1 A_1 v_1}{A_2 v_2}$ . Converting all quantities to units of meters and seconds and substituting into the equation above gives

### Equation:

$$n_2 = \frac{(1)(\pi)(10 \times 10^{-3} \text{ m})^2(0.27 \text{ m/s})}{(\pi)(4.0 \times 10^{-6} \text{ m})^2(0.33 \times 10^{-3} \text{ m/s})} = 5.0 \times 10^9 \text{ capillaries}.$$

### Discussion

Note that the speed of flow in the capillaries is considerably reduced relative to the speed in the aorta due to the significant increase in the total cross-sectional area at the capillaries. This low speed is to allow sufficient

time for effective exchange to occur although it is equally important for the flow not to become stationary in order to avoid the possibility of clotting. Does this large number of capillaries in the body seem reasonable? In active muscle, one finds about 200 capillaries per  $\text{mm}^3$ , or about  $200 \times 10^6$  per 1 kg of muscle. For 20 kg of muscle, this amounts to about  $4 \times 10^9$  capillaries.

## Section Summary

- Flow rate  $Q$  is defined to be the volume  $V$  flowing past a point in time  $t$ , or  $Q = \frac{V}{t}$  where  $V$  is volume and  $t$  is time.
- The SI unit of volume is  $\text{m}^3$ .
- Another common unit is the liter (L), which is  $10^{-3} \text{ m}^3$ .
- Flow rate and velocity are related by  $Q = Av$  where  $A$  is the cross-sectional area of the flow and  $v$  is its average velocity.
- For incompressible fluids, flow rate at various points is constant. That is,

**Equation:**

$$\begin{aligned}Q_1 &= Q_2 \\A_1 v_1 &= A_2 v_2 \quad . \\n_1 A_1 v_1 &= n_2 A_2 v_2\end{aligned}$$

## Conceptual Questions

**Exercise:**

**Problem:**

What is the difference between flow rate and fluid velocity? How are they related?

**Exercise:**

**Problem:**

Many figures in the text show streamlines. Explain why fluid velocity is greatest where streamlines are closest together. (Hint: Consider the relationship between fluid velocity and the cross-sectional area through which it flows.)

**Exercise:****Problem:**

Identify some substances that are incompressible and some that are not.

**Problems & Exercises****Exercise:****Problem:**

What is the average flow rate in  $\text{cm}^3/\text{s}$  of gasoline to the engine of a car traveling at 100 km/h if it averages 10.0 km/L?

---

**Solution:**

$$2.78 \text{ cm}^3/\text{s}$$

**Exercise:****Problem:**

The heart of a resting adult pumps blood at a rate of 5.00 L/min. (a) Convert this to  $\text{cm}^3/\text{s}$ . (b) What is this rate in  $\text{m}^3/\text{s}$ ?

**Exercise:****Problem:**

Blood is pumped from the heart at a rate of 5.0 L/min into the aorta (of radius 1.0 cm). Determine the speed of blood through the aorta.

---

**Solution:**

27 cm/s

**Exercise:****Problem:**

Blood is flowing through an artery of radius 2 mm at a rate of 40 cm/s. Determine the flow rate and the volume that passes through the artery in a period of 30 s.

**Exercise:****Problem:**

The Huka Falls on the Waikato River is one of New Zealand's most visited natural tourist attractions (see [\[link\]](#)). On average the river has a flow rate of about 300,000 L/s. At the gorge, the river narrows to 20 m wide and averages 20 m deep. (a) What is the average speed of the river in the gorge? (b) What is the average speed of the water in the river downstream of the falls when it widens to 60 m and its depth increases to an average of 40 m?



The Huka Falls in Taupo,  
New Zealand, demonstrate  
flow rate. (credit:  
RaviGogna, Flickr)

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**Solution:**

(a) 0.75 m/s

(b) 0.13 m/s

**Exercise:****Problem:**

A major artery with a cross-sectional area of  $1.00 \text{ cm}^2$  branches into 18 smaller arteries, each with an average cross-sectional area of  $0.400 \text{ cm}^2$ . By what factor is the average velocity of the blood reduced when it passes into these branches?

**Exercise:****Problem:**

(a) As blood passes through the capillary bed in an organ, the capillaries join to form venules (small veins). If the blood speed increases by a factor of 4.00 and the total cross-sectional area of the venules is  $10.0 \text{ cm}^2$ , what is the total cross-sectional area of the capillaries feeding these venules? (b) How many capillaries are involved if their average diameter is  $10.0 \mu\text{m}$ ?

---

**Solution:**

(a)  $40.0 \text{ cm}^2$

(b)  $5.09 \times 10^7$

**Exercise:****Problem:**

The human circulation system has approximately  $1 \times 10^9$  capillary vessels. Each vessel has a diameter of about  $8 \mu\text{m}$ . Assuming cardiac output is 5 L/min, determine the average velocity of blood flow through each capillary vessel.

**Exercise:****Problem:**

(a) Estimate the time it would take to fill a private swimming pool with a capacity of 80,000 L using a garden hose delivering 60 L/min. (b) How long would it take to fill if you could divert a moderate size river, flowing at  $5000 \text{ m}^3/\text{s}$ , into it?

---

**Solution:**

(a) 22 h

(b) 0.016 s

**Exercise:****Problem:**

The flow rate of blood through a  $2.00 \times 10^{-6}\text{-m}$  -radius capillary is  $3.80 \times 10^{-9} \text{ cm}^3/\text{s}$ . (a) What is the speed of the blood flow? (This small speed allows time for diffusion of materials to and from the blood.) (b) Assuming all the blood in the body passes through capillaries, how many of them must there be to carry a total flow of  $90.0 \text{ cm}^3/\text{s}$ ? (The large number obtained is an overestimate, but it is still reasonable.)

**Exercise:****Problem:**

(a) What is the fluid speed in a fire hose with a 9.00-cm diameter carrying 80.0 L of water per second? (b) What is the flow rate in cubic meters per second? (c) Would your answers be different if salt water replaced the fresh water in the fire hose?

---

**Solution:**

(a) 12.6 m/s

(b)  $0.0800 \text{ m}^3/\text{s}$

(c) No, independent of density.

**Exercise:**

**Problem:**

The main uptake air duct of a forced air gas heater is  $0.300 \text{ m}$  in diameter. What is the average speed of air in the duct if it carries a volume equal to that of the house's interior every  $15 \text{ min}$ ? The inside volume of the house is equivalent to a rectangular solid  $13.0 \text{ m}$  wide by  $20.0 \text{ m}$  long by  $2.75 \text{ m}$  high.

**Exercise:**

**Problem:**

Water is moving at a velocity of  $2.00 \text{ m/s}$  through a hose with an internal diameter of  $1.60 \text{ cm}$ . (a) What is the flow rate in liters per second? (b) The fluid velocity in this hose's nozzle is  $15.0 \text{ m/s}$ . What is the nozzle's inside diameter?

---

**Solution:**

(a)  $0.402 \text{ L/s}$

(b)  $0.584 \text{ cm}$

**Exercise:**

**Problem:**

Prove that the speed of an incompressible fluid through a constriction, such as in a Venturi tube, increases by a factor equal to the square of the factor by which the diameter decreases. (The converse applies for flow out of a constriction into a larger-diameter region.)

**Exercise:**

**Problem:**

Water emerges straight down from a faucet with a 1.80-cm diameter at a speed of 0.500 m/s. (Because of the construction of the faucet, there is no variation in speed across the stream.) (a) What is the flow rate in  $\text{cm}^3/\text{s}$ ? (b) What is the diameter of the stream 0.200 m below the faucet? Neglect any effects due to surface tension.

---

**Solution:**

(a)  $127 \text{ cm}^3/\text{s}$

(b) 0.890 cm

**Exercise:****Problem: Unreasonable Results**

A mountain stream is 10.0 m wide and averages 2.00 m in depth. During the spring runoff, the flow in the stream reaches  $100,000 \text{ m}^3/\text{s}$ . (a) What is the average velocity of the stream under these conditions? (b) What is unreasonable about this velocity? (c) What is unreasonable or inconsistent about the premises?

**Glossary**

flow rate

abbreviated  $Q$ , it is the volume  $V$  that flows past a particular point during a time  $t$ , or  $Q = V/t$

liter

a unit of volume, equal to  $10^{-3} \text{ m}^3$

## Bernoulli's Equation

- Explain the terms in Bernoulli's equation.
- Explain how Bernoulli's equation is related to conservation of energy.
- Explain how to derive Bernoulli's principle from Bernoulli's equation.
- Calculate with Bernoulli's principle.
- List some applications of Bernoulli's principle.

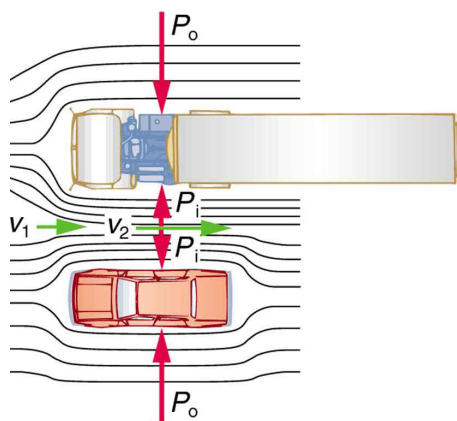
When a fluid flows into a narrower channel, its speed increases. That means its kinetic energy also increases. Where does that change in kinetic energy come from? The increased kinetic energy comes from the net work done on the fluid to push it into the channel and the work done on the fluid by the gravitational force, if the fluid changes vertical position. Recall the work-energy theorem,

**Equation:**

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

There is a pressure difference when the channel narrows. This pressure difference results in a net force on the fluid: recall that pressure times area equals force. The net work done increases the fluid's kinetic energy. As a result, the *pressure will drop in a rapidly-moving fluid*, whether or not the fluid is confined to a tube.

There are a number of common examples of pressure dropping in rapidly-moving fluids. Shower curtains have a disagreeable habit of bulging into the shower stall when the shower is on. The high-velocity stream of water and air creates a region of lower pressure inside the shower, and standard atmospheric pressure on the other side. The pressure difference results in a net force inward pushing the curtain in. You may also have noticed that when passing a truck on the highway, your car tends to veer toward it. The reason is the same—the high velocity of the air between the car and the truck creates a region of lower pressure, and the vehicles are pushed together by greater pressure on the outside. (See [\[link\]](#).) This effect was observed as far back as the mid-1800s, when it was found that trains passing in opposite directions tipped precariously toward one another.



An overhead view of a car passing a truck on a highway. Air passing between the vehicles flows in a narrower channel and must increase its speed ( $v_2$  is greater than  $v_1$ ), causing the pressure between them to drop ( $P_i$  is less than  $P_o$ ). Greater pressure on the outside pushes the car and truck together.

**Note:**

**Making Connections: Take-Home Investigation with a Sheet of Paper**  
Hold the short edge of a sheet of paper parallel to your mouth with one hand on each side of your mouth. The page should slant downward over your hands. Blow over the top of the page. Describe what happens and explain the reason for this behavior.

## Bernoulli's Equation

The relationship between pressure and velocity in fluids is described quantitatively by **Bernoulli's equation**, named after its discoverer, the Swiss scientist Daniel Bernoulli (1700–1782). Bernoulli's equation states that for an incompressible, frictionless fluid, the following sum is constant:  
**Equation:**

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant},$$

where  $P$  is the absolute pressure,  $\rho$  is the fluid density,  $v$  is the velocity of the fluid,  $h$  is the height above some reference point, and  $g$  is the acceleration due to gravity. If we follow a small volume of fluid along its path, various quantities in the sum may change, but the total remains constant. Let the subscripts 1 and 2 refer to any two points along the path that the bit of fluid follows; Bernoulli's equation becomes

**Equation:**

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2.$$

Bernoulli's equation is a form of the conservation of energy principle. Note that the second and third terms are the kinetic and potential energy with  $m$  replaced by  $\rho$ . In fact, each term in the equation has units of energy per unit volume. We can prove this for the second term by substituting  $\rho = m/V$  into it and gathering terms:

**Equation:**

$$\frac{1}{2}\rho v^2 = \frac{\frac{1}{2}mv^2}{V} = \frac{\text{KE}}{V}.$$

So  $\frac{1}{2}\rho v^2$  is the kinetic energy per unit volume. Making the same substitution into the third term in the equation, we find

**Equation:**

$$\rho gh = \frac{mgh}{V} = \frac{PE_g}{V},$$

so  $\rho gh$  is the gravitational potential energy per unit volume. Note that pressure  $P$  has units of energy per unit volume, too. Since  $P = F/A$ , its units are  $\text{N}/\text{m}^2$ . If we multiply these by  $\text{m}/\text{m}$ , we obtain  $\text{N} \cdot \text{m}/\text{m}^3 = \text{J}/\text{m}^3$ , or energy per unit volume. Bernoulli's equation is, in fact, just a convenient statement of conservation of energy for an incompressible fluid in the absence of friction.

**Note:**

**Making Connections: Conservation of Energy**

Conservation of energy applied to fluid flow produces Bernoulli's equation. The net work done by the fluid's pressure results in changes in the fluid's KE and  $PE_g$  per unit volume. If other forms of energy are involved in fluid flow, Bernoulli's equation can be modified to take these forms into account. Such forms of energy include thermal energy dissipated because of fluid viscosity.

The general form of Bernoulli's equation has three terms in it, and it is broadly applicable. To understand it better, we will look at a number of specific situations that simplify and illustrate its use and meaning.

## Bernoulli's Equation for Static Fluids

Let us first consider the very simple situation where the fluid is static—that is,  $v_1 = v_2 = 0$ . Bernoulli's equation in that case is

**Equation:**

$$P_1 + \rho gh_1 = P_2 + \rho gh_2.$$

We can further simplify the equation by taking  $h_2 = 0$  (we can always choose some height to be zero, just as we often have done for other situations involving the gravitational force, and take all other heights to be relative to this). In that case, we get

**Equation:**

$$P_2 = P_1 + \rho gh_1 .$$

This equation tells us that, in static fluids, pressure increases with depth. As we go from point 1 to point 2 in the fluid, the depth increases by  $h_1$ , and consequently,  $P_2$  is greater than  $P_1$  by an amount  $\rho gh_1$ . In the very simplest case,  $P_1$  is zero at the top of the fluid, and we get the familiar relationship  $P = \rho gh$ . (Recall that  $P = \rho gh$  and  $\Delta PE_g = mgh$ .)

Bernoulli's equation includes the fact that the pressure due to the weight of a fluid is  $\rho gh$ . Although we introduce Bernoulli's equation for fluid flow, it includes much of what we studied for static fluids in the preceding chapter.

## **Bernoulli's Principle—Bernoulli's Equation at Constant Depth**

Another important situation is one in which the fluid moves but its depth is constant—that is,  $h_1 = h_2$ . Under that condition, Bernoulli's equation becomes

**Equation:**

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$$

Situations in which fluid flows at a constant depth are so important that this equation is often called **Bernoulli's principle**. It is Bernoulli's equation for fluids at constant depth. (Note again that this applies to a small volume of fluid as we follow it along its path.) As we have just discussed, pressure drops as speed increases in a moving fluid. We can see this from Bernoulli's principle. For example, if  $v_2$  is greater than  $v_1$  in the equation, then  $P_2$  must be less than  $P_1$  for the equality to hold.

**Example:****Calculating Pressure: Pressure Drops as a Fluid Speeds Up**

In [\[link\]](#), we found that the speed of water in a hose increased from 1.96 m/s to 25.5 m/s going from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is  $1.01 \times 10^5 \text{ N/m}^2$  (atmospheric, as it must be) and assuming level, frictionless flow.

**Strategy**

Level flow means constant depth, so Bernoulli's principle applies. We use the subscript 1 for values in the hose and 2 for those in the nozzle. We are thus asked to find  $P_1$ .

**Solution**

Solving Bernoulli's principle for  $P_1$  yields

**Equation:**

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho(v_2^2 - v_1^2).$$

Substituting known values,

**Equation:**

$$\begin{aligned} P_1 &= 1.01 \times 10^5 \text{ N/m}^2 \\ &\quad + \frac{1}{2}(10^3 \text{ kg/m}^3)[(25.5 \text{ m/s})^2 - (1.96 \text{ m/s})^2] \\ &= 4.24 \times 10^5 \text{ N/m}^2. \end{aligned}$$

**Discussion**

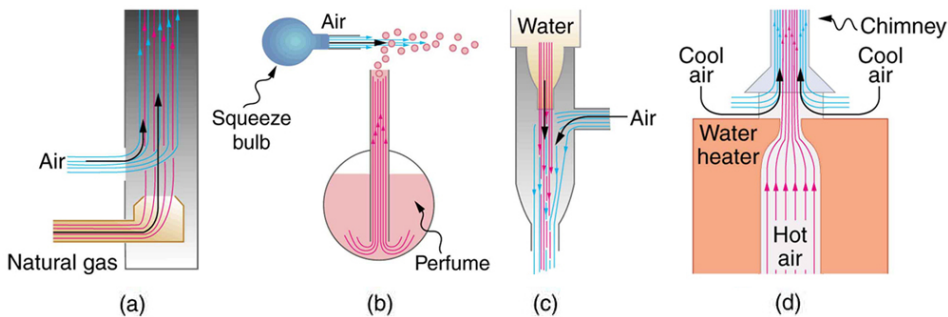
This absolute pressure in the hose is greater than in the nozzle, as expected since  $v$  is greater in the nozzle. The pressure  $P_2$  in the nozzle must be atmospheric since it emerges into the atmosphere without other changes in conditions.

## Applications of Bernoulli's Principle

There are a number of devices and situations in which fluid flows at a constant height and, thus, can be analyzed with Bernoulli's principle.

## Entrainment

People have long put the Bernoulli principle to work by using reduced pressure in high-velocity fluids to move things about. With a higher pressure on the outside, the high-velocity fluid forces other fluids into the stream. This process is called *entrainment*. Entrainment devices have been in use since ancient times, particularly as pumps to raise water small heights, as in draining swamps, fields, or other low-lying areas. Some other devices that use the concept of entrainment are shown in [\[link\]](#).



Examples of entrainment devices that use increased fluid speed to create low pressures, which then entrain one fluid into another. (a) A Bunsen burner uses an adjustable gas nozzle, entraining air for proper combustion. (b) An atomizer uses a squeeze bulb to create a jet of air that entrains drops of perfume. Paint sprayers and carburetors use very similar techniques to move their respective liquids. (c) A common aspirator uses a high-speed stream of water to create a region of lower pressure. Aspirators may be used as suction pumps in dental and surgical situations or for draining a flooded basement or producing a reduced pressure in a vessel. (d) The chimney of a water heater is designed to entrain air into the pipe leading through the ceiling.

## Wings and Sails

The airplane wing is a beautiful example of Bernoulli's principle in action. [\[link\]](#)(a) shows the characteristic shape of a wing. The wing is tilted upward at a small angle and the upper surface is longer, causing air to flow faster over it. The pressure on top of the wing is therefore reduced, creating a net upward force or lift. (Wings can also gain lift by pushing air downward, utilizing the conservation of momentum principle. The deflected air molecules result in an upward force on the wing — Newton's third law.) Sails also have the characteristic shape of a wing. (See [\[link\]](#)(b).) The pressure on the front side of the sail,  $P_{\text{front}}$ , is lower than the pressure on the back of the sail,  $P_{\text{back}}$ . This results in a forward force and even allows you to sail into the wind.

### Note:

#### Making Connections: Take-Home Investigation with Two Strips of Paper

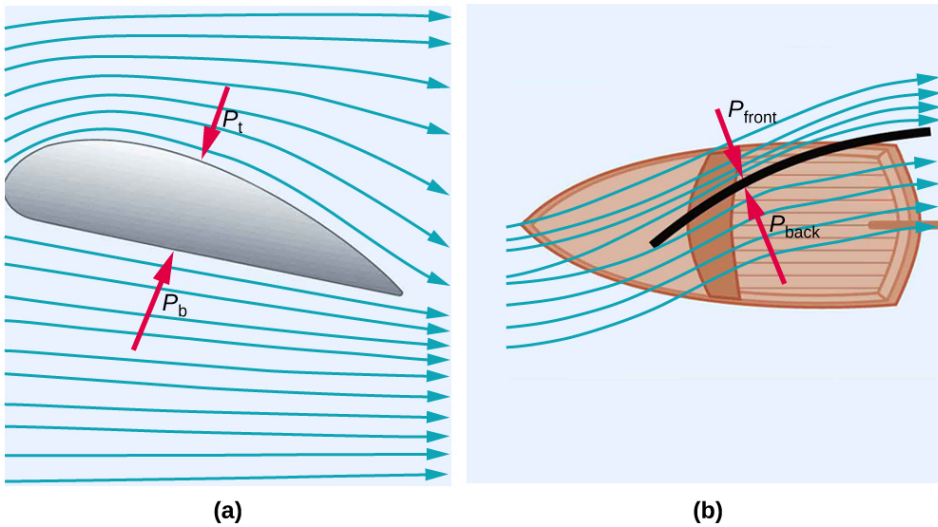
For a good illustration of Bernoulli's principle, make two strips of paper, each about 15 cm long and 4 cm wide. Hold the small end of one strip up to your lips and let it drape over your finger. Blow across the paper. What happens? Now hold two strips of paper up to your lips, separated by your fingers. Blow between the strips. What happens?

## Velocity measurement

[\[link\]](#) shows two devices that measure fluid velocity based on Bernoulli's principle. The manometer in [\[link\]](#)(a) is connected to two tubes that are small enough not to appreciably disturb the flow. The tube facing the oncoming fluid creates a dead spot having zero velocity ( $v_1 = 0$ ) in front of it, while fluid passing the other tube has velocity  $v_2$ . This means that Bernoulli's principle as stated in  $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$  becomes

### Equation:

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2.$$



(a) The Bernoulli principle helps explain lift generated by a wing. (b) Sails use the same technique to generate part of their thrust.

Thus pressure  $P_2$  over the second opening is reduced by  $\frac{1}{2}\rho v_2^2$ , and so the fluid in the manometer rises by  $h$  on the side connected to the second opening, where

**Equation:**

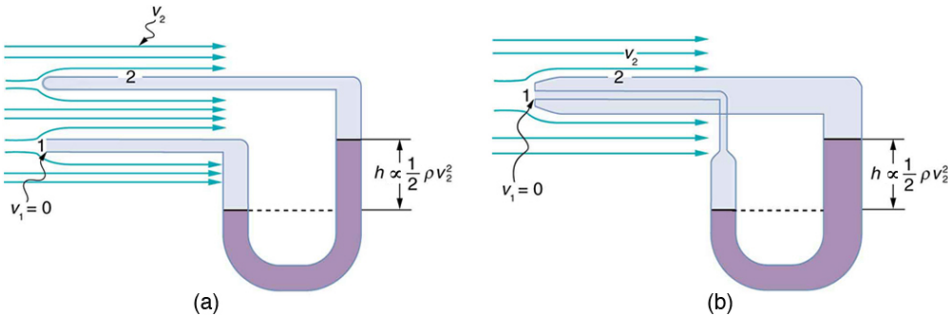
$$h \propto \frac{1}{2}\rho v_2^2.$$

(Recall that the symbol  $\propto$  means “proportional to.”) Solving for  $v_2$ , we see that

**Equation:**

$$v_2 \propto \sqrt{h}.$$

[\[link\]](#)(b) shows a version of this device that is in common use for measuring various fluid velocities; such devices are frequently used as air speed indicators in aircraft.



Measurement of fluid speed based on Bernoulli's principle. (a) A manometer is connected to two tubes that are close together and small enough not to disturb the flow. Tube 1 is open at the end facing the flow. A dead spot having zero speed is created there. Tube 2 has an opening on the side, and so the fluid has a speed  $v$  across the opening; thus, pressure there drops. The difference in pressure at the manometer is  $\frac{1}{2} \rho v_2^2$ , and so  $h$  is proportional to  $\frac{1}{2} \rho v_2^2$ . (b) This type of velocity measuring device is a Prandtl tube, also known as a pitot tube.

## Summary

- Bernoulli's equation states that the sum on each side of the following equation is constant, or the same at any two points in an incompressible frictionless fluid:

**Equation:**

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2.$$

- Bernoulli's principle is Bernoulli's equation applied to situations in which depth is constant. The terms involving depth (or height  $h$ ) subtract out, yielding

**Equation:**

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$$

- Bernoulli's principle has many applications, including entrainment, wings and sails, and velocity measurement.

## Conceptual Questions

**Exercise:**

**Problem:**

You can squirt water a considerably greater distance by placing your thumb over the end of a garden hose and then releasing, than by leaving it completely uncovered. Explain how this works.

**Exercise:**

**Problem:**

Water is shot nearly vertically upward in a decorative fountain and the stream is observed to broaden as it rises. Conversely, a stream of water falling straight down from a faucet narrows. Explain why, and discuss whether surface tension enhances or reduces the effect in each case.

**Exercise:**

**Problem:**

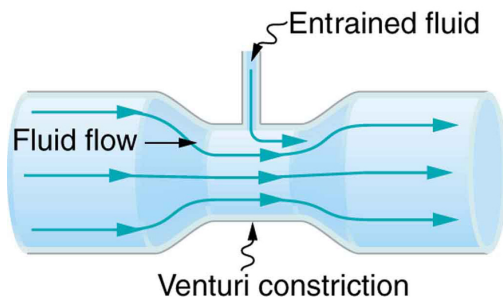
Look back to [\[link\]](#). Answer the following two questions. Why is  $P_o$  less than atmospheric? Why is  $P_o$  greater than  $P_i$ ?

**Exercise:**

**Problem:** Give an example of entrainment not mentioned in the text.

**Exercise:****Problem:**

Many entrainment devices have a constriction, called a Venturi, such as shown in [\[link\]](#). How does this bolster entrainment?



A tube with a narrow segment designed to enhance entrainment is called a Venturi. These are very commonly used in carburetors and aspirators.

**Exercise:****Problem:**

Some chimney pipes have a T-shape, with a crosspiece on top that helps draw up gases whenever there is even a slight breeze. Explain how this works in terms of Bernoulli's principle.

**Exercise:****Problem:**

Is there a limit to the height to which an entrainment device can raise a fluid? Explain your answer.

**Exercise:**

**Problem:**

Why is it preferable for airplanes to take off into the wind rather than with the wind?

**Exercise:**

**Problem:**

Roofs are sometimes pushed off vertically during a tropical cyclone, and buildings sometimes explode outward when hit by a tornado. Use Bernoulli's principle to explain these phenomena.

**Exercise:**

**Problem:** Why does a sailboat need a keel?

**Exercise:**

**Problem:**

It is dangerous to stand close to railroad tracks when a rapidly moving commuter train passes. Explain why atmospheric pressure would push you toward the moving train.

**Exercise:**

**Problem:**

Water pressure inside a hose nozzle can be less than atmospheric pressure due to the Bernoulli effect. Explain in terms of energy how the water can emerge from the nozzle against the opposing atmospheric pressure.

**Exercise:**

**Problem:**

A perfume bottle or atomizer sprays a fluid that is in the bottle. ([link](#).) How does the fluid rise up in the vertical tube in the bottle?



Atomizer:  
perfume  
bottle with  
tube to carry  
perfume up  
through the  
bottle.  
(credit:  
Antonia Foy,  
Flickr)

### **Exercise:**

#### **Problem:**

If you lower the window on a car while moving, an empty plastic bag can sometimes fly out the window. Why does this happen?

## **Problems & Exercises**

### **Exercise:**

**Problem:** Verify that pressure has units of energy per unit volume.

---

**Solution:**

$$\begin{aligned} P &= \frac{\text{Force}}{\text{Area}}, \\ (P)_{\text{units}} &= \text{N/m}^2 = \text{N} \cdot \text{m/m}^3 = \text{J/m}^3 \\ &= \text{energy/volume} \end{aligned}$$

**Exercise:****Problem:**

Suppose you have a wind speed gauge like the pitot tube shown in [\[link\]](#)(b). By what factor must wind speed increase to double the value of  $h$  in the manometer? Is this independent of the moving fluid and the fluid in the manometer?

**Exercise:****Problem:**

If the pressure reading of your pitot tube is 15.0 mm Hg at a speed of 200 km/h, what will it be at 700 km/h at the same altitude?

---

**Solution:**

184 mm Hg

**Exercise:****Problem:**

Calculate the maximum height to which water could be squirted with the hose in [\[link\]](#) example if it: (a) Emerges from the nozzle. (b) Emerges with the nozzle removed, assuming the same flow rate.

**Exercise:**

**Problem:**

Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s (about 100 mi/h) when the jet stream descends during early spring. Approximately what is the force due to the Bernoulli effect on a roof having an area of 220 m<sup>2</sup>? Typical air density in Boulder is 1.14 kg/m<sup>3</sup>, and the corresponding atmospheric pressure is  $8.89 \times 10^4$  N/m<sup>2</sup>. (Bernoulli's principle as stated in the text assumes laminar flow. Using the principle here produces only an approximate result, because there is significant turbulence.)

---

**Solution:**

$$2.54 \times 10^5 \text{ N}$$

**Exercise:****Problem:**

(a) Calculate the approximate force on a square meter of sail, given the horizontal velocity of the wind is 6.00 m/s parallel to its front surface and 3.50 m/s along its back surface. Take the density of air to be 1.29 kg/m<sup>3</sup>. (The calculation, based on Bernoulli's principle, is approximate due to the effects of turbulence.) (b) Discuss whether this force is great enough to be effective for propelling a sailboat.

**Exercise:****Problem:**

(a) What is the pressure drop due to the Bernoulli effect as water goes into a 3.00-cm-diameter nozzle from a 9.00-cm-diameter fire hose while carrying a flow of 40.0 L/s? (b) To what maximum height above the nozzle can this water rise? (The actual height will be significantly smaller due to air resistance.)

---

**Solution:**

(a)  $1.58 \times 10^6 \text{ N/m}^2$

(b) 163 m

**Exercise:**

**Problem:**

(a) Using Bernoulli's equation, show that the measured fluid speed  $v$  for a pitot tube, like the one in [\[link\]](#)(b), is given by

**Equation:**

$$v = \left( \frac{2\rho'gh}{\rho} \right)^{1/2},$$

where  $h$  is the height of the manometer fluid,  $\rho'$  is the density of the manometer fluid,  $\rho$  is the density of the moving fluid, and  $g$  is the acceleration due to gravity. (Note that  $v$  is indeed proportional to the square root of  $h$ , as stated in the text.) (b) Calculate  $v$  for moving air if a mercury manometer's  $h$  is 0.200 m.

## Glossary

**Bernoulli's equation**

the equation resulting from applying conservation of energy to an incompressible frictionless fluid:  $P + 1/2\rho v^2 + \rho gh = \text{constant}$ , through the fluid

**Bernoulli's principle**

Bernoulli's equation applied at constant depth:  $P_1 + 1/2\rho v_1^2 = P_2 + 1/2\rho v_2^2$

## Viscosity and Laminar Flow; Poiseuille's Law

- Define laminar flow and turbulent flow.
- Explain what viscosity is.
- Calculate flow and resistance with Poiseuille's law.
- Explain how pressure drops due to resistance.

### Laminar Flow and Viscosity

When you pour yourself a glass of juice, the liquid flows freely and quickly. But when you pour syrup on your pancakes, that liquid flows slowly and sticks to the pitcher. The difference is fluid friction, both within the fluid itself and between the fluid and its surroundings. We call this property of fluids *viscosity*. Juice has low viscosity, whereas syrup has high viscosity. In the previous sections we have considered ideal fluids with little or no viscosity. In this section, we will investigate what factors, including viscosity, affect the rate of fluid flow.

The precise definition of viscosity is based on *laminar*, or nonturbulent, flow. Before we can define viscosity, then, we need to define laminar flow and turbulent flow.

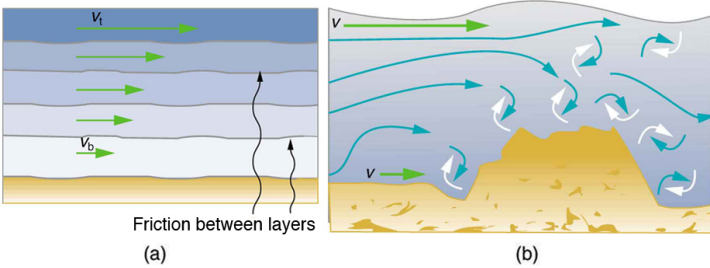
[\[link\]](#) shows both types of flow. **Laminar** flow is characterized by the smooth flow of the fluid in layers that do not mix. Turbulent flow, or **turbulence**, is characterized by eddies and swirls that mix layers of fluid together.



Smoke rises smoothly for a while and then

begins to form  
swirls and eddies.  
The smooth flow is  
called laminar flow,  
whereas the swirls  
and eddies typify  
turbulent flow. If  
you watch the  
smoke (being  
careful not to  
breathe on it), you  
will notice that it  
rises more rapidly  
when flowing  
smoothly than after  
it becomes  
turbulent, implying  
that turbulence  
poses more  
resistance to flow.  
(credit:  
Creativity103)

[\[link\]](#) shows schematically how laminar and turbulent flow differ. Layers flow without mixing when flow is laminar. When there is turbulence, the layers mix, and there are significant velocities in directions other than the overall direction of flow. The lines that are shown in many illustrations are the paths followed by small volumes of fluids. These are called *streamlines*. Streamlines are smooth and continuous when flow is laminar, but break up and mix when flow is turbulent. Turbulence has two main causes. First, any obstruction or sharp corner, such as in a faucet, creates turbulence by imparting velocities perpendicular to the flow. Second, high speeds cause turbulence. The drag both between adjacent layers of fluid and between the fluid and its surroundings forms swirls and eddies, if the speed is great enough. We shall concentrate on laminar flow for the remainder of this section, leaving certain aspects of turbulence for later sections.



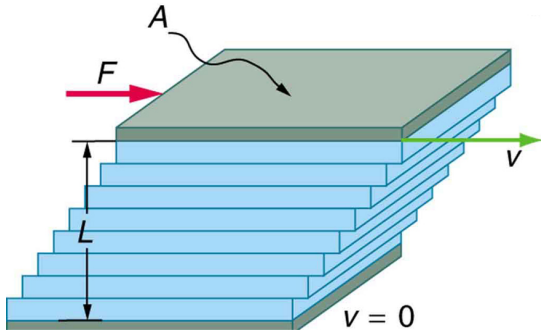
(a) Laminar flow occurs in layers without mixing. Notice that viscosity causes drag between layers as well as with the fixed surface. (b) An obstruction in the vessel produces turbulence. Turbulent flow mixes the fluid. There is more interaction, greater heating, and more resistance than in laminar flow.

**Note:**

**Making Connections: Take-Home Experiment: Go Down to the River**

Try dropping simultaneously two sticks into a flowing river, one near the edge of the river and one near the middle. Which one travels faster? Why?

[\[link\]](#) shows how viscosity is measured for a fluid. Two parallel plates have the specific fluid between them. The bottom plate is held fixed, while the top plate is moved to the right, dragging fluid with it. The layer (or lamina) of fluid in contact with either plate does not move relative to the plate, and so the top layer moves at  $v$  while the bottom layer remains at rest. Each successive layer from the top down exerts a force on the one below it, trying to drag it along, producing a continuous variation in speed from  $v$  to 0 as shown. Care is taken to insure that the flow is laminar; that is, the layers do not mix. The motion in [\[link\]](#) is like a continuous shearing motion. Fluids have zero shear strength, but the *rate* at which they are sheared is related to the same geometrical factors  $A$  and  $L$  as is shear deformation for solids.



The graphic shows laminar flow of fluid between two plates of area  $A$ . The bottom plate is fixed. When the top plate is pushed to the right, it drags the fluid along with it.

A force  $F$  is required to keep the top plate in [link](#) moving at a constant velocity  $v$ , and experiments have shown that this force depends on four factors. First,  $F$  is directly proportional to  $v$  (until the speed is so high that turbulence occurs—then a much larger force is needed, and it has a more complicated dependence on  $v$ ). Second,  $F$  is proportional to the area  $A$  of the plate. This relationship seems reasonable, since  $A$  is directly proportional to the amount of fluid being moved. Third,  $F$  is inversely proportional to the distance between the plates  $L$ . This relationship is also reasonable;  $L$  is like a lever arm, and the greater the lever arm, the less force that is needed. Fourth,  $F$  is directly proportional to *the coefficient of viscosity*,  $\eta$ . The greater the viscosity, the greater the force required. These dependencies are combined into the equation

**Equation:**

$$F = \eta \frac{vA}{L},$$

which gives us a working definition of fluid **viscosity**  $\eta$ . Solving for  $\eta$  gives

**Equation:**

$$\eta = \frac{FL}{vA},$$

which defines viscosity in terms of how it is measured. The SI unit of viscosity is  $\text{N} \cdot \text{m}/[(\text{m}/\text{s})\text{m}^2] = (\text{N}/\text{m}^2)\text{s}$  or  $\text{Pa} \cdot \text{s}$ . [\[link\]](#) lists the coefficients of viscosity for various fluids.

Viscosity varies from one fluid to another by several orders of magnitude. As you might expect, the viscosities of gases are much less than those of liquids, and these viscosities are often temperature dependent. The viscosity of blood can be reduced by aspirin consumption, allowing it to flow more easily around the body. (When used over the long term in low doses, aspirin can help prevent heart attacks, and reduce the risk of blood clotting.)

## Laminar Flow Confined to Tubes—Poiseuille’s Law

What causes flow? The answer, not surprisingly, is pressure difference. In fact, there is a very simple relationship between horizontal flow and pressure. Flow rate  $Q$  is in the direction from high to low pressure. The greater the pressure differential between two points, the greater the flow rate. This relationship can be stated as

**Equation:**

$$Q = \frac{P_2 - P_1}{R},$$

where  $P_1$  and  $P_2$  are the pressures at two points, such as at either end of a tube, and  $R$  is the resistance to flow. The resistance  $R$  includes everything, except pressure, that affects flow rate. For example,  $R$  is greater for a long tube than for a short one. The greater the viscosity of a fluid, the greater the value of  $R$ . Turbulence greatly increases  $R$ , whereas increasing the diameter of a tube decreases  $R$ .

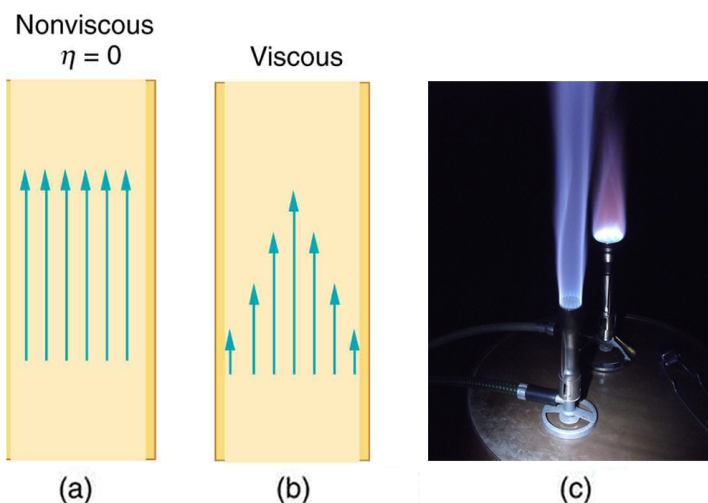
If viscosity is zero, the fluid is frictionless and the resistance to flow is also zero. Comparing frictionless flow in a tube to viscous flow, as in [\[link\]](#), we see that for a viscous fluid, speed is greatest at midstream because of drag at the boundaries. We can see the effect of viscosity in a Bunsen burner flame, even though the viscosity of natural gas is small.

The resistance  $R$  to laminar flow of an incompressible fluid having viscosity  $\eta$  through a horizontal tube of uniform radius  $r$  and length  $l$ , such as the one in [\[link\]](#), is given by

**Equation:**

$$R = \frac{8\eta l}{\pi r^4}.$$

This equation is called **Poiseuille's law for resistance** after the French scientist J. L. Poiseuille (1799–1869), who derived it in an attempt to understand the flow of blood, an often turbulent fluid.



(a) If fluid flow in a tube has negligible resistance, the speed is the same all across the tube. (b) When a viscous fluid flows through a tube, its speed at the walls is zero, increasing steadily to its maximum at the center of the tube. (c) The shape of the Bunsen burner flame is due to the velocity profile across the tube. (credit: Jason Woodhead)

Let us examine Poiseuille's expression for  $R$  to see if it makes good intuitive sense. We see that resistance is directly proportional to both fluid viscosity  $\eta$  and the length  $l$  of a tube. After all, both of these directly affect the amount of friction encountered—the greater either is, the greater the resistance and the smaller the flow. The radius  $r$  of a tube affects the resistance, which again makes sense, because the greater the radius, the greater the flow (all other factors remaining the same). But it is surprising that  $r$  is raised to the *fourth* power in Poiseuille's law. This exponent means that any change in the radius of a tube has a very large effect on resistance. For example, doubling the radius of a tube decreases resistance by a factor of  $2^4 = 16$ .

Taken together,  $Q = \frac{P_2 - P_1}{R}$  and  $R = \frac{8\eta l}{\pi r^4}$  give the following expression for flow rate:

**Equation:**

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}.$$

This equation describes laminar flow through a tube. It is sometimes called Poiseuille's law for laminar flow, or simply **Poiseuille's law**.

**Example:**

**Using Flow Rate: Plaque Deposits Reduce Blood Flow**

Suppose the flow rate of blood in a coronary artery has been reduced to half its normal value by plaque deposits. By what factor has the radius of the artery been reduced, assuming no turbulence occurs?

**Strategy**

Assuming laminar flow, Poiseuille's law states that

**Equation:**

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}.$$

We need to compare the artery radius before and after the flow rate reduction.

**Solution**

With a constant pressure difference assumed and the same length and viscosity, along the artery we have

**Equation:**

$$\frac{Q_1}{r_1^4} = \frac{Q_2}{r_2^4}.$$

So, given that  $Q_2 = 0.5Q_1$ , we find that  $r_2^4 = 0.5r_1^4$ .

Therefore,  $r_2 = (0.5)^{0.25}r_1 = 0.841r_1$ , a decrease in the artery radius of 16%.

**Discussion**

This decrease in radius is surprisingly small for this situation. To restore the blood flow in spite of this buildup would require an increase in the pressure difference ( $P_2 - P_1$ ) of a factor of two, with subsequent strain on the heart.

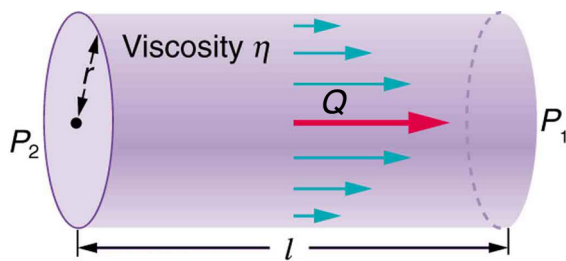
Fluid	Temperature (°C)	Viscosity $\eta$ (mPa·s)
<i>Gases</i>		
Air	0	0.0171
	20	0.0181
	40	0.0190
	100	0.0218
Ammonia	20	0.00974
Carbon dioxide	20	0.0147
Helium	20	0.0196
Hydrogen	0	0.0090
Mercury	20	0.0450
Oxygen	20	0.0203
Steam	100	0.0130
<i>Liquids</i>		
Water	0	1.792
	20	1.002
	37	0.6947
	40	0.653
	100	0.282
Whole blood <a href="#">[footnote]</a>	20	3.015

Fluid	Temperature (°C)	Viscosity $\eta$ (mPa·s)
The ratios of the viscosities of blood to water are nearly constant between 0°C and 37°C.	37	2.084
Blood plasma <a href="#">[footnote]</a> See note on Whole Blood.	20	1.810
	37	1.257
Ethyl alcohol	20	1.20
Methanol	20	0.584
Oil (heavy machine)	20	660
Oil (motor, SAE 10)	30	200
Oil (olive)	20	138
Glycerin	20	1500
Honey	20	2000–10000
Maple Syrup	20	2000–3000
Milk	20	3.0
Oil (Corn)	20	65

### Coefficients of Viscosity of Various Fluids

The circulatory system provides many examples of Poiseuille's law in action—with blood flow regulated by changes in vessel size and blood pressure. Blood vessels are not rigid but elastic. Adjustments to blood flow are primarily made by varying the size of the vessels, since the resistance is so sensitive to the radius. During vigorous exercise, blood vessels are selectively dilated to important muscles and organs and blood pressure increases. This creates both greater overall blood flow and increased flow to specific areas. Conversely, decreases in vessel radii, perhaps from plaques in

the arteries, can greatly reduce blood flow. If a vessel's radius is reduced by only 5% (to 0.95 of its original value), the flow rate is reduced to about  $(0.95)^4 = 0.81$  of its original value. A 19% decrease in flow is caused by a 5% decrease in radius. The body may compensate by increasing blood pressure by 19%, but this presents hazards to the heart and any vessel that has weakened walls. Another example comes from automobile engine oil. If you have a car with an oil pressure gauge, you may notice that oil pressure is high when the engine is cold. Motor oil has greater viscosity when cold than when warm, and so pressure must be greater to pump the same amount of cold oil.



Poiseuille's law applies to laminar flow of an incompressible fluid of viscosity  $\eta$  through a tube of length  $l$  and radius  $r$ . The direction of flow is from greater to lower pressure.

Flow rate  $Q$  is directly proportional to the pressure difference  $P_2 - P_1$ , and inversely proportional to the length  $l$  of the tube and viscosity  $\eta$  of the fluid. Flow rate increases with  $r^4$ , the fourth power of the radius.

**Example:**  
**What Pressure Produces This Flow Rate?**

An intravenous (IV) system is supplying saline solution to a patient at the rate of  $0.120 \text{ cm}^3/\text{s}$  through a needle of radius  $0.150 \text{ mm}$  and length  $2.50 \text{ cm}$ . What pressure is needed at the entrance of the needle to cause this flow, assuming the viscosity of the saline solution to be the same as that of water? The gauge pressure of the blood in the patient's vein is  $8.00 \text{ mm Hg}$ . (Assume that the temperature is  $20^\circ\text{C}$ .)

**Strategy**

Assuming laminar flow, Poiseuille's law applies. This is given by

**Equation:**

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l},$$

where  $P_2$  is the pressure at the entrance of the needle and  $P_1$  is the pressure in the vein. The only unknown is  $P_2$ .

**Solution**

Solving for  $P_2$  yields

**Equation:**

$$P_2 = \frac{8\eta l}{\pi r^4} Q + P_1.$$

$P_1$  is given as  $8.00 \text{ mm Hg}$ , which converts to  $1.066 \times 10^3 \text{ N/m}^2$ . Substituting this and the other known values yields

**Equation:**

$$\begin{aligned} P_2 &= \left[ \frac{8(1.00 \times 10^{-3} \text{ N}\cdot\text{s/m}^2)(2.50 \times 10^{-2} \text{ m})}{\pi(0.150 \times 10^{-3} \text{ m})^4} \right] (1.20 \times 10^{-7} \text{ m}^3/\text{s}) + 1.066 \times 10^3 \text{ N/m}^2 \\ &= 1.62 \times 10^4 \text{ N/m}^2. \end{aligned}$$

**Discussion**

This pressure could be supplied by an IV bottle with the surface of the saline solution  $1.61 \text{ m}$  above the entrance to the needle (this is left for you to solve in this chapter's Problems and Exercises), assuming that there is negligible pressure drop in the tubing leading to the needle.

## Flow and Resistance as Causes of Pressure Drops

You may have noticed that water pressure in your home might be lower than normal on hot summer days when there is more use. This pressure drop occurs in the water

main before it reaches your home. Let us consider flow through the water main as illustrated in [\[link\]](#). We can understand why the pressure  $P_1$  to the home drops during times of heavy use by rearranging

**Equation:**

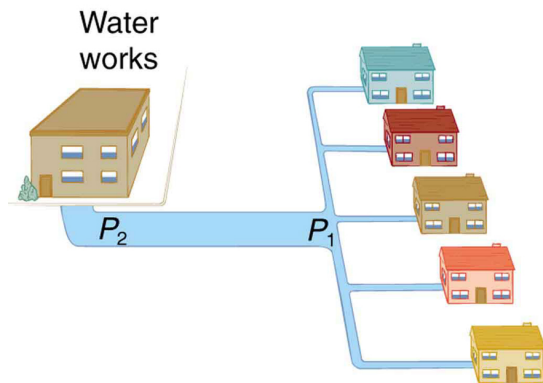
$$Q = \frac{P_2 - P_1}{R}$$

to

**Equation:**

$$P_2 - P_1 = RQ,$$

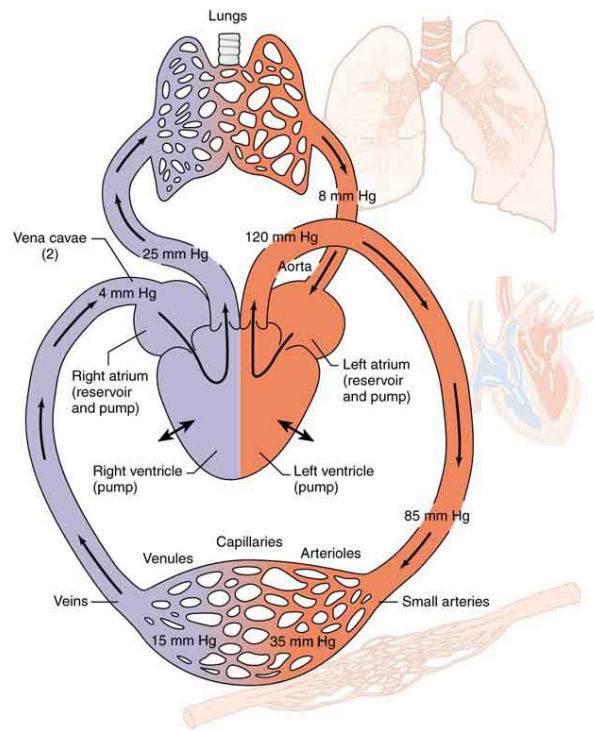
where, in this case,  $P_2$  is the pressure at the water works and  $R$  is the resistance of the water main. During times of heavy use, the flow rate  $Q$  is large. This means that  $P_2 - P_1$  must also be large. Thus  $P_1$  must decrease. It is correct to think of flow and resistance as causing the pressure to drop from  $P_2$  to  $P_1$ .  $P_2 - P_1 = RQ$  is valid for both laminar and turbulent flows.



During times of heavy use, there is a significant pressure drop in a water main, and  $P_1$  supplied to users is significantly less than  $P_2$  created at the water works. If the flow is very small, then the pressure drop is negligible, and  $P_2 \approx P_1$ .

We can use  $P_2 - P_1 = RQ$  to analyze pressure drops occurring in more complex systems in which the tube radius is not the same everywhere. Resistance will be much greater in narrow places, such as an obstructed coronary artery. For a given flow rate  $Q$ , the pressure drop will be greatest where the tube is most narrow. This is how water faucets control flow. Additionally,  $R$  is greatly increased by turbulence, and a constriction that creates turbulence greatly reduces the pressure downstream. Plaque in an artery reduces pressure and hence flow, both by its resistance and by the turbulence it creates.

[\[link\]](#) is a schematic of the human circulatory system, showing average blood pressures in its major parts for an adult at rest. Pressure created by the heart's two pumps, the right and left ventricles, is reduced by the resistance of the blood vessels as the blood flows through them. The left ventricle increases arterial blood pressure that drives the flow of blood through all parts of the body except the lungs. The right ventricle receives the lower pressure blood from two major veins and pumps it through the lungs for gas exchange with atmospheric gases – the disposal of carbon dioxide from the blood and the replenishment of oxygen. Only one major organ is shown schematically, with typical branching of arteries to ever smaller vessels, the smallest of which are the capillaries, and rejoining of small veins into larger ones. Similar branching takes place in a variety of organs in the body, and the circulatory system has considerable flexibility in flow regulation to these organs by the dilation and constriction of the arteries leading to them and the capillaries within them. The sensitivity of flow to tube radius makes this flexibility possible over a large range of flow rates.



Schematic of the circulatory system.

Pressure difference is created by the two pumps in the heart and is reduced by resistance in the vessels. Branching of vessels into capillaries allows blood to reach individual cells and exchange substances, such as oxygen and waste products, with them. The system has an impressive ability to regulate flow to individual organs, accomplished largely by varying vessel diameters.

Each branching of larger vessels into smaller vessels increases the total cross-sectional area of the tubes through which the blood flows. For example, an artery with a cross section of  $1 \text{ cm}^2$  may branch into 20 smaller arteries, each with cross sections of  $0.5 \text{ cm}^2$ , with a total of  $10 \text{ cm}^2$ . In that manner, the resistance of the branchings is reduced so that pressure is not entirely lost. Moreover, because  $Q = Av$  and  $A$  increases through branching, the average velocity of the blood in the smaller vessels is reduced. The blood velocity in the aorta (diameter = 1 cm) is about 25 cm/s, while in the capillaries ( $20 \mu\text{m}$  in diameter) the velocity is about 1

mm/s. This reduced velocity allows the blood to exchange substances with the cells in the capillaries and alveoli in particular.

## Section Summary

- Laminar flow is characterized by smooth flow of the fluid in layers that do not mix.
- Turbulence is characterized by eddies and swirls that mix layers of fluid together.
- Fluid viscosity  $\eta$  is due to friction within a fluid. Representative values are given in [\[link\]](#). Viscosity has units of  $(\text{N}/\text{m}^2)\text{s}$  or  $\text{Pa} \cdot \text{s}$ .
- Flow is proportional to pressure difference and inversely proportional to resistance:

**Equation:**

$$Q = \frac{P_2 - P_1}{R}.$$

- For laminar flow in a tube, Poiseuille's law for resistance states that

**Equation:**

$$R = \frac{8\eta l}{\pi r^4}.$$

- Poiseuille's law for flow in a tube is

**Equation:**

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}.$$

- The pressure drop caused by flow and resistance is given by

**Equation:**

$$P_2 - P_1 = RQ.$$

## Conceptual Questions

**Exercise:**

**Problem:**

Explain why the viscosity of a liquid decreases with temperature—that is, how might increased temperature reduce the effects of cohesive forces in a liquid? Also explain why the viscosity of a gas increases with temperature—that is, how does increased gas temperature create more collisions between atoms and molecules?

**Exercise:**

**Problem:**

When paddling a canoe upstream, it is wisest to travel as near to the shore as possible. When canoeing downstream, it may be best to stay near the middle. Explain why.

**Exercise:**

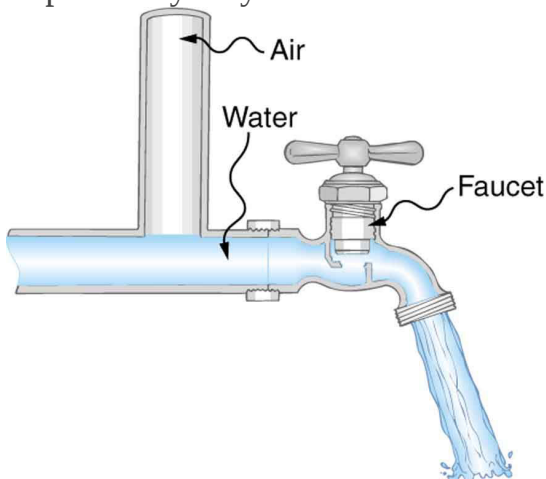
**Problem:**

Why does flow decrease in your shower when someone flushes the toilet?

**Exercise:**

**Problem:**

Plumbing usually includes air-filled tubes near water faucets, as shown in [\[link\]](#). Explain why they are needed and how they work.



The vertical tube near the water tap remains full of air and serves a useful purpose.

## Problems & Exercises

### Exercise:

#### Problem:

(a) Calculate the retarding force due to the viscosity of the air layer between a cart and a level air track given the following information—air temperature is  $20^\circ\text{C}$ , the cart is moving at  $0.400\text{ m/s}$ , its surface area is  $2.50 \times 10^{-2}\text{ m}^2$ , and the thickness of the air layer is  $6.00 \times 10^{-5}\text{ m}$ . (b) What is the ratio of this force to the weight of the  $0.300\text{-kg}$  cart?

---

#### Solution:

(a)  $3.02 \times 10^{-3}\text{ N}$

(b)  $1.03 \times 10^{-3}$

### Exercise:

#### Problem:

What force is needed to pull one microscope slide over another at a speed of  $1.00\text{ cm/s}$ , if there is a  $0.500\text{-mm}$ -thick layer of  $20^\circ\text{C}$  water between them and the contact area is  $8.00\text{ cm}^2$ ?

### Exercise:

#### Problem:

A glucose solution being administered with an IV has a flow rate of  $4.00\text{ cm}^3/\text{min}$ . What will the new flow rate be if the glucose is replaced by whole blood having the same density but a viscosity 2.50 times that of the glucose? All other factors remain constant.

---

#### Solution:

$1.60\text{ cm}^3/\text{min}$

### Exercise:

**Problem:**

The pressure drop along a length of artery is 100 Pa, the radius is 10 mm, and the flow is laminar. The average speed of the blood is 15 mm/s. (a) What is the net force on the blood in this section of artery? (b) What is the power expended maintaining the flow?

**Exercise:****Problem:**

A small artery has a length of  $1.1 \times 10^{-3}$  m and a radius of  $2.5 \times 10^{-5}$  m. If the pressure drop across the artery is 1.3 kPa, what is the flow rate through the artery? (Assume that the temperature is 37° C.)

---

**Solution:**

$$8.7 \times 10^{-11} \text{ m}^3/\text{s}$$

**Exercise:****Problem:**

Fluid originally flows through a tube at a rate of 100 cm<sup>3</sup>/s. To illustrate the sensitivity of flow rate to various factors, calculate the new flow rate for the following changes with all other factors remaining the same as in the original conditions. (a) Pressure difference increases by a factor of 1.50. (b) A new fluid with 3.00 times greater viscosity is substituted. (c) The tube is replaced by one having 4.00 times the length. (d) Another tube is used with a radius 0.100 times the original. (e) Yet another tube is substituted with a radius 0.100 times the original and half the length, *and* the pressure difference is increased by a factor of 1.50.

**Exercise:****Problem:**

The arterioles (small arteries) leading to an organ, constrict in order to decrease flow to the organ. To shut down an organ, blood flow is reduced naturally to 1.00% of its original value. By what factor did the radii of the arterioles constrict? Penguins do this when they stand on ice to reduce the blood flow to their feet.

---

**Solution:**

0.316

**Exercise:**

**Problem:**

Angioplasty is a technique in which arteries partially blocked with plaque are dilated to increase blood flow. By what factor must the radius of an artery be increased in order to increase blood flow by a factor of 10?

**Exercise:**

**Problem:**

(a) Suppose a blood vessel's radius is decreased to 90.0% of its original value by plaque deposits and the body compensates by increasing the pressure difference along the vessel to keep the flow rate constant. By what factor must the pressure difference increase? (b) If turbulence is created by the obstruction, what additional effect would it have on the flow rate?

---

**Solution:**

(a) 1.52

(b) Turbulence will decrease the flow rate of the blood, which would require an even larger increase in the pressure difference, leading to higher blood pressure.

**Exercise:**

**Problem:**

A spherical particle falling at a terminal speed in a liquid must have the gravitational force balanced by the drag force and the buoyant force. The buoyant force is equal to the weight of the displaced fluid, while the drag force is assumed to be given by Stokes Law,  $F_s = 6\pi r\eta v$ . Show that the terminal speed is given by

**Equation:**

$$v = \frac{2R^2g}{9\eta}(\rho_s - \rho_1),$$

where  $R$  is the radius of the sphere,  $\rho_s$  is its density, and  $\rho_1$  is the density of the fluid and  $\eta$  the coefficient of viscosity.

**Exercise:**

**Problem:**

Using the equation of the previous problem, find the viscosity of motor oil in which a steel ball of radius 0.8 mm falls with a terminal speed of 4.32 cm/s. The densities of the ball and the oil are 7.86 and 0.88 g/mL, respectively.

---

**Solution:****Equation:**

$$225 \text{ mPa} \cdot \text{s}$$

**Exercise:****Problem:**

A skydiver will reach a terminal velocity when the air drag equals their weight. For a skydiver with high speed and a large body, turbulence is a factor. The drag force then is approximately proportional to the square of the velocity. Taking the drag force to be  $F_D = \frac{1}{2} \rho A v^2$  and setting this equal to the person's weight, find the terminal speed for a person falling "spread eagle." Find both a formula and a number for  $v_t$ , with assumptions as to size.

**Exercise:****Problem:**

A layer of oil 1.50 mm thick is placed between two microscope slides. Researchers find that a force of  $5.50 \times 10^{-4} \text{ N}$  is required to glide one over the other at a speed of 1.00 cm/s when their contact area is  $6.00 \text{ cm}^2$ . What is the oil's viscosity? What type of oil might it be?

---

**Solution:****Equation:**

$$0.138 \text{ Pa} \cdot \text{s},$$

or

Olive oil.

**Exercise:**

**Problem:**

(a) Verify that a 19.0% decrease in laminar flow through a tube is caused by a 5.00% decrease in radius, assuming that all other factors remain constant, as stated in the text. (b) What increase in flow is obtained from a 5.00% increase in radius, again assuming all other factors remain constant?

**Exercise:****Problem:**

[\[link\]](#) dealt with the flow of saline solution in an IV system. (a) Verify that a pressure of  $1.62 \times 10^4 \text{ N/m}^2$  is created at a depth of 1.61 m in a saline solution, assuming its density to be that of sea water. (b) Calculate the new flow rate if the height of the saline solution is decreased to 1.50 m. (c) At what height would the direction of flow be reversed? (This reversal can be a problem when patients stand up.)

---

**Solution:**

(a)  $1.62 \times 10^4 \text{ N/m}^2$

(b)  $0.111 \text{ cm}^3/\text{s}$

(c) 10.6 cm

**Exercise:****Problem:**

When physicians diagnose arterial blockages, they quote the reduction in flow rate. If the flow rate in an artery has been reduced to 10.0% of its normal value by a blood clot and the average pressure difference has increased by 20.0%, by what factor has the clot reduced the radius of the artery?

**Exercise:****Problem:**

During a marathon race, a runner's blood flow increases to 10.0 times her resting rate. Her blood's viscosity has dropped to 95.0% of its normal value, and the blood pressure difference across the circulatory system has increased by 50.0%. By what factor has the average radii of her blood vessels increased?

---

**Solution:**

**Exercise:****Problem:**

Water supplied to a house by a water main has a pressure of  $3.00 \times 10^5 \text{ N/m}^2$  early on a summer day when neighborhood use is low. This pressure produces a flow of 20.0 L/min through a garden hose. Later in the day, pressure at the exit of the water main and entrance to the house drops, and a flow of only 8.00 L/min is obtained through the same hose. (a) What pressure is now being supplied to the house, assuming resistance is constant? (b) By what factor did the flow rate in the water main increase in order to cause this decrease in delivered pressure? The pressure at the entrance of the water main is  $5.00 \times 10^5 \text{ N/m}^2$ , and the original flow rate was 200 L/min. (c) How many more users are there, assuming each would consume 20.0 L/min in the morning?

**Exercise:****Problem:**

An oil gusher shoots crude oil 25.0 m into the air through a pipe with a 0.100-m diameter. Neglecting air resistance but not the resistance of the pipe, and assuming laminar flow, calculate the gauge pressure at the entrance of the 50.0-m-long vertical pipe. Take the density of the oil to be  $900 \text{ kg/m}^3$  and its viscosity to be  $1.00 (\text{N/m}^2) \cdot \text{s}$  (or  $1.00 \text{ Pa} \cdot \text{s}$ ). Note that you must take into account the pressure due to the 50.0-m column of oil in the pipe.

**Solution:**

$$2.95 \times 10^6 \text{ N/m}^2 (\text{gauge pressure})$$

**Exercise:****Problem:**

Concrete is pumped from a cement mixer to the place it is being laid, instead of being carried in wheelbarrows. The flow rate is 200.0 L/min through a 50.0-m-long, 8.00-cm-diameter hose, and the pressure at the pump is  $8.00 \times 10^6 \text{ N/m}^2$ . (a) Calculate the resistance of the hose. (b) What is the viscosity of the concrete, assuming the flow is laminar? (c) How much power is being supplied, assuming the point of use is at the same level as the pump? You may neglect the power supplied to increase the concrete's velocity.

## Exercise:

### Problem: Construct Your Own Problem

Consider a coronary artery constricted by arteriosclerosis. Construct a problem in which you calculate the amount by which the diameter of the artery is decreased, based on an assessment of the decrease in flow rate.

## Exercise:

### Problem:

Consider a river that spreads out in a delta region on its way to the sea. Construct a problem in which you calculate the average speed at which water moves in the delta region, based on the speed at which it was moving up river. Among the things to consider are the size and flow rate of the river before it spreads out and its size once it has spread out. You can construct the problem for the river spreading out into one large river or into multiple smaller rivers.

## Glossary

laminar

a type of fluid flow in which layers do not mix

turbulence

fluid flow in which layers mix together via eddies and swirls

viscosity

the friction in a fluid, defined in terms of the friction between layers

Poiseuille's law for resistance

the resistance to laminar flow of an incompressible fluid in a tube:  $R = 8\eta l / \pi r^4$

Poiseuille's law

the rate of laminar flow of an incompressible fluid in a tube:  $Q = (P_2 - P_1) \pi r^4 / 8\eta l$

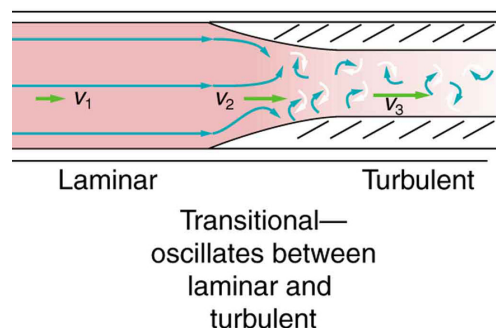
## The Onset of Turbulence

- Calculate Reynolds number.
- Use the Reynolds number for a system to determine whether it is laminar or turbulent.

Sometimes we can predict if flow will be laminar or turbulent. We know that flow in a very smooth tube or around a smooth, streamlined object will be laminar at low velocity. We also know that at high velocity, even flow in a smooth tube or around a smooth object will experience turbulence. In between, it is more difficult to predict. In fact, at intermediate velocities, flow may oscillate back and forth indefinitely between laminar and turbulent.

An occlusion, or narrowing, of an artery, such as shown in [\[link\]](#), is likely to cause turbulence because of the irregularity of the blockage, as well as the complexity of blood as a fluid. Turbulence in the circulatory system is noisy and can sometimes be detected with a stethoscope, such as when measuring diastolic pressure in the upper arm's partially collapsed brachial artery. These turbulent sounds, at the onset of blood flow when the cuff pressure becomes sufficiently small, are called *Korotkoff sounds*.

Aneurysms, or ballooning of arteries, create significant turbulence and can sometimes be detected with a stethoscope. Heart murmurs, consistent with their name, are sounds produced by turbulent flow around damaged and insufficiently closed heart valves. Ultrasound can also be used to detect turbulence as a medical indicator in a process analogous to Doppler-shift radar used to detect storms.



Flow is laminar in the large part of this blood

vessel and turbulent in the part narrowed by plaque, where velocity is high. In the transition region, the flow can oscillate chaotically between laminar and turbulent flow.

An indicator called the **Reynolds number**  $N_R$  can reveal whether flow is laminar or turbulent. For flow in a tube of uniform diameter, the Reynolds number is defined as

**Equation:**

$$N_R = \frac{2\rho v r}{\eta} (\text{flow in tube}),$$

where  $\rho$  is the fluid density,  $v$  its speed,  $\eta$  its viscosity, and  $r$  the tube radius. The Reynolds number is a unitless quantity. Experiments have revealed that  $N_R$  is related to the onset of turbulence. For  $N_R$  below about 2000, flow is laminar. For  $N_R$  above about 3000, flow is turbulent. For values of  $N_R$  between about 2000 and 3000, flow is unstable—that is, it can be laminar, but small obstructions and surface roughness can make it turbulent, and it may oscillate randomly between being laminar and turbulent. The blood flow through most of the body is a quiet, laminar flow. The exception is in the aorta, where the speed of the blood flow rises above a critical value of 35 m/s and becomes turbulent.

**Example:**

**Is This Flow Laminar or Turbulent?**

Calculate the Reynolds number for flow in the needle considered in [Example 12.8](#) to verify the assumption that the flow is laminar. Assume

that the density of the saline solution is  $1025 \text{ kg/m}^3$ .

**Strategy**

We have all of the information needed, except the fluid speed  $v$ , which can be calculated from  $v = Q/A = 1.70 \text{ m/s}$  (verification of this is in this chapter's Problems and Exercises).

**Solution**

Entering the known values into  $N_R = \frac{2\rho v r}{\eta}$  gives

**Equation:**

$$\begin{aligned} N_R &= \frac{2\rho v r}{\eta} \\ &= \frac{2(1025 \text{ kg/m}^3)(1.70 \text{ m/s})(0.150 \times 10^{-3} \text{ m})}{1.00 \times 10^{-3} \text{ N}\cdot\text{s/m}^2} \\ &= 523. \end{aligned}$$

**Discussion**

Since  $N_R$  is well below 2000, the flow should indeed be laminar.

**Note:**

**Take-Home Experiment: Inhalation**

Under the conditions of normal activity, an adult inhales about 1 L of air during each inhalation. With the aid of a watch, determine the time for one of your own inhalations by timing several breaths and dividing the total length by the number of breaths. Calculate the average flow rate  $Q$  of air traveling through the trachea during each inhalation.

The topic of chaos has become quite popular over the last few decades. A system is defined to be *chaotic* when its behavior is so sensitive to some factor that it is extremely difficult to predict. The field of *chaos* is the study of chaotic behavior. A good example of chaotic behavior is the flow of a fluid with a Reynolds number between 2000 and 3000. Whether or not the flow is turbulent is difficult, but not impossible, to predict—the difficulty lies in the extremely sensitive dependence on factors like roughness and

obstructions on the nature of the flow. A tiny variation in one factor has an exaggerated (or nonlinear) effect on the flow. Phenomena as disparate as turbulence, the orbit of Pluto, and the onset of irregular heartbeats are chaotic and can be analyzed with similar techniques.

## Section Summary

- The Reynolds number  $N_R$  can reveal whether flow is laminar or turbulent. It is

**Equation:**

$$N_R = \frac{2\rho vr}{\eta}.$$

- For  $N_R$  below about 2000, flow is laminar. For  $N_R$  above about 3000, flow is turbulent. For values of  $N_R$  between 2000 and 3000, it may be either or both.

## Conceptual Questions

**Exercise:**

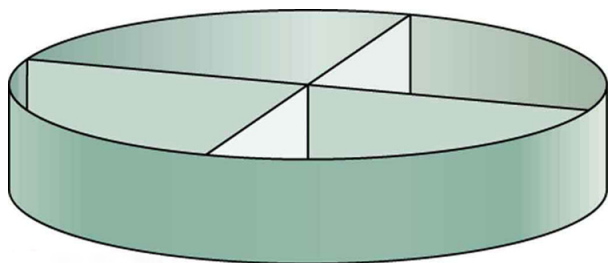
**Problem:**

Doppler ultrasound can be used to measure the speed of blood in the body. If there is a partial constriction of an artery, where would you expect blood speed to be greatest, at or nearby the constriction? What are the two distinct causes of higher resistance in the constriction?

**Exercise:**

**Problem:**

Sink drains often have a device such as that shown in [\[link\]](#) to help speed the flow of water. How does this work?



You will find devices such as this in many drains. They significantly increase flow rate.

**Exercise:**

**Problem:**

Some ceiling fans have decorative wicker reeds on their blades. Discuss whether these fans are as quiet and efficient as those with smooth blades.

## Problems & Exercises

**Exercise:**

**Problem:**

Verify that the flow of oil is laminar (barely) for an oil gusher that shoots crude oil 25.0 m into the air through a pipe with a 0.100-m diameter. The vertical pipe is 50 m long. Take the density of the oil to be  $900 \text{ kg/m}^3$  and its viscosity to be  $1.00 \text{ (N/m}^2) \cdot \text{s}$  (or  $1.00 \text{ Pa} \cdot \text{s}$ ).

---

**Solution:**

$$N_R = 1.99 \times 10^2 < 2000$$

**Exercise:**

**Problem:**

Show that the Reynolds number  $N_R$  is unitless by substituting units for all the quantities in its definition and cancelling.

**Exercise:****Problem:**

Calculate the Reynolds numbers for the flow of water through (a) a nozzle with a radius of 0.250 cm and (b) a garden hose with a radius of 0.900 cm, when the nozzle is attached to the hose. The flow rate through hose and nozzle is 0.500 L/s. Can the flow in either possibly be laminar?

---

**Solution:**

(a) nozzle:  $1.27 \times 10^5$ , not laminar

(b) hose:  $3.51 \times 10^4$ , not laminar.

**Exercise:****Problem:**

A fire hose has an inside diameter of 6.40 cm. Suppose such a hose carries a flow of 40.0 L/s starting at a gauge pressure of  $1.62 \times 10^6 \text{ N/m}^2$ . The hose goes 10.0 m up a ladder to a nozzle having an inside diameter of 3.00 cm. Calculate the Reynolds numbers for flow in the fire hose and nozzle to show that the flow in each must be turbulent.

**Exercise:**

**Problem:**

Concrete is pumped from a cement mixer to the place it is being laid, instead of being carried in wheelbarrows. The flow rate is 200.0 L/min through a 50.0-m-long, 8.00-cm-diameter hose, and the pressure at the pump is  $8.00 \times 10^6 \text{ N/m}^2$ . Verify that the flow of concrete is laminar taking concrete's viscosity to be  $48.0 \text{ (N/m}^2) \cdot \text{s}$ , and given its density is  $2300 \text{ kg/m}^3$ .

---

**Solution:**

$2.54 \ll 2000$ , laminar.

**Exercise:****Problem:**

At what flow rate might turbulence begin to develop in a water main with a 0.200-m diameter? Assume a  $20^\circ \text{ C}$  temperature.

**Exercise:****Problem:**

What is the greatest average speed of blood flow at  $37^\circ \text{ C}$  in an artery of radius 2.00 mm if the flow is to remain laminar? What is the corresponding flow rate? Take the density of blood to be  $1025 \text{ kg/m}^3$ .

---

**Solution:**

1.02 m/s

$1.28 \times 10^{-2} \text{ L/s}$

**Exercise:**

**Problem:**

In [Take-Home Experiment: Inhalation](#), we measured the average flow rate  $Q$  of air traveling through the trachea during each inhalation. Now calculate the average air speed in meters per second through your trachea during each inhalation. The radius of the trachea in adult humans is approximately  $10^{-2}$  m. From the data above, calculate the Reynolds number for the air flow in the trachea during inhalation. Do you expect the air flow to be laminar or turbulent?

**Exercise:****Problem:**

Gasoline is piped underground from refineries to major users. The flow rate is  $3.00 \times 10^{-2} \text{ m}^3/\text{s}$  (about 500 gal/min), the viscosity of gasoline is  $1.00 \times 10^{-3} (\text{N}/\text{m}^2) \cdot \text{s}$ , and its density is  $680 \text{ kg}/\text{m}^3$ . (a) What minimum diameter must the pipe have if the Reynolds number is to be less than 2000? (b) What pressure difference must be maintained along each kilometer of the pipe to maintain this flow rate?

---

**Solution:**

(a)  $\geq 13.0 \text{ m}$

(b)  $2.68 \times 10^{-6} \text{ N}/\text{m}^2$

**Exercise:****Problem:**

Assuming that blood is an ideal fluid, calculate the critical flow rate at which turbulence is a certainty in the aorta. Take the diameter of the aorta to be 2.50 cm. (Turbulence will actually occur at lower average flow rates, because blood is not an ideal fluid. Furthermore, since blood flow pulses, turbulence may occur during only the high-velocity part of each heartbeat.)

**Exercise:**

### **Problem: Unreasonable Results**

A fairly large garden hose has an internal radius of 0.600 cm and a length of 23.0 m. The nozzleless horizontal hose is attached to a faucet, and it delivers 50.0 L/s. (a) What water pressure is supplied by the faucet? (b) What is unreasonable about this pressure? (c) What is unreasonable about the premise? (d) What is the Reynolds number for the given flow? (Take the viscosity of water as  $1.005 \times 10^{-3} \text{ (N/m}^2) \cdot \text{s}$ .)

---

### **Solution:**

- (a) 23.7 atm or 344 lb/in<sup>2</sup>
- (b) The pressure is much too high.
- (c) The assumed flow rate is very high for a garden hose.
- (d)  $5.27 \times 10^6 \gg 3000$ , turbulent, contrary to the assumption of laminar flow when using this equation.

### **Glossary**

Reynolds number

a dimensionless parameter that can reveal whether a particular flow is laminar or turbulent

## Introduction to Heat and Heat Transfer Methods

class="introduction"

(a) The chilling effect of a clear breezy night is produced by the wind and by radiative heat transfer to cold outer space. (b)

There was once great controversy about the Earth's age, but it is now generally accepted to be about 4.5 billion years old. Much of the debate is centered on the Earth's molten interior.

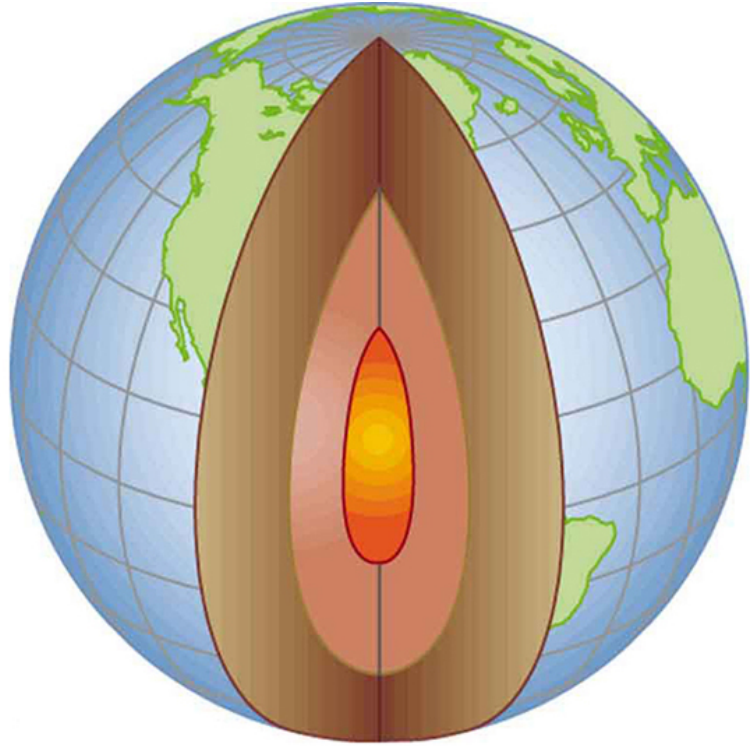
According to our understanding of heat transfer, if the Earth is really that old, its

center should  
have cooled  
off long ago.

The  
discovery of  
radioactivity  
in rocks  
revealed the  
source of  
energy that  
keeps the  
Earth's  
interior  
molten,  
despite heat  
transfer to the  
surface, and  
from there to  
cold outer  
space.



(a)



(b)

Energy can exist in many forms and heat is one of the most intriguing. Heat is often hidden, as it only exists when in transit, and is transferred by a number of distinctly different methods. Heat transfer touches every aspect of our lives and helps us understand how the universe functions. It explains the chill we feel on a clear breezy night, or why Earth's core has yet to cool. This chapter defines and explores heat transfer, its effects, and the methods by which heat is transferred. These topics are fundamental, as well as practical, and will often be referred to in the chapters ahead.

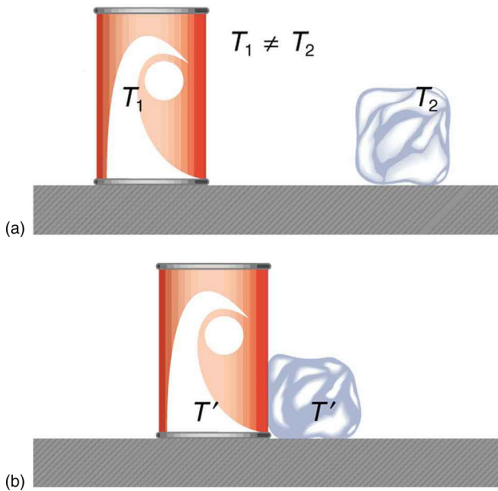
## Heat

- Define heat as transfer of energy.

In [Work, Energy, and Energy Resources](#), we defined work as force times distance and learned that work done on an object changes its kinetic energy. We also saw in [Temperature, Kinetic Theory, and the Gas Laws](#) that temperature is proportional to the (average) kinetic energy of atoms and molecules. We say that a thermal system has a certain internal energy: its internal energy is higher if the temperature is higher. If two objects at different temperatures are brought in contact with each other, energy is transferred from the hotter to the colder object until equilibrium is reached and the bodies reach thermal equilibrium (i.e., they are at the same temperature). No work is done by either object, because no force acts through a distance. The transfer of energy is caused by the temperature difference, and ceases once the temperatures are equal. These observations lead to the following definition of **heat**: Heat is the spontaneous transfer of energy due to a temperature difference.

As noted in [Temperature, Kinetic Theory, and the Gas Laws](#), heat is often confused with temperature. For example, we may say the heat was unbearable, when we actually mean that the temperature was high. Heat is a form of energy, whereas temperature is not. The misconception arises because we are sensitive to the flow of heat, rather than the temperature.

Owing to the fact that heat is a form of energy, it has the SI unit of *joule* (J). The *calorie* (cal) is a common unit of energy, defined as the energy needed to change the temperature of 1.00 g of water by 1.00°C —specifically, between 14.5°C and 15.5°C, since there is a slight temperature dependence. Perhaps the most common unit of heat is the **kilocalorie** (kcal), which is the energy needed to change the temperature of 1.00 kg of water by 1.00°C. Since mass is most often specified in kilograms, kilocalorie is commonly used. Food calories (given the notation Cal, and sometimes called “big calorie”) are actually kilocalories (1 kilocalorie = 1000 calories), a fact not easily determined from package labeling.



In figure (a) the soft drink and the ice have different temperatures,  $T_1$  and  $T_2$ , and are not in thermal equilibrium. In figure (b), when the soft drink and ice are allowed to interact, energy is transferred until they reach the same temperature  $T'$ , achieving equilibrium. Heat transfer occurs due to the difference in temperatures. In fact, since the soft drink and ice are both in contact with the surrounding air and bench, the equilibrium temperature will be the same for both.

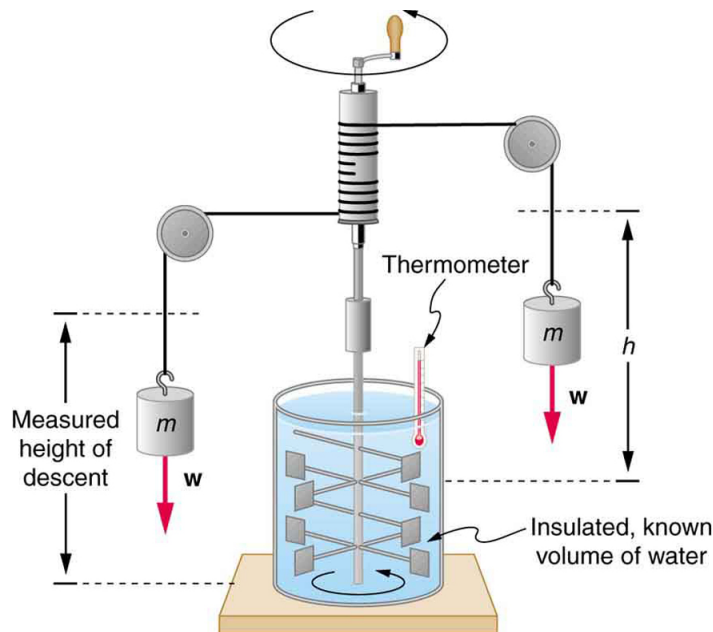
## Mechanical Equivalent of Heat

It is also possible to change the temperature of a substance by doing work. Work can transfer energy into or out of a system. This realization helped establish the fact that heat is a form of energy. James Prescott Joule (1818–1889) performed many experiments to establish the **mechanical equivalent of heat**—*the work needed to produce the same effects as heat transfer*. In terms of the units used for these two terms, the best modern value for this equivalence is

**Equation:**

$$1.000 \text{ kcal} = 4186 \text{ J.}$$

We consider this equation as the conversion between two different units of energy.



Schematic depiction of Joule's experiment that established the equivalence of heat and work.

The figure above shows one of Joule's most famous experimental setups for demonstrating the mechanical equivalent of heat. It demonstrated that work and heat can produce the same effects, and helped establish the principle of conservation of energy. Gravitational potential energy (PE) (work done by the gravitational force) is converted into kinetic energy (KE), and then randomized by viscosity and turbulence into increased average kinetic energy of atoms and molecules in the system, producing a temperature increase. His contributions to the field of thermodynamics were so significant that the SI unit of energy was named after him.

Heat added or removed from a system changes its internal energy and thus its temperature. Such a temperature increase is observed while cooking. However, adding heat does not necessarily increase the temperature. An example is melting of ice; that is, when a substance changes from one phase to another. Work done on the system or by the system can also change the internal energy of the system. Joule demonstrated that the temperature of a system can be increased by stirring. If an ice cube is rubbed against a rough surface, work is done by the frictional force. A system has a well-defined internal energy, but we cannot say that it has a certain "heat content" or "work content". We use the phrase "heat transfer" to emphasize its nature.

### **Exercise:**

#### **Check Your Understanding**

##### **Problem:**

Two samples (A and B) of the same substance are kept in a lab. Someone adds 10 kilojoules (kJ) of heat to one sample, while 10 kJ of work is done on the other sample. How can you tell to which sample the heat was added?

---

##### **Solution:**

Heat and work both change the internal energy of the substance. However, the properties of the sample only depend on the internal energy so that it is impossible to tell whether heat was added to sample A or B.

## Summary

- Heat and work are the two distinct methods of energy transfer.
- Heat is energy transferred solely due to a temperature difference.
- Any energy unit can be used for heat transfer, and the most common are kilocalorie (kcal) and joule (J).
- Kilocalorie is defined to be the energy needed to change the temperature of 1.00 kg of water between 14.5°C and 15.5°C.
- The mechanical equivalent of this heat transfer is  
 $1.00 \text{ kcal} = 4186 \text{ J}$ .

## Conceptual Questions

### Exercise:

**Problem:** How is heat transfer related to temperature?

### Exercise:

#### Problem:

Describe a situation in which heat transfer occurs. What are the resulting forms of energy?

### Exercise:

#### Problem:

When heat transfers into a system, is the energy stored as heat? Explain briefly.

## Glossary

heat

the spontaneous transfer of energy due to a temperature difference

kilocalorie

1 kilocalorie = 1000 calories

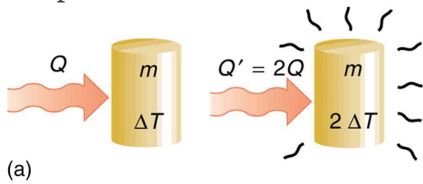
mechanical equivalent of heat

the work needed to produce the same effects as heat transfer

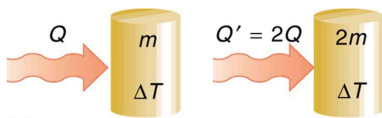
## Temperature Change and Heat Capacity

- Observe heat transfer and change in temperature and mass.
- Calculate final temperature after heat transfer between two objects.

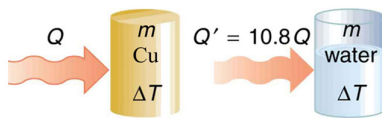
One of the major effects of heat transfer is temperature change: heating increases the temperature while cooling decreases it. We assume that there is no phase change and that no work is done on or by the system. Experiments show that the transferred heat depends on three factors—the change in temperature, the mass of the system, and the substance and phase of the substance.



(a)



(b)



(c)

The heat  $Q$  transferred to cause a temperature change depends on the magnitude of the temperature change, the mass of the system, and the substance and phase involved. (a) The amount of heat transferred is directly proportional to the temperature change.

To double the temperature change of a mass  $m$ , you need to add twice the heat. (b) The amount of heat transferred is also directly proportional to the mass.

To cause an equivalent temperature change in a

doubled mass, you need to add twice the heat. (c)

The amount of heat transferred depends on the substance and its phase. If it takes an amount  $Q$  of heat to cause a temperature change  $\Delta T$  in a given mass of copper, it will take 10.8 times that amount of heat to cause the equivalent temperature change in the same mass of water assuming no phase change in either substance.

The dependence on temperature change and mass are easily understood. Owing to the fact that the (average) kinetic energy of an atom or molecule is proportional to the absolute temperature, the internal energy of a system is proportional to the absolute temperature and the number of atoms or molecules. Owing to the fact that the transferred heat is equal to the change in the internal energy, the heat is proportional to the mass of the substance and the temperature change. The transferred heat also depends on the substance so that, for example, the heat necessary to raise the temperature is less for alcohol than for water. For the same substance, the transferred heat also depends on the phase (gas, liquid, or solid).

**Note:**

**Heat Transfer and Temperature Change**

The quantitative relationship between heat transfer and temperature change contains all three factors:

**Equation:**

$$Q = mc\Delta T,$$

where  $Q$  is the symbol for heat transfer,  $m$  is the mass of the substance, and  $\Delta T$  is the change in temperature. The symbol  $c$  stands for **specific heat** and depends on the material and phase. The specific heat is the amount of heat necessary to change the

temperature of 1.00 kg of mass by 1.00°C. The specific heat  $c$  is a property of the substance; its SI unit is J/(kg · K) or J/(kg · °C). Recall that the temperature change ( $\Delta T$ ) is the same in units of kelvin and degrees Celsius. If heat transfer is measured in kilocalories, then *the unit of specific heat* is kcal/(kg · °C).

Values of specific heat must generally be looked up in tables, because there is no simple way to calculate them. In general, the specific heat also depends on the temperature. [\[link\]](#) lists representative values of specific heat for various substances. Except for gases, the temperature and volume dependence of the specific heat of most substances is weak. We see from this table that the specific heat of water is five times that of glass and ten times that of iron, which means that it takes five times as much heat to raise the temperature of water the same amount as for glass and ten times as much heat to raise the temperature of water as for iron. In fact, water has one of the largest specific heats of any material, which is important for sustaining life on Earth.

### **Example:**

#### **Calculating the Required Heat: Heating Water in an Aluminum Pan**

A 0.500 kg aluminum pan on a stove is used to heat 0.250 liters of water from 20.0°C to 80.0°C. (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

#### **Strategy**

The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is increased by the same amount. We use the equation for the heat transfer for the given temperature change and mass of water and aluminum. The specific heat values for water and aluminum are given in [\[link\]](#).

#### **Solution**

Because water is in thermal contact with the aluminum, the pan and the water are at the same temperature.

1. Calculate the temperature difference:

#### **Equation:**

$$\Delta T = T_f - T_i = 60.0^\circ\text{C}.$$

2. Calculate the mass of water. Because the density of water is 1000 kg/m<sup>3</sup>, one liter of water has a mass of 1 kg, and the mass of 0.250 liters of water is  $m_w = 0.250$  kg.
3. Calculate the heat transferred to the water. Use the specific heat of water in [\[link\]](#):  
**Equation:**

$$Q_w = m_w c_w \Delta T = (0.250 \text{ kg})(4186 \text{ J/kg}^\circ\text{C})(60.0^\circ\text{C}) = 62.8 \text{ kJ}.$$

4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in [\[link\]](#):

**Equation:**

$$Q_{Al} = m_{Al} c_{Al} \Delta T = (0.500 \text{ kg})(900 \text{ J/kg}^\circ\text{C})(60.0^\circ\text{C}) = 27.0 \times 10^4 \text{ J} = 27.0 \text{ kJ}.$$

5. Compare the percentage of heat going into the pan versus that going into the water. First, find the total transferred heat:

**Equation:**

$$Q_{\text{Total}} = Q_W + Q_{Al} = 62.8 \text{ kJ} + 27.0 \text{ kJ} = 89.8 \text{ kJ}.$$

Thus, the amount of heat going into heating the pan is

**Equation:**

$$\frac{27.0 \text{ kJ}}{89.8 \text{ kJ}} \times 100\% = 30.1\%,$$

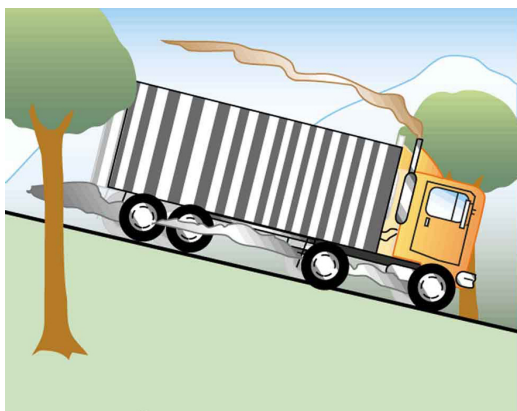
and the amount going into heating the water is

**Equation:**

$$\frac{62.8 \text{ kJ}}{89.8 \text{ kJ}} \times 100\% = 69.9\%.$$

### Discussion

In this example, the heat transferred to the container is a significant fraction of the total transferred heat. Although the mass of the pan is twice that of the water, the specific heat of water is over four times greater than that of aluminum. Therefore, it takes a bit more than twice the heat to achieve the given temperature change for the water as compared to the aluminum pan.



The smoking brakes on this truck are a visible evidence of the mechanical equivalent of heat.

**Example:**

**Calculating the Temperature Increase from the Work Done on a Substance: Truck Brakes Overheat on Downhill Runs**

Truck brakes used to control speed on a downhill run do work, converting gravitational potential energy into increased internal energy (higher temperature) of the brake material. This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck. The problem is that the mass of the truck is large compared with that of the brake material absorbing the energy, and the temperature increase may occur too fast for sufficient heat to transfer from the brakes to the environment.

Calculate the temperature increase of 100 kg of brake material with an average specific heat of  $800 \text{ J/kg} \cdot ^\circ\text{C}$  if the material retains 10% of the energy from a 10,000-kg truck descending 75.0 m (in vertical displacement) at a constant speed.

**Strategy**

If the brakes are not applied, gravitational potential energy is converted into kinetic energy. When brakes are applied, gravitational potential energy is converted into internal energy of the brake material. We first calculate the gravitational potential energy ( $Mgh$ ) that the entire truck loses in its descent and then find the temperature increase produced in the brake material alone.

**Solution**

1. Calculate the change in gravitational potential energy as the truck goes downhill

**Equation:**

$$Mgh = (10,000 \text{ kg}) (9.80 \text{ m/s}^2) (75.0 \text{ m}) = 7.35 \times 10^6 \text{ J.}$$

2. Calculate the temperature from the heat transferred using  $Q=Mgh$  and

**Equation:**

$$\Delta T = \frac{Q}{mc},$$

where  $m$  is the mass of the brake material. Insert the values  $m = 100 \text{ kg}$  and  $c = 800 \text{ J/kg} \cdot ^\circ\text{C}$  to find

**Equation:**

$$\Delta T = \frac{(7.35 \times 10^5 \text{ J})}{(100 \text{ kg})(800 \text{ J/kg}^\circ\text{C})} = 9.2^\circ\text{C.}$$

### Discussion

This same idea underlies the recent hybrid technology of cars, where mechanical energy (gravitational potential energy) is converted by the brakes into electrical energy (battery).

Substances	Specific heat (c)	
Solids	J/kg·°C	kcal/kg·°C <a href="#">[footnote]</a> These values are identical in units of cal/g ·°C.
Aluminum	900	0.215
Asbestos	800	0.19
Concrete, granite (average)	840	0.20
Copper	387	0.0924
Glass	840	0.20

Substances	Specific heat ( $c$ )	
Gold	129	0.0308
Human body (average at 37 °C)	3500	0.83
Ice (average, -50°C to 0°C)	2090	0.50
Iron, steel	452	0.108
Lead	128	0.0305
Silver	235	0.0562
Wood	1700	0.4
<i>Liquids</i>		
Benzene	1740	0.415
Ethanol	2450	0.586
Glycerin	2410	0.576
Mercury	139	0.0333
Water (15.0 °C)	4186	1.000
<i>Gases</i> <a href="#">[footnote]</a> $c_v$ at constant volume and at 20.0°C, except as noted, and at 1.00 atm average pressure. Values in parentheses are $c_p$ at a constant pressure of 1.00 atm.		
Air (dry)	721 (1015)	0.172 (0.242)
Ammonia	1670 (2190)	0.399 (0.523)
Carbon dioxide	638 (833)	0.152 (0.199)

Substances	Specific heat (c)	
Nitrogen	739 (1040)	0.177 (0.248)
Oxygen	651 (913)	0.156 (0.218)
Steam (100°C)	1520 (2020)	0.363 (0.482)

Specific Heats<sup>[footnote]</sup> of Various Substances

The values for solids and liquids are at constant volume and at 25°C, except as noted.

Note that [\[link\]](#) is an illustration of the mechanical equivalent of heat. Alternatively, the temperature increase could be produced by a blow torch instead of mechanically.

### Example:

#### Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan

Suppose you pour 0.250 kg of 20.0°C water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of 150°C. Assume that the pan is placed on an insulated pad and that a negligible amount of water boils off. What is the temperature when the water and pan reach thermal equilibrium a short time later?

#### Strategy

The pan is placed on an insulated pad so that little heat transfer occurs with the surroundings. Originally the pan and water are not in thermal equilibrium: the pan is at a higher temperature than the water. Heat transfer then restores thermal equilibrium once the water and pan are in contact. Because heat transfer between the pan and water takes place rapidly, the mass of evaporated water is negligible and the magnitude of the heat lost by the pan is equal to the heat gained by the water. The exchange of heat stops once a thermal equilibrium between the pan and the water is achieved. The heat exchange can be written as  $|Q_{\text{hot}}| = Q_{\text{cold}}$ .

#### Solution

1. Use the equation for heat transfer  $Q = mc\Delta T$  to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:

#### Equation:

$$Q_{\text{hot}} = m_{\text{Al}}c_{\text{Al}}(T_{\text{f}} - 150^{\circ}\text{C}).$$

2. Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water and the final temperature:

**Equation:**

$$Q_{\text{cold}} = m_{\text{W}}c_{\text{W}}(T_{\text{f}} - 20.0^{\circ}\text{C}).$$

3. Note that  $Q_{\text{hot}} < 0$  and  $Q_{\text{cold}} > 0$  and that they must sum to zero because the heat lost by the hot pan must be the same as the heat gained by the cold water:

**Equation:**

$$\begin{aligned}Q_{\text{cold}} + Q_{\text{hot}} &= 0, \\Q_{\text{cold}} &= -Q_{\text{hot}}, \\m_{\text{W}}c_{\text{W}}(T_{\text{f}} - 20.0^{\circ}\text{C}) &= -m_{\text{Al}}c_{\text{Al}}(T_{\text{f}} - 150^{\circ}\text{C}).\end{aligned}$$

4. This an equation for the unknown final temperature,  $T_{\text{f}}$   
5. Bring all terms involving  $T_{\text{f}}$  on the left hand side and all other terms on the right hand side. Solve for  $T_{\text{f}}$ ,

**Equation:**

$$T_{\text{f}} = \frac{m_{\text{Al}}c_{\text{Al}}(150^{\circ}\text{C}) + m_{\text{W}}c_{\text{W}}(20.0^{\circ}\text{C})}{m_{\text{Al}}c_{\text{Al}} + m_{\text{W}}c_{\text{W}}},$$

and insert the numerical values:

**Equation:**

$$\begin{aligned}T_{\text{f}} &= \frac{(0.500 \text{ kg})(900 \text{ J/kg}^{\circ}\text{C})(150^{\circ}\text{C}) + (0.250 \text{ kg})(4186 \text{ J/kg}^{\circ}\text{C})(20.0^{\circ}\text{C})}{(0.500 \text{ kg})(900 \text{ J/kg}^{\circ}\text{C}) + (0.250 \text{ kg})(4186 \text{ J/kg}^{\circ}\text{C})} \\&= \frac{88430 \text{ J}}{1496.5 \text{ J}^{\circ}\text{C}} \\&= 59.1^{\circ}\text{C}.\end{aligned}$$

## Discussion

This is a typical *calorimetry* problem—two bodies at different temperatures are brought in contact with each other and exchange heat until a common temperature is reached. Why is the final temperature so much closer to  $20.0^{\circ}\text{C}$  than  $150^{\circ}\text{C}$ ? The reason is that water has a greater specific heat than most common substances and thus undergoes a small temperature change for a given heat transfer. A large body of water, such as a lake, requires a large amount of heat to increase its temperature appreciably. This explains why the temperature of a lake stays relatively constant during a day even when the temperature change of the air is large. However, the water temperature does change over longer times (e.g., summer to winter).

**Note:****Take-Home Experiment: Temperature Change of Land and Water**

What heats faster, land or water?

To study differences in heat capacity:

- Place equal masses of dry sand (or soil) and water at the same temperature into two small jars. (The average density of soil or sand is about 1.6 times that of water, so you can achieve approximately equal masses by using 50% more water by volume.)
- Heat both (using an oven or a heat lamp) for the same amount of time.
- Record the final temperature of the two masses.
- Now bring both jars to the same temperature by heating for a longer period of time.
- Remove the jars from the heat source and measure their temperature every 5 minutes for about 30 minutes.

Which sample cools off the fastest? This activity replicates the phenomena responsible for land breezes and sea breezes.

**Exercise:****Check Your Understanding****Problem:**

If 25 kJ is necessary to raise the temperature of a block from 25°C to 30°C, how much heat is necessary to heat the block from 45°C to 50°C?

**Solution:**

The heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case.

**Summary**

- The transfer of heat  $Q$  that leads to a change  $\Delta T$  in the temperature of a body with mass  $m$  is  $Q = mc\Delta T$ , where  $c$  is the specific heat of the material. This relationship can also be considered as the definition of specific heat.

**Conceptual Questions****Exercise:**

**Problem:**

What three factors affect the heat transfer that is necessary to change an object's temperature?

**Exercise:****Problem:**

The brakes in a car increase in temperature by  $\Delta T$  when bringing the car to rest from a speed  $v$ . How much greater would  $\Delta T$  be if the car initially had twice the speed? You may assume the car to stop sufficiently fast so that no heat transfers out of the brakes.

**Problems & Exercises****Exercise:****Problem:**

On a hot day, the temperature of an 80,000-L swimming pool increases by  $1.50^\circ\text{C}$ . What is the net heat transfer during this heating? Ignore any complications, such as loss of water by evaporation.

---

**Solution:****Equation:**

$$5.02 \times 10^8 \text{ J}$$

**Exercise:**

**Problem:** Show that  $1 \text{ cal/g} \cdot ^\circ\text{C} = 1 \text{ kcal/kg} \cdot ^\circ\text{C}$ .

**Exercise:****Problem:**

To sterilize a 50.0-g glass baby bottle, we must raise its temperature from  $22.0^\circ\text{C}$  to  $95.0^\circ\text{C}$ . How much heat transfer is required?

---

**Solution:****Equation:**

$$3.07 \times 10^3 \text{ J}$$

**Exercise:****Problem:**

The same heat transfer into identical masses of different substances produces different temperature changes. Calculate the final temperature when 1.00 kcal of heat transfers into 1.00 kg of the following, originally at 20.0°C: (a) water; (b) concrete; (c) steel; and (d) mercury.

**Exercise:****Problem:**

Rubbing your hands together warms them by converting work into thermal energy. If a woman rubs her hands back and forth for a total of 20 rubs, at a distance of 7.50 cm per rub, and with an average frictional force of 40.0 N, what is the temperature increase? The mass of tissues warmed is only 0.100 kg, mostly in the palms and fingers.

---

**Solution:****Equation:**

$$0.171^{\circ}\text{C}$$

**Exercise:****Problem:**

A 0.250-kg block of a pure material is heated from 20.0°C to 65.0°C by the addition of 4.35 kJ of energy. Calculate its specific heat and identify the substance of which it is most likely composed.

**Exercise:****Problem:**

Suppose identical amounts of heat transfer into different masses of copper and water, causing identical changes in temperature. What is the ratio of the mass of copper to water?

---

**Solution:**

$$10.8$$

**Exercise:**

**Problem:**

(a) The number of kilocalories in food is determined by calorimetry techniques in which the food is burned and the amount of heat transfer is measured. How many kilocalories per gram are there in a 5.00-g peanut if the energy from burning it is transferred to 0.500 kg of water held in a 0.100-kg aluminum cup, causing a  $54.9^{\circ}\text{C}$  temperature increase? (b) Compare your answer to labeling information found on a package of peanuts and comment on whether the values are consistent.

**Exercise:****Problem:**

Following vigorous exercise, the body temperature of an 80.0-kg person is  $40.0^{\circ}\text{C}$ . At what rate in watts must the person transfer thermal energy to reduce the the body temperature to  $37.0^{\circ}\text{C}$  in 30.0 min, assuming the body continues to produce energy at the rate of 150 W? (1 watt = 1 joule/second or  $1\text{ W} = 1\text{ J/s}$  ).

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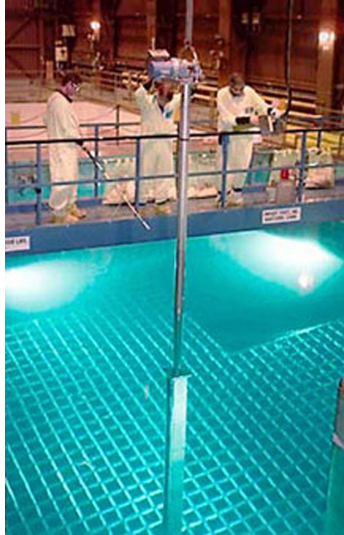
**Solution:**

617 W

**Exercise:****Problem:**

Even when shut down after a period of normal use, a large commercial nuclear reactor transfers thermal energy at the rate of 150 MW by the radioactive decay of fission products. This heat transfer causes a rapid increase in temperature if the cooling system fails

(1 watt = 1 joule/second or  $1\text{ W} = 1\text{ J/s}$  and  $1\text{ MW} = 1\text{ megawatt}$ ). (a) Calculate the rate of temperature increase in degrees Celsius per second ( $^{\circ}\text{C/s}$ ) if the mass of the reactor core is  $1.60 \times 10^5\text{ kg}$  and it has an average specific heat of  $0.3349\text{ kJ/kg}^{\circ}\text{C}$ . (b) How long would it take to obtain a temperature increase of  $2000^{\circ}\text{C}$ , which could cause some metals holding the radioactive materials to melt? (The initial rate of temperature increase would be greater than that calculated here because the heat transfer is concentrated in a smaller mass. Later, however, the temperature increase would slow down because the  $5 \times 10^5\text{-kg}$  steel containment vessel would also begin to heat up.)



Radioactive spent-fuel pool at a nuclear power plant. Spent fuel stays hot for a long time. (credit: U.S. Department of Energy)

## Glossary

specific heat

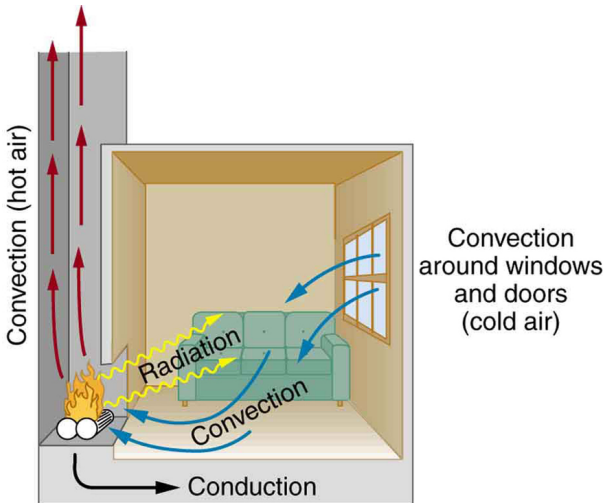
the amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 °C

## Heat Transfer Methods

- Discuss the different methods of heat transfer.

Equally as interesting as the effects of heat transfer on a system are the methods by which this occurs. Whenever there is a temperature difference, heat transfer occurs. Heat transfer may occur rapidly, such as through a cooking pan, or slowly, such as through the walls of a picnic ice chest. We can control rates of heat transfer by choosing materials (such as thick wool clothing for the winter), controlling air movement (such as the use of weather stripping around doors), or by choice of color (such as a white roof to reflect summer sunlight). So many processes involve heat transfer, so that it is hard to imagine a situation where no heat transfer occurs. Yet every process involving heat transfer takes place by only three methods:

1. **Conduction** is heat transfer through stationary matter by physical contact. (The matter is stationary on a macroscopic scale—we know there is thermal motion of the atoms and molecules at any temperature above absolute zero.) Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction.
2. **Convection** is the heat transfer by the macroscopic movement of a fluid. This type of transfer takes place in a forced-air furnace and in weather systems, for example.
3. Heat transfer by **radiation** occurs when microwaves, infrared radiation, visible light, or another form of electromagnetic radiation is emitted or absorbed. An obvious example is the warming of the Earth by the Sun. A less obvious example is thermal radiation from the human body.



In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but at a much slower rate. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney.

We examine these methods in some detail in the three following modules. Each method has unique and interesting characteristics, but all three do have one thing in common: they transfer heat solely because of a temperature difference [\[link\]](#).

**Exercise:**

**Check Your Understanding**

**Problem:**

Name an example from daily life (different from the text) for each mechanism of heat transfer.

---

**Solution:**

Conduction: Heat transfers into your hands as you hold a hot cup of coffee.

Convection: Heat transfers as the barista “steams” cold milk to make hot *cocoa*.

Radiation: Reheating a cold cup of coffee in a microwave oven.

**Summary**

- Heat is transferred by three different methods: conduction, convection, and radiation.

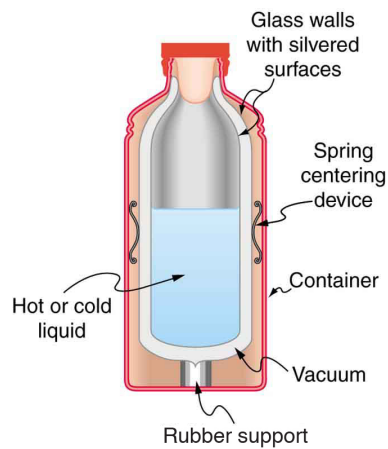
**Conceptual Questions****Exercise:****Problem:**

What are the main methods of heat transfer from the hot core of Earth to its surface? From Earth’s surface to outer space?

When our bodies get too warm, they respond by sweating and increasing blood circulation to the surface to transfer thermal energy away from the core. What effect will this have on a person in a 40.0°C hot tub?

[\[link\]](#) shows a cut-away drawing of a thermos bottle (also known as a Dewar flask), which is a device designed specifically to slow down all forms of heat transfer. Explain the functions of the various parts, such as the

vacuum, the silvering of the walls, the thin-walled long glass neck, the rubber support, the air layer, and the stopper.



The construction of a thermos bottle is designed to inhibit all methods of heat transfer.

## Glossary

### conduction

heat transfer through stationary matter by physical contact

### convection

heat transfer by the macroscopic movement of fluid

### radiation

heat transfer which occurs when microwaves, infrared radiation, visible light, or other electromagnetic radiation is emitted or absorbed

## Conduction

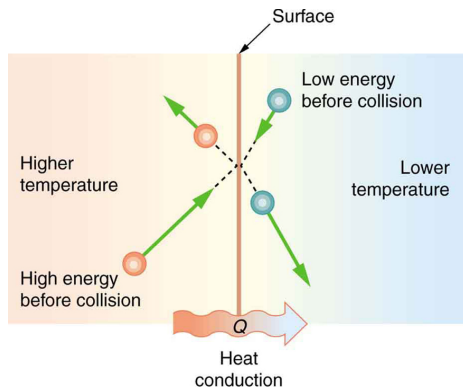
- Calculate thermal conductivity.
- Observe conduction of heat in collisions.
- Study thermal conductivities of common substances.



Insulation is used to limit the conduction of heat from the inside to the outside (in winters) and from the outside to the inside (in summers).  
(credit: Giles Douglas)

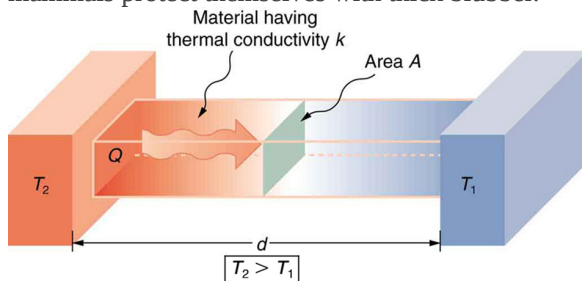
Your feet feel cold as you walk barefoot across the living room carpet in your cold house and then step onto the kitchen tile floor. This result is intriguing, since the carpet and tile floor are both at the same temperature. The different sensation you feel is explained by the different rates of heat transfer: the heat loss during the same time interval is greater for skin in contact with the tiles than with the carpet, so the temperature drop is greater on the tiles.

Some materials conduct thermal energy faster than others. In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors. [\[link\]](#) shows molecules in two bodies at different temperatures. The (average) kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, an energy transfer from the molecule with greater kinetic energy to the molecule with less kinetic energy occurs. The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. The heat flux thus depends on the temperature difference  $\Delta T = T_{\text{hot}} - T_{\text{cold}}$ . Therefore, you will get a more severe burn from boiling water than from hot tap water. Conversely, if the temperatures are the same, the net heat transfer rate falls to zero, and equilibrium is achieved. Owing to the fact that the number of collisions increases with increasing area, heat conduction depends on the cross-sectional area. If you touch a cold wall with your palm, your hand cools faster than if you just touch it with your fingertip.



The molecules in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions. In this illustration, a molecule in the lower temperature region (right side) has low energy before collision, but its energy increases after colliding with the contact surface. In contrast, a molecule in the higher temperature region (left side) has high energy before collision, but its energy decreases after colliding with the contact surface.

A third factor in the mechanism of conduction is the thickness of the material through which heat transfers. The figure below shows a slab of material with different temperatures on either side. Suppose that  $T_2$  is greater than  $T_1$ , so that heat is transferred from left to right. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. This model explains why thick clothing is warmer than thin clothing in winters, and why Arctic mammals protect themselves with thick blubber.



Heat conduction occurs through any material, represented here by a rectangular bar, whether window glass or walrus blubber. The temperature of the material is  $T_2$  on the left and  $T_1$  on the right, where  $T_2$  is greater than  $T_1$ .

The rate of heat transfer by conduction is directly proportional to the surface area  $A$ , the temperature difference  $T_2 - T_1$ , and the substance's conductivity  $k$ . The rate of heat transfer is inversely proportional to the thickness  $d$ .

Lastly, the heat transfer rate depends on the material properties described by the coefficient of thermal conductivity. All four factors are included in a simple equation that was deduced from and is confirmed by experiments. The **rate of conductive heat transfer** through a slab of material, such as the one in [\[link\]](#), is given by

**Equation:**

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d},$$

where  $Q/t$  is the rate of heat transfer in watts or kilocalories per second,  $k$  is the **thermal conductivity** of the material,  $A$  and  $d$  are its surface area and thickness, as shown in [\[link\]](#), and  $(T_2 - T_1)$  is the temperature difference across the slab. [\[link\]](#) gives representative values of thermal conductivity.

**Example:**

#### Calculating Heat Transfer Through Conduction: Conduction Rate Through an Ice Box

A Styrofoam ice box has a total area of  $0.950 \text{ m}^2$  and walls with an average thickness of  $2.50 \text{ cm}$ . The box contains ice, water, and canned beverages at  $0^\circ\text{C}$ . The inside of the box is kept cold by melting ice. How much ice melts in one day if the ice box is kept in the trunk of a car at  $35.0^\circ\text{C}$ ?

**Strategy**

This question involves both heat for a phase change (melting of ice) and the transfer of heat by conduction. To find the amount of ice melted, we must find the net heat transferred. This value can be obtained by calculating the rate of heat transfer by conduction and multiplying by time.

**Solution**

1. Identify the knowns.

**Equation:**

$$A = 0.950 \text{ m}^2; d = 2.50 \text{ cm} = 0.0250 \text{ m}; T_1 = 0^\circ\text{C}; T_2 = 35.0^\circ\text{C}; t = 1 \text{ day} = 24 \text{ hours} = 86,400 \text{ s}.$$

2. Identify the unknowns. We need to solve for the mass of the ice,  $m$ . We will also need to solve for the net heat transferred to melt the ice,  $Q$ .
3. Determine which equations to use. The rate of heat transfer by conduction is given by

**Equation:**

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}.$$

4. The heat is used to melt the ice:  $Q = mL_f$ .
5. Insert the known values:

**Equation:**

$$\frac{Q}{t} = \frac{(0.010 \text{ J/s} \cdot \text{m} \cdot ^\circ\text{C})(0.950 \text{ m}^2)(35.0^\circ\text{C} - 0^\circ\text{C})}{0.0250 \text{ m}} = 13.3 \text{ J/s}.$$

6. Multiply the rate of heat transfer by the time (1 day = 86,400 s):

**Equation:**

$$Q = (Q/t)t = (13.3 \text{ J/s})(86,400 \text{ s}) = 1.15 \times 10^6 \text{ J}.$$

7. Set this equal to the heat transferred to melt the ice:  $Q = mL_f$ . Solve for the mass  $m$ :

**Equation:**

$$m = \frac{Q}{L_f} = \frac{1.15 \times 10^6 \text{ J}}{334 \times 10^3 \text{ J/kg}} = 3.44 \text{ kg}.$$

### Discussion

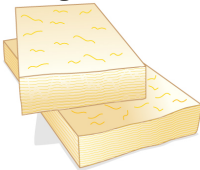
The result of 3.44 kg, or about 7.6 lbs, seems about right, based on experience. You might expect to use about a 4 kg (7–10 lb) bag of ice per day. A little extra ice is required if you add any warm food or beverages. Inspecting the conductivities in [\[link\]](#) shows that Styrofoam is a very poor conductor and thus a good insulator. Other good insulators include fiberglass, wool, and goose-down feathers. Like Styrofoam, these all incorporate many small pockets of air, taking advantage of air's poor thermal conductivity.

Substance	Thermal conductivity $k$ (J/s·m·°C)
Silver	420
Copper	390
Gold	318
Aluminum	220
Steel iron	80
Steel (stainless)	14
Ice	2.2
Glass (average)	0.84
Concrete brick	0.84
Water	0.6
Fatty tissue (without blood)	0.2
Asbestos	0.16
Plasterboard	0.16
Wood	0.08–0.16

Substance	Thermal conductivity $k$ (J/s·m·°C)
Snow (dry)	0.10
Cork	0.042
Glass wool	0.042
Wool	0.04
Down feathers	0.025
Air	0.023
Styrofoam	0.010

Thermal Conductivities of Common Substances[\[footnote\]](#)  
At temperatures near 0°C.

A combination of material and thickness is often manipulated to develop good insulators—the smaller the conductivity  $k$  and the larger the thickness  $d$ , the better. The ratio of  $d/k$  will thus be large for a good insulator. The ratio  $d/k$  is called the  **$R$  factor**. The rate of conductive heat transfer is inversely proportional to  $R$ . The larger the value of  $R$ , the better the insulation.  $R$  factors are most commonly quoted for household insulation, refrigerators, and the like—unfortunately, it is still in non-metric units of  $\text{ft}^2 \cdot ^\circ\text{F} \cdot \text{h} / \text{Btu}$ , although the unit usually goes unstated (1 British thermal unit [Btu] is the amount of energy needed to change the temperature of 1.0 lb of water by 1.0 °F). A couple of representative values are an  $R$  factor of 11 for 3.5-in-thick fiberglass batts (pieces) of insulation and an  $R$  factor of 19 for 6.5-in-thick fiberglass batts. Walls are usually insulated with 3.5-in batts, while ceilings are usually insulated with 6.5-in batts. In cold climates, thicker batts may be used in ceilings and walls.



The fiberglass batt is used for insulation of walls and ceilings to prevent heat transfer between the inside of the building and the outside environment.

Note that in [\[link\]](#), the best thermal conductors—silver, copper, gold, and aluminum—are also the best electrical conductors, again related to the density of free electrons in them. Cooking utensils are typically made

from good conductors.

**Example:**

**Calculating the Temperature Difference Maintained by a Heat Transfer: Conduction Through an Aluminum Pan**

Water is boiling in an aluminum pan placed on an electrical element on a stovetop. The sauce pan has a bottom that is 0.800 cm thick and 14.0 cm in diameter. The boiling water is evaporating at the rate of 1.00 g/s. What is the temperature difference across (through) the bottom of the pan?

**Strategy**

Conduction through the aluminum is the primary method of heat transfer here, and so we use the equation for the rate of heat transfer and solve for the temperature difference.

**Equation:**

$$T_2 - T_1 = \frac{Q}{t} \left( \frac{d}{kA} \right).$$

**Solution**

1. Identify the knowns and convert them to the SI units.

The thickness of the pan,  $d = 0.800 \text{ cm} = 8.0 \times 10^{-3} \text{ m}$ , the area of the pan,  $A = \pi(0.14/2)^2 \text{ m}^2 = 1.54 \times 10^{-2} \text{ m}^2$ , and the thermal conductivity,  $k = 220 \text{ J/s} \cdot \text{m} \cdot ^\circ\text{C}$ .

2. Calculate the necessary heat of vaporization of 1 g of water:

**Equation:**

$$Q = mL_v = (1.00 \times 10^{-3} \text{ kg})(2256 \times 10^3 \text{ J/kg}) = 2256 \text{ J}.$$

3. Calculate the rate of heat transfer given that 1 g of water melts in one second:

**Equation:**

$$Q/t = 2256 \text{ J/s or } 2.26 \text{ kW}.$$

4. Insert the knowns into the equation and solve for the temperature difference:

**Equation:**

$$T_2 - T_1 = \frac{Q}{t} \left( \frac{d}{kA} \right) = (2256 \text{ J/s}) \frac{8.00 \times 10^{-3} \text{ m}}{(220 \text{ J/s} \cdot \text{m} \cdot ^\circ\text{C})(1.54 \times 10^{-2} \text{ m}^2)} = 5.33^\circ\text{C}.$$

**Discussion**

The value for the heat transfer  $Q/t = 2.26 \text{ kW}$  or  $2256 \text{ J/s}$  is typical for an electric stove. This value gives a remarkably small temperature difference between the stove and the pan. Consider that the stove burner is red hot while the inside of the pan is nearly  $100^\circ\text{C}$  because of its contact with boiling water. This contact effectively cools the bottom of the pan in spite of its proximity to the very hot stove burner. Aluminum is such a good conductor that it only takes this small temperature difference to produce a heat transfer of  $2.26 \text{ kW}$  into the pan.

Conduction is caused by the random motion of atoms and molecules. As such, it is an ineffective mechanism for heat transport over macroscopic distances and short time distances. Take, for example, the temperature on the Earth, which would be unbearably cold during the night and extremely hot during the day if heat transport in the atmosphere was to be only through conduction. In another example, car engines would overheat unless there was a more efficient way to remove excess heat from the pistons.

**Exercise:**  
**Check Your Understanding**

**Problem:**

How does the rate of heat transfer by conduction change when all spatial dimensions are doubled?

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**Solution:**

Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubled ( $A_{\text{final}} = (2d)^2 = 4d^2 = 4A_{\text{initial}}$ ). The distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two, or two:

**Equation:**

$$\left(\frac{Q}{t}\right)_{\text{final}} = \frac{kA_{\text{final}}(T_2 - T_1)}{d_{\text{final}}} = \frac{k(4A_{\text{initial}})(T_2 - T_1)}{2d_{\text{initial}}} = 2 \frac{kA_{\text{initial}}(T_2 - T_1)}{d_{\text{initial}}} = 2 \left(\frac{Q}{t}\right)_{\text{initial}}.$$

**Summary**

- Heat conduction is the transfer of heat between two objects in direct contact with each other.
- The rate of heat transfer  $Q/t$  (energy per unit time) is proportional to the temperature difference  $T_2 - T_1$  and the contact area  $A$  and inversely proportional to the distance  $d$  between the objects:

**Equation:**

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}.$$

**Conceptual Questions**

**Exercise:**

**Problem:**

Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A pot placed over a heating element will be heated, while it is safe to touch the surface only a few centimeters away. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?

**Exercise:**

**Problem:**

Loose-fitting white clothing covering most of the body is ideal for desert dwellers, both in the hot Sun and during cold evenings. Explain how such clothing is advantageous during both day and night.



A jellabiya is worn by many men in Egypt. (credit: Zerida)

## Problems & Exercises

### Exercise:

#### Problem:

- (a) Calculate the rate of heat conduction through house walls that are 13.0 cm thick and that have an average thermal conductivity twice that of glass wool. Assume there are no windows or doors. The surface area of the walls is  $120 \text{ m}^2$  and their inside surface is at  $18.0^\circ\text{C}$ , while their outside surface is at  $5.00^\circ\text{C}$ .
- (b) How many 1-kW room heaters would be needed to balance the heat transfer due to conduction?

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#### Solution:

- (a)  $1.01 \times 10^3 \text{ W}$
- (b) One

### Exercise:

#### Problem:

The rate of heat conduction out of a window on a winter day is rapid enough to chill the air next to it. To see just how rapidly the windows transfer heat by conduction, calculate the rate of conduction in watts through a  $3.00\text{-m}^2$  window that is 0.635 cm thick (1/4 in) if the temperatures of the inner and outer surfaces are  $5.00^\circ\text{C}$  and  $-10.0^\circ\text{C}$ , respectively. This rapid rate will not be maintained—the inner surface will cool, and even result in frost formation.

### Exercise:

#### Problem:

Calculate the rate of heat conduction out of the human body, assuming that the core internal temperature is  $37.0^\circ\text{C}$ , the skin temperature is  $34.0^\circ\text{C}$ , the thickness of the tissues between averages 1.00 cm, and the surface area is  $1.40 \text{ m}^2$ .

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**Solution:**

84.0 W

**Exercise:****Problem:**

Suppose you stand with one foot on ceramic flooring and one foot on a wool carpet, making contact over an area of  $80.0 \text{ cm}^2$  with each foot. Both the ceramic and the carpet are 2.00 cm thick and are  $10.0^\circ\text{C}$  on their bottom sides. At what rate must heat transfer occur from each foot to keep the top of the ceramic and carpet at  $33.0^\circ\text{C}$ ?

**Exercise:****Problem:**

A man consumes 3000 kcal of food in one day, converting most of it to maintain body temperature. If he loses half this energy by evaporating water (through breathing and sweating), how many kilograms of water evaporate?

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**Solution:**

2.59 kg

**Exercise:****Problem:**

(a) A firewalker runs across a bed of hot coals without sustaining burns. Calculate the heat transferred by conduction into the sole of one foot of a firewalker given that the bottom of the foot is a 3.00-mm-thick callus with a conductivity at the low end of the range for wood and its density is  $300 \text{ kg/m}^3$ . The area of contact is  $25.0 \text{ cm}^2$ , the temperature of the coals is  $700^\circ\text{C}$ , and the time in contact is 1.00 s.

(b) What temperature increase is produced in the  $25.0 \text{ cm}^3$  of tissue affected?

(c) What effect do you think this will have on the tissue, keeping in mind that a callus is made of dead cells?

**Exercise:****Problem:**

(a) What is the rate of heat conduction through the 3.00-cm-thick fur of a large animal having a  $1.40\text{-m}^2$  surface area? Assume that the animal's skin temperature is  $32.0^\circ\text{C}$ , that the air temperature is  $-5.00^\circ\text{C}$ , and that fur has the same thermal conductivity as air. (b) What food intake will the animal need in one day to replace this heat transfer?

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**Solution:**

(a) 39.7 W

(b) 820 kcal

**Exercise:**

**Problem:**

A walrus transfers energy by conduction through its blubber at the rate of 150 W when immersed in  $-1.00^{\circ}\text{C}$  water. The walrus's internal core temperature is  $37.0^{\circ}\text{C}$ , and it has a surface area of  $2.00\text{ m}^2$ . What is the average thickness of its blubber, which has the conductivity of fatty tissues without blood?



Walrus on ice. (credit: Captain Budd Christman, NOAA Corps)

**Exercise:****Problem:**

Compare the rate of heat conduction through a 13.0-cm-thick wall that has an area of  $10.0\text{ m}^2$  and a thermal conductivity twice that of glass wool with the rate of heat conduction through a window that is 0.750 cm thick and that has an area of  $2.00\text{ m}^2$ , assuming the same temperature difference across each.

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**Solution:**

35 to 1, window to wall

**Exercise:****Problem:**

Suppose a person is covered head to foot by wool clothing with average thickness of 2.00 cm and is transferring energy by conduction through the clothing at the rate of 50.0 W. What is the temperature difference across the clothing, given the surface area is  $1.40\text{ m}^2$ ?

**Exercise:****Problem:**

Some stove tops are smooth ceramic for easy cleaning. If the ceramic is 0.600 cm thick and heat conduction occurs through the same area and at the same rate as computed in [\[link\]](#), what is the temperature difference across it? Ceramic has the same thermal conductivity as glass and brick.

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**Solution:**

$1.05 \times 10^3\text{ K}$

**Exercise:**

**Problem:**

One easy way to reduce heating (and cooling) costs is to add extra insulation in the attic of a house. Suppose the house already had 15 cm of fiberglass insulation in the attic and in all the exterior surfaces. If you added an extra 8.0 cm of fiberglass to the attic, then by what percentage would the heating cost of the house drop? Take the single story house to be of dimensions 10 m by 15 m by 3.0 m. Ignore air infiltration and heat loss through windows and doors.

**Exercise:****Problem:**

(a) Calculate the rate of heat conduction through a double-paned window that has a  $1.50\text{-m}^2$  area and is made of two panes of 0.800-cm-thick glass separated by a 1.00-cm air gap. The inside surface temperature is  $15.0^\circ\text{C}$ , while that on the outside is  $-10.0^\circ\text{C}$ . (Hint: There are identical temperature drops across the two glass panes. First find these and then the temperature drop across the air gap. This problem ignores the increased heat transfer in the air gap due to convection.)

(b) Calculate the rate of heat conduction through a 1.60-cm-thick window of the same area and with the same temperatures. Compare your answer with that for part (a).

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**Solution:**

(a) 83 W

(b) 24 times that of a double pane window.

**Exercise:****Problem:**

Many decisions are made on the basis of the payback period: the time it will take through savings to equal the capital cost of an investment. Acceptable payback times depend upon the business or philosophy one has. (For some industries, a payback period is as small as two years.) Suppose you wish to install the extra insulation in [\[link\]](#). If energy cost \$1.00 per million joules and the insulation was \$4.00 per square meter, then calculate the simple payback time. Take the average  $\Delta T$  for the 120 day heating season to be  $15.0^\circ\text{C}$ .

**Exercise:****Problem:**

For the human body, what is the rate of heat transfer by conduction through the body's tissue with the following conditions: the tissue thickness is 3.00 cm, the change in temperature is  $2.00^\circ\text{C}$ , and the skin area is  $1.50\text{ m}^2$ . How does this compare with the average heat transfer rate to the body resulting from an energy intake of about 2400 kcal per day? (No exercise is included.)

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**Solution:**

20.0 W, 17.2% of 2400 kcal per day

**Glossary**

*R* factor

the ratio of thickness to the conductivity of a material

rate of conductive heat transfer

rate of heat transfer from one material to another

thermal conductivity

the property of a material's ability to conduct heat

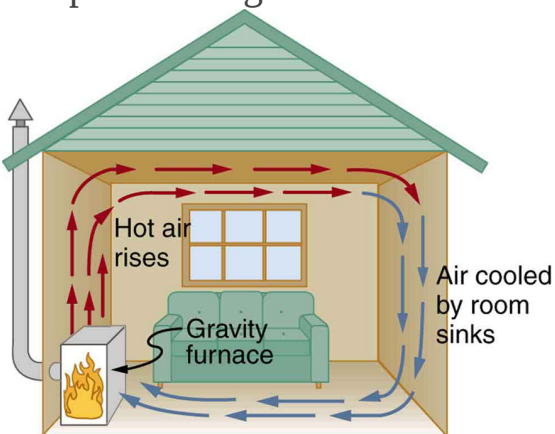
## Convection

- Discuss the method of heat transfer by convection.

Convection is driven by large-scale flow of matter. In the case of Earth, the atmospheric circulation is caused by the flow of hot air from the tropics to the poles, and the flow of cold air from the poles toward the tropics. (Note that Earth's rotation causes the observed easterly flow of air in the northern hemisphere). Car engines are kept cool by the flow of water in the cooling system, with the water pump maintaining a flow of cool water to the pistons. The circulatory system is used the body: when the body overheats, the blood vessels in the skin expand (dilate), which increases the blood flow to the skin where it can be cooled by sweating. These vessels become smaller when it is cold outside and larger when it is hot (so more fluid flows, and more energy is transferred).

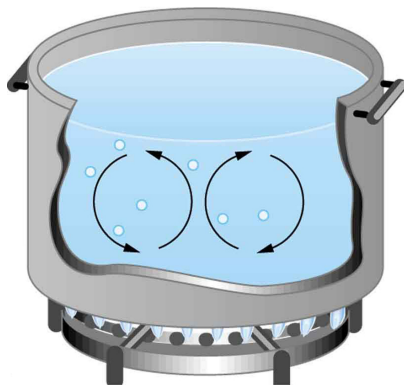
The body also loses a significant fraction of its heat through the breathing process.

While convection is usually more complicated than conduction, we can describe convection and do some straightforward, realistic calculations of its effects. Natural convection is driven by buoyant forces: hot air rises because density decreases as temperature increases. The house in [\[link\]](#) is kept warm in this manner, as is the pot of water on the stove in [\[link\]](#). Ocean currents and large-scale atmospheric circulation transfer energy from one part of the globe to another. Both are examples of natural convection.



Air heated by the so-called

gravity furnace expands and rises, forming a convective loop that transfers energy to other parts of the room. As the air is cooled at the ceiling and outside walls, it contracts, eventually becoming denser than room air and sinking to the floor. A properly designed heating system using natural convection, like this one, can be quite efficient in uniformly heating a home.



Convection plays an important role in heat transfer inside this pot of water. Once conducted to the inside, heat transfer to other parts of the pot is mostly by convection. The hotter water expands, decreases

in density, and rises to transfer heat to other regions of the water, while colder water sinks to the bottom. This process keeps repeating.

**Note:**

**Take-Home Experiment: Convection Rolls in a Heated Pan**

Take two small pots of water and use an eye dropper to place a drop of food coloring near the bottom of each. Leave one on a bench top and heat the other over a stovetop. Watch how the color spreads and how long it takes the color to reach the top. Watch how convective loops form.

**Example:**

**Calculating Heat Transfer by Convection: Convection of Air Through the Walls of a House**

Most houses are not airtight: air goes in and out around doors and windows, through cracks and crevices, following wiring to switches and outlets, and so on. The air in a typical house is completely replaced in less than an hour. Suppose that a moderately-sized house has inside dimensions  $12.0\text{m} \times 18.0\text{m} \times 3.00\text{m}$  high, and that all air is replaced in 30.0 min. Calculate the heat transfer per unit time in watts needed to warm the incoming cold air by  $10.0^\circ\text{C}$ , thus replacing the heat transferred by convection alone.

**Strategy**

Heat is used to raise the temperature of air so that  $Q = mc\Delta T$ . The rate of heat transfer is then  $Q/t$ , where  $t$  is the time for air turnover. We are given that  $\Delta T$  is  $10.0^\circ\text{C}$ , but we must still find values for the mass of air and its

specific heat before we can calculate  $Q$ . The specific heat of air is a weighted average of the specific heats of nitrogen and oxygen, which gives  $c = c_p \cong 1000 \text{ J/kg} \cdot ^\circ \text{C}$  from [\[link\]](#) (note that the specific heat at constant pressure must be used for this process).

### Solution

1. Determine the mass of air from its density and the given volume of the house. The density is given from the density  $\rho$  and the volume

**Equation:**

$$m = \rho V = (1.29 \text{ kg/m}^3)(12.0 \text{ m} \times 18.0 \text{ m} \times 3.00 \text{ m}) = 836 \text{ kg}.$$

2. Calculate the heat transferred from the change in air temperature:

$$Q = mc\Delta T \text{ so that}$$

**Equation:**

$$Q = (836 \text{ kg})(1000 \text{ J/kg} \cdot ^\circ \text{C})(10.0^\circ \text{C}) = 8.36 \times 10^6 \text{ J}.$$

3. Calculate the heat transfer from the heat  $Q$  and the turnover time  $t$ . Since air is turned over in  $t = 0.500 \text{ h} = 1800 \text{ s}$ , the heat transferred per unit time is

**Equation:**

$$\frac{Q}{t} = \frac{8.36 \times 10^6 \text{ J}}{1800 \text{ s}} = 4.64 \text{ kW}.$$

### Discussion

This rate of heat transfer is equal to the power consumed by about forty-six 100-W light bulbs. Newly constructed homes are designed for a turnover time of 2 hours or more, rather than 30 minutes for the house of this example. Weather stripping, caulking, and improved window seals are commonly employed. More extreme measures are sometimes taken in very cold (or hot) climates to achieve a tight standard of more than 6 hours for one air turnover. Still longer turnover times are unhealthy, because a minimum amount of fresh air is necessary to supply oxygen for breathing and to dilute household pollutants. The term used for the process by which

outside air leaks into the house from cracks around windows, doors, and the foundation is called “air infiltration.”

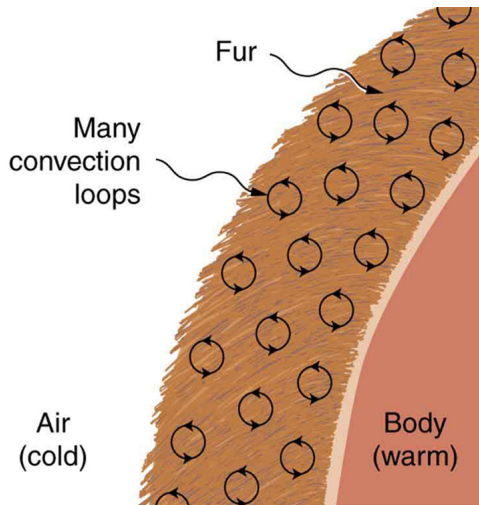
A cold wind is much more chilling than still cold air, because convection combines with conduction in the body to increase the rate at which energy is transferred away from the body. The table below gives approximate wind-chill factors, which are the temperatures of still air that produce the same rate of cooling as air of a given temperature and speed. Wind-chill factors are a dramatic reminder of convection’s ability to transfer heat faster than conduction. For example, a 15.0 m/s wind at 0°C has the chilling equivalent of still air at about  $-18^{\circ}\text{C}$ .

<b>Moving air temperature</b>	<b>Wind speed (m/s)</b>				
<b>(°C)</b>	<b>2</b>	<b>5</b>	<b>10</b>	<b>15</b>	<b>20</b>
<b>5</b>	3	−1	−8	−10	−12
<b>2</b>	0	−7	−12	−16	−18
<b>0</b>	−2	−9	−15	−18	−20

<b>Moving air temperature</b>	<b>Wind speed (m/s)</b>				
<b>−5</b>	<b>−7</b>	<b>−15</b>	<b>−22</b>	<b>−26</b>	<b>−29</b>
<b>−10</b>	<b>−12</b>	<b>−21</b>	<b>−29</b>	<b>−34</b>	<b>−36</b>
<b>−20</b>	<b>−23</b>	<b>−34</b>	<b>−44</b>	<b>−50</b>	<b>−52</b>
<b>−40</b>	<b>−44</b>	<b>−59</b>	<b>−73</b>	<b>−82</b>	<b>−84</b>

### Wind-Chill Factors

Although air can transfer heat rapidly by convection, it is a poor conductor and thus a good insulator. The amount of available space for airflow determines whether air acts as an insulator or conductor. The space between the inside and outside walls of a house, for example, is about 9 cm (3.5 in)—large enough for convection to work effectively. The addition of wall insulation prevents airflow, so heat loss (or gain) is decreased. Similarly, the gap between the two panes of a double-paned window is about 1 cm, which prevents convection and takes advantage of air's low conductivity to prevent greater loss. Fur, fiber, and fiberglass also take advantage of the low conductivity of air by trapping it in spaces too small to support convection, as shown in the figure. Fur and feathers are lightweight and thus ideal for the protection of animals.



Fur is filled with air, breaking it up into many small pockets. Convection is very slow here, because the loops are so small. The low conductivity of air makes fur a very good lightweight insulator.

Some interesting phenomena happen *when convection is accompanied by a phase change*. It allows us to cool off by sweating, even if the temperature of the surrounding air exceeds body temperature. Heat from the skin is required for sweat to evaporate from the skin, but without air flow, the air becomes saturated and evaporation stops. Air flow caused by convection replaces the saturated air by dry air and evaporation continues.

**Example:**

**Calculate the Flow of Mass during Convection: Sweat-Heat Transfer away from the Body**

The average person produces heat at the rate of about 120 W when at rest. At what rate must water evaporate from the body to get rid of all this energy? (This evaporation might occur when a person is sitting in the shade and surrounding temperatures are the same as skin temperature, eliminating heat transfer by other methods.)

**Strategy**

Energy is needed for a phase change ( $Q = mL_v$ ). Thus, the energy loss per unit time is

**Equation:**

$$\frac{Q}{t} = \frac{mL_v}{t} = 120 \text{ W} = 120 \text{ J/s}.$$

We divide both sides of the equation by  $L_v$  to find that the mass evaporated per unit time is

**Equation:**

$$\frac{m}{t} = \frac{120 \text{ J/s}}{L_v}.$$

**Solution**

(1) Insert the value of the latent heat from [\[link\]](#),  $L_v = 2430 \text{ kJ/kg} = 2430 \text{ J/g}$ . This yields

**Equation:**

$$\frac{m}{t} = \frac{120 \text{ J/s}}{2430 \text{ J/g}} = 0.0494 \text{ g/s} = 2.96 \text{ g/min}.$$

**Discussion**

Evaporating about 3 g/min seems reasonable. This would be about 180 g (about 7 oz) per hour. If the air is very dry, the sweat may evaporate without even being noticed. A significant amount of evaporation also takes place in the lungs and breathing passages.

Another important example of the combination of phase change and convection occurs when water evaporates from the oceans. Heat is removed

from the ocean when water evaporates. If the water vapor condenses in liquid droplets as clouds form, heat is released in the atmosphere. Thus, there is an overall transfer of heat from the ocean to the atmosphere. This process is the driving power behind thunderheads, those great cumulus clouds that rise as much as 20.0 km into the stratosphere. Water vapor carried in by convection condenses, releasing tremendous amounts of energy. This energy causes the air to expand and rise, where it is colder. More condensation occurs in these colder regions, which in turn drives the cloud even higher. Such a mechanism is called positive feedback, since the process reinforces and accelerates itself. These systems sometimes produce violent storms, with lightning and hail, and constitute the mechanism driving hurricanes.



Cumulus clouds  
are caused by  
water vapor that  
rises because of  
convection. The  
rise of clouds is  
driven by a  
positive  
feedback  
mechanism.  
(credit: Mike  
Love)



Convection  
accompanied by a  
phase change  
releases the energy  
needed to drive this  
thunderhead into the  
stratosphere. (credit:  
Gerardo García  
Moretti )



The phase change that  
occurs when this  
iceberg melts involves  
tremendous heat

transfer. (credit:  
Dominic Alves)

The movement of icebergs is another example of convection accompanied by a phase change. Suppose an iceberg drifts from Greenland into warmer Atlantic waters. Heat is removed from the warm ocean water when the ice melts and heat is released to the land mass when the iceberg forms on Greenland.

**Exercise:**

**Check Your Understanding**

**Problem:** Explain why using a fan in the summer feels refreshing!

---

**Solution:**

Using a fan increases the flow of air: warm air near your body is replaced by cooler air from elsewhere. Convection increases the rate of heat transfer so that moving air “feels” cooler than still air.

**Summary**

- Convection is heat transfer by the macroscopic movement of mass. Convection can be natural or forced and generally transfers thermal energy faster than conduction. [\[link\]](#) gives wind-chill factors, indicating that moving air has the same chilling effect of much colder stationary air. *Convection that occurs along with a phase change* can transfer energy from cold regions to warm ones.

**Conceptual Questions**

**Exercise:**

**Problem:**

One way to make a fireplace more energy efficient is to have an external air supply for the combustion of its fuel. Another is to have room air circulate around the outside of the fire box and back into the room. Detail the methods of heat transfer involved in each.

**Exercise:****Problem:**

On cold, clear nights horses will sleep under the cover of large trees. How does this help them keep warm?

**Problems & Exercises****Exercise:****Problem:**

At what wind speed does  $-10^{\circ}\text{C}$  air cause the same chill factor as still air at  $-29^{\circ}\text{C}$ ?

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**Solution:**

10 m/s

**Exercise:****Problem:**

At what temperature does still air cause the same chill factor as  $-5^{\circ}\text{C}$  air moving at 15 m/s?

**Exercise:**

**Problem:**

The “steam” above a freshly made cup of instant coffee is really water vapor droplets condensing after evaporating from the hot coffee. What is the final temperature of 250 g of hot coffee initially at  $90.0^{\circ}\text{C}$  if 2.00 g evaporates from it? The coffee is in a Styrofoam cup, so other methods of heat transfer can be neglected.

---

**Solution:**

$85.7^{\circ}\text{C}$

**Exercise:****Problem:**

(a) How many kilograms of water must evaporate from a 60.0-kg woman to lower her body temperature by  $0.750^{\circ}\text{C}$ ?

(b) Is this a reasonable amount of water to evaporate in the form of perspiration, assuming the relative humidity of the surrounding air is low?

**Exercise:****Problem:**

On a hot dry day, evaporation from a lake has just enough heat transfer to balance the  $1.00\text{ kW}/\text{m}^2$  of incoming heat from the Sun. What mass of water evaporates in 1.00 h from each square meter? Explicitly show how you follow the steps in the [Problem-Solving Strategies for the Effects of Heat Transfer](#).

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**Solution:**

1.48 kg

**Exercise:**

**Problem:**

One winter day, the climate control system of a large university classroom building malfunctions. As a result,  $500 \text{ m}^3$  of excess cold air is brought in each minute. At what rate in kilowatts must heat transfer occur to warm this air by  $10.0^\circ\text{C}$  (that is, to bring the air to room temperature)?

**Exercise:****Problem:**

The Kilauea volcano in Hawaii is the world's most active, disgorging about  $5 \times 10^5 \text{ m}^3$  of  $1200^\circ\text{C}$  lava per day. What is the rate of heat transfer out of Earth by convection if this lava has a density of  $2700 \text{ kg/m}^3$  and eventually cools to  $30^\circ\text{C}$ ? Assume that the specific heat of lava is the same as that of granite.



Lava flow on Kilauea volcano in Hawaii. (credit: J. P. Eaton, U.S. Geological Survey)

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**Solution:**

$$2 \times 10^4 \text{ MW}$$

**Exercise:****Problem:**

During heavy exercise, the body pumps 2.00 L of blood per minute to the surface, where it is cooled by  $2.00^{\circ}\text{C}$ . What is the rate of heat transfer from this forced convection alone, assuming blood has the same specific heat as water and its density is  $1050\text{ kg/m}^3$ ?

**Exercise:****Problem:**

A person inhales and exhales 2.00 L of  $37.0^{\circ}\text{C}$  air, evaporating  $4.00 \times 10^{-2}\text{ g}$  of water from the lungs and breathing passages with each breath.

- (a) How much heat transfer occurs due to evaporation in each breath?
- (b) What is the rate of heat transfer in watts if the person is breathing at a moderate rate of 18.0 breaths per minute?
- (c) If the inhaled air had a temperature of  $20.0^{\circ}\text{C}$ , what is the rate of heat transfer for warming the air?
- (d) Discuss the total rate of heat transfer as it relates to typical metabolic rates. Will this breathing be a major form of heat transfer for this person?

---

**Solution:**

- (a) 97.2 J
- (b) 29.2 W
- (c) 9.49 W
- (d) The total rate of heat loss would be  $29.2\text{ W} + 9.49\text{ W} = 38.7\text{ W}$ . While sleeping, our body consumes 83 W of power, while sitting it

consumes 120 to 210 W. Therefore, the total rate of heat loss from breathing will not be a major form of heat loss for this person.

**Exercise:**

**Problem:**

A glass coffee pot has a circular bottom with a 9.00-cm diameter in contact with a heating element that keeps the coffee warm with a continuous heat transfer rate of 50.0 W

(a) What is the temperature of the bottom of the pot, if it is 3.00 mm thick and the inside temperature is 60.0°C?

(b) If the temperature of the coffee remains constant and all of the heat transfer is removed by evaporation, how many grams per minute evaporate? Take the heat of vaporization to be 2340 kJ/kg.

## Radiation

- Discuss heat transfer by radiation.
- Explain the power of different materials.

You can feel the heat transfer from a fire and from the Sun. Similarly, you can sometimes tell that the oven is hot without touching its door or looking inside—it may just warm you as you walk by. The space between the Earth and the Sun is largely empty, without any possibility of heat transfer by convection or conduction. In these examples, heat is transferred by radiation. That is, the hot body emits electromagnetic waves that are absorbed by our skin: no medium is required for electromagnetic waves to propagate. Different names are used for electromagnetic waves of different wavelengths: radio waves, microwaves, infrared **radiation**, visible light, ultraviolet radiation, X-rays, and gamma rays.

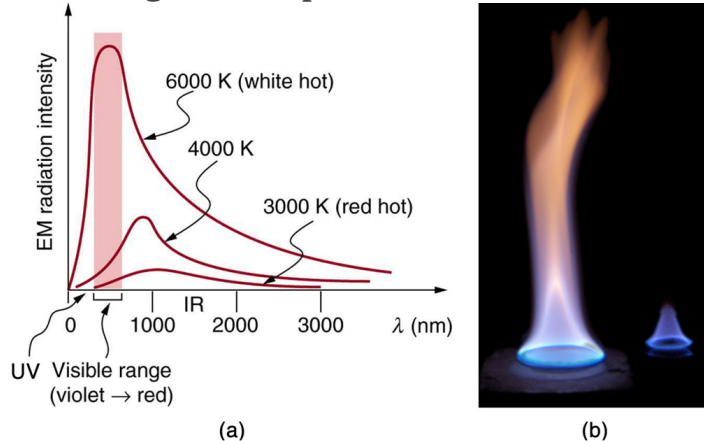


Most of the heat transfer from this fire to the observers is through infrared radiation. The visible light, although dramatic, transfers relatively little thermal energy. Convection transfers energy away from the observers as hot air rises, while conduction is negligibly slow here. Skin is very sensitive to infrared radiation, so that you can sense the presence of a fire

without looking at it directly. (credit:  
Daniel X. O'Neil)

The energy of electromagnetic radiation depends on the wavelength (color) and varies over a wide range: a smaller wavelength (or higher frequency) corresponds to a higher energy. Because more heat is radiated at higher temperatures, a temperature change is accompanied by a color change. Take, for example, an electrical element on a stove, which glows from red to orange, while the higher-temperature steel in a blast furnace glows from yellow to white. The radiation you feel is mostly infrared, which corresponds to a lower temperature than that of the electrical element and the steel. The radiated energy depends on its intensity, which is represented in the figure below by the height of the distribution.

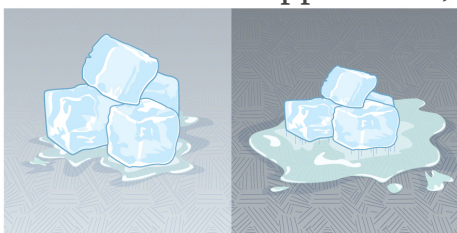
[Electromagnetic Waves](#) explains more about the electromagnetic spectrum and [Introduction to Quantum Physics](#) discusses how the decrease in wavelength corresponds to an increase in energy.



(a) A graph of the spectra of electromagnetic waves emitted from an ideal radiator at three different temperatures. The intensity or rate of radiation emission increases dramatically with temperature, and the spectrum shifts toward the visible and ultraviolet parts of the spectrum. The

shaded portion denotes the visible part of the spectrum. It is apparent that the shift toward the ultraviolet with temperature makes the visible appearance shift from red to white to blue as temperature increases. (b)  
Note the variations in color corresponding to variations in flame temperature. (credit: Tuohirulla)

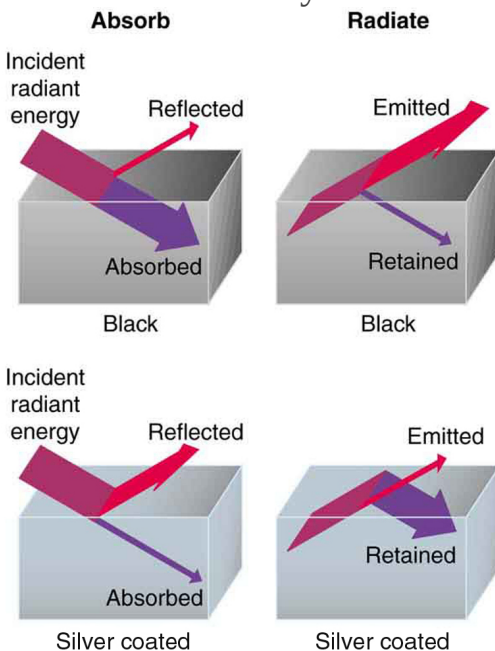
All objects absorb and emit electromagnetic radiation. The rate of heat transfer by radiation is largely determined by the color of the object. Black is the most effective, and white is the least effective. People living in hot climates generally avoid wearing black clothing, for instance (see [\[link\]](#)). Similarly, black asphalt in a parking lot will be hotter than adjacent gray sidewalk on a summer day, because black absorbs better than gray. The reverse is also true—black radiates better than gray. Thus, on a clear summer night, the asphalt will be colder than the gray sidewalk, because black radiates the energy more rapidly than gray. An *ideal radiator* is the same color as an *ideal absorber*, and captures all the radiation that falls on it. In contrast, white is a poor absorber and is also a poor radiator. A white object reflects all radiation, like a mirror. (A perfect, polished white surface is mirror-like in appearance, and a crushed mirror looks white.)



This illustration shows that the darker pavement is hotter than the lighter pavement (much more of the ice on the right has

melted), although both have been in the sunlight for the same time. The thermal conductivities of the pavements are the same.

Gray objects have a uniform ability to absorb all parts of the electromagnetic spectrum. Colored objects behave in similar but more complex ways, which gives them a particular color in the visible range and may make them special in other ranges of the nonvisible spectrum. Take, for example, the strong absorption of infrared radiation by the skin, which allows us to be very sensitive to it.



A black object is a good absorber and a good radiator, while a white (or silver) object is a poor absorber and a poor radiator. It is as if

radiation from the inside is reflected back into the silver object, whereas radiation from the inside of the black object is “absorbed” when it hits the surface and finds itself on the outside and is strongly emitted.

The rate of heat transfer by emitted radiation is determined by the **Stefan-Boltzmann law of radiation**:

**Equation:**

$$\frac{Q}{t} = \sigma e A T^4,$$

where  $\sigma = 5.67 \times 10^{-8} \text{ J/s} \cdot \text{m}^2 \cdot \text{K}^4$  is the Stefan-Boltzmann constant,  $A$  is the surface area of the object, and  $T$  is its absolute temperature in kelvin. The symbol  $e$  stands for the **emissivity** of the object, which is a measure of how well it radiates. An ideal jet-black (or black body) radiator has  $e = 1$ , whereas a perfect reflector has  $e = 0$ . Real objects fall between these two values. Take, for example, tungsten light bulb filaments which have an  $e$  of about 0.5, and carbon black (a material used in printer toner), which has the (greatest known) emissivity of about 0.99.

The radiation rate is directly proportional to the *fourth power* of the absolute temperature—a remarkably strong temperature dependence. Furthermore, the radiated heat is proportional to the surface area of the object. If you knock apart the coals of a fire, there is a noticeable increase in radiation due to an increase in radiating surface area.



A thermograph of part of a building shows temperature variations, indicating where heat transfer to the outside is most severe. Windows are a major region of heat transfer to the outside of homes. (credit: U.S. Army)

Skin is a remarkably good absorber and emitter of infrared radiation, having an emissivity of 0.97 in the infrared spectrum. Thus, we are all nearly (jet) black in the infrared, in spite of the obvious variations in skin color. This high infrared emissivity is why we can so easily feel radiation on our skin. It is also the basis for the use of night scopes used by law enforcement and the military to detect human beings. Even small temperature variations can be detected because of the  $T^4$  dependence. Images, called *thermographs*, can be used medically to detect regions of abnormally high temperature in the body, perhaps indicative of disease. Similar techniques can be used to detect heat leaks in homes [\[link\]](#), optimize performance of blast furnaces, improve comfort levels in work environments, and even remotely map the Earth's temperature profile.

All objects emit and absorb radiation. The *net* rate of heat transfer by radiation (absorption minus emission) is related to both the temperature of the object and the temperature of its surroundings. Assuming that an object

with a temperature  $T_1$  is surrounded by an environment with uniform temperature  $T_2$ , the **net rate of heat transfer by radiation** is

**Equation:**

$$\frac{Q_{\text{net}}}{t} = \sigma e A (T_2^4 - T_1^4),$$

where  $e$  is the emissivity of the object alone. In other words, it does not matter whether the surroundings are white, gray, or black; the balance of radiation into and out of the object depends on how well it emits and absorbs radiation. When  $T_2 > T_1$ , the quantity  $Q_{\text{net}}/t$  is positive; that is, the net heat transfer is from hot to cold.

**Note:**

**Take-Home Experiment: Temperature in the Sun**

Place a thermometer out in the sunshine and shield it from direct sunlight using an aluminum foil. What is the reading? Now remove the shield, and note what the thermometer reads. Take a handkerchief soaked in nail polish remover, wrap it around the thermometer and place it in the sunshine. What does the thermometer read?

**Example:**

**Calculate the Net Heat Transfer of a Person: Heat Transfer by Radiation**

What is the rate of heat transfer by radiation, with an unclothed person standing in a dark room whose ambient temperature is  $22.0^\circ\text{C}$ . The person has a normal skin temperature of  $33.0^\circ\text{C}$  and a surface area of  $1.50 \text{ m}^2$ . The emissivity of skin is 0.97 in the infrared, where the radiation takes place.

**Strategy**

We can solve this by using the equation for the rate of radiative heat transfer.

**Solution**

Insert the temperatures values  $T_2 = 295 \text{ K}$  and  $T_1 = 306 \text{ K}$ , so that

**Equation:**

$$\frac{Q}{t} = \sigma e A (T_2^4 - T_1^4)$$

**Equation:**

$$= (5.67 \times 10^{-8} \text{ J/s} \cdot \text{m}^2 \cdot \text{K}^4)(0.97)(1.50 \text{ m}^2) \left[ (295 \text{ K})^4 - (306 \text{ K})^4 \right]$$

**Equation:**

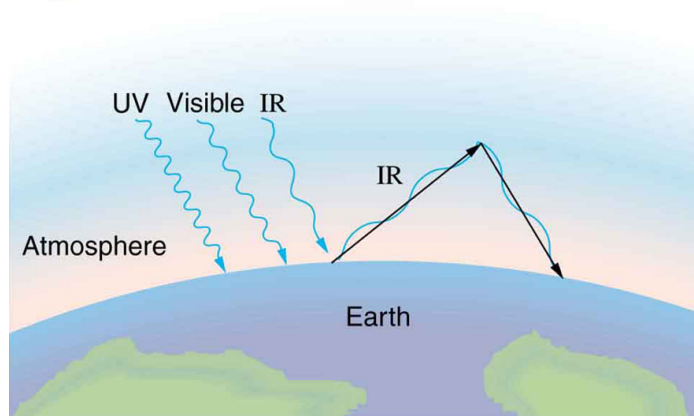
$$= -99 \text{ J/s} = -99 \text{ W}.$$

### Discussion

This value is a significant rate of heat transfer to the environment (note the minus sign), considering that a person at rest may produce energy at the rate of 125 W and that conduction and convection will also be transferring energy to the environment. Indeed, we would probably expect this person to feel cold. Clothing significantly reduces heat transfer to the environment by many methods, because clothing slows down both conduction and convection, and has a lower emissivity (especially if it is white) than skin.

The Earth receives almost all its energy from radiation of the Sun and reflects some of it back into outer space. Because the Sun is hotter than the Earth, the net energy flux is from the Sun to the Earth. However, the rate of energy transfer is less than the equation for the radiative heat transfer would predict because the Sun does not fill the sky. The average emissivity ( $e$ ) of the Earth is about 0.65, but the calculation of this value is complicated by the fact that the highly reflective cloud coverage varies greatly from day to day. There is a negative feedback (one in which a change produces an effect that opposes that change) between clouds and heat transfer; greater temperatures evaporate more water to form more clouds, which reflect more radiation back into space, reducing the temperature. The often mentioned **greenhouse effect** is directly related to the variation of the Earth's emissivity with radiation type (see the figure given below). The greenhouse

effect is a natural phenomenon responsible for providing temperatures suitable for life on Earth. The Earth's relatively constant temperature is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by carbon dioxide ( $\text{CO}_2$ ) and water ( $\text{H}_2\text{O}$ ) in the atmosphere and then re-radiated back to the Earth or into outer space. Re-radiation back to the Earth maintains its surface temperature about  $40^\circ\text{C}$  higher than it would be if there was no atmosphere, similar to the way glass increases temperatures in a greenhouse.

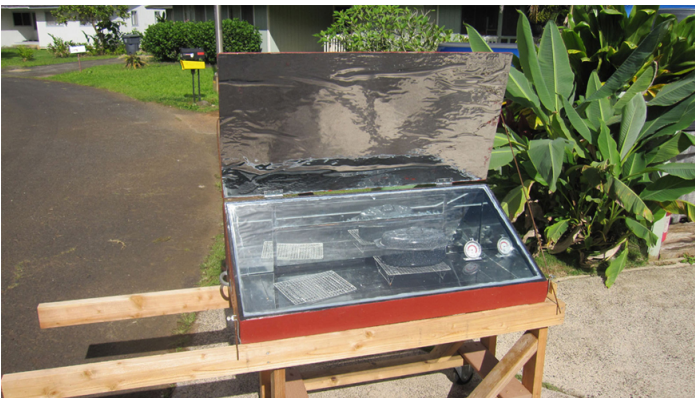


The greenhouse effect is a name given to the trapping of energy in the Earth's atmosphere by a process similar to that used in greenhouses. The atmosphere, like window glass, is transparent to incoming visible radiation and most of the Sun's infrared. These wavelengths are absorbed by the Earth and re-emitted as infrared. Since Earth's temperature is much lower than that of the Sun, the infrared radiated by the Earth has a much longer wavelength. The atmosphere, like glass, traps these longer infrared rays, keeping the Earth warmer than it would otherwise

be. The amount of trapping depends on concentrations of trace gases like carbon dioxide, and a change in the concentration of these gases is believed to affect the Earth's surface temperature.

The greenhouse effect is also central to the discussion of global warming due to emission of carbon dioxide and methane (and other so-called greenhouse gases) into the Earth's atmosphere from industrial production and farming. Changes in global climate could lead to more intense storms, precipitation changes (affecting agriculture), reduction in rain forest biodiversity, and rising sea levels.

Heating and cooling are often significant contributors to energy use in individual homes. Current research efforts into developing environmentally friendly homes quite often focus on reducing conventional heating and cooling through better building materials, strategically positioning windows to optimize radiation gain from the Sun, and opening spaces to allow convection. It is possible to build a zero-energy house that allows for comfortable living in most parts of the United States with hot and humid summers and cold winters.



This simple but effective solar cooker uses the greenhouse effect and reflective material to trap and retain solar energy. Made of inexpensive,

durable materials, it saves money and labor, and is of particular economic value in energy-poor developing countries. (credit: E.B. Kauai)

Conversely, dark space is very cold, about 3K ( $-454^{\circ}\text{F}$ ), so that the Earth radiates energy into the dark sky. Owing to the fact that clouds have lower emissivity than either oceans or land masses, they reflect some of the radiation back to the surface, greatly reducing heat transfer into dark space, just as they greatly reduce heat transfer into the atmosphere during the day. The rate of heat transfer from soil and grasses can be so rapid that frost may occur on clear summer evenings, even in warm latitudes.

**Exercise:**

**Check Your Understanding**

**Problem:**

What is the change in the rate of the radiated heat by a body at the temperature  $T_1 = 20^{\circ}\text{C}$  compared to when the body is at the temperature  $T_2 = 40^{\circ}\text{C}$ ?

---

**Solution:**

The radiated heat is proportional to the fourth power of the *absolute temperature*. Because  $T_1 = 293\text{ K}$  and  $T_2 = 313\text{ K}$ , the rate of heat transfer increases by about 30 percent of the original rate.

**Note:**

**Career Connection: Energy Conservation Consultation**

The cost of energy is generally believed to remain very high for the foreseeable future. Thus, passive control of heat loss in both commercial and domestic housing will become increasingly important. Energy consultants measure and analyze the flow of energy into and out of houses

and ensure that a healthy exchange of air is maintained inside the house. The job prospects for an energy consultant are strong.

**Note:**

**Problem-Solving Strategies for the Methods of Heat Transfer**

1. *Examine the situation to determine what type of heat transfer is involved.*
2. *Identify the type(s) of heat transfer—conduction, convection, or radiation.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is very useful.*
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).*
5. *Solve the appropriate equation for the quantity to be determined (the unknown).*
6. For conduction, equation  $\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}$  is appropriate. [\[link\]](#) lists thermal conductivities. For convection, determine the amount of matter moved and use equation  $Q = mc\Delta T$ , to calculate the heat transfer involved in the temperature change of the fluid. If a phase change accompanies convection, equation  $Q = mL_f$  or  $Q = mL_v$  is appropriate to find the heat transfer involved in the phase change. [\[link\]](#) lists information relevant to phase change. For radiation, equation  $\frac{Q_{\text{net}}}{t} = \sigma eA(T_2^4 - T_1^4)$  gives the net heat transfer rate.
7. *Insert the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.*
8. *Check the answer to see if it is reasonable. Does it make sense?*

## Summary

- Radiation is the rate of heat transfer through the emission or absorption of electromagnetic waves.

- The rate of heat transfer depends on the surface area and the fourth power of the absolute temperature:

**Equation:**

$$\frac{Q}{t} = \sigma e A T^4,$$

where  $\sigma = 5.67 \times 10^{-8} \text{ J/s} \cdot \text{m}^2 \cdot \text{K}^4$  is the Stefan-Boltzmann constant and  $e$  is the emissivity of the body. For a black body,  $e = 1$  whereas a shiny white or perfect reflector has  $e = 0$ , with real objects having values of  $e$  between 1 and 0. The net rate of heat transfer by radiation is

**Equation:**

$$\frac{Q_{\text{net}}}{t} = \sigma e A (T_2^4 - T_1^4)$$

where  $T_1$  is the temperature of an object surrounded by an environment with uniform temperature  $T_2$  and  $e$  is the emissivity of the *object*.

## Conceptual Questions

**Exercise:**

**Problem:**

When watching a daytime circus in a large, dark-colored tent, you sense significant heat transfer from the tent. Explain why this occurs.

**Exercise:**

**Problem:**

Satellites designed to observe the radiation from cold (3 K) dark space have sensors that are shaded from the Sun, Earth, and Moon and that are cooled to very low temperatures. Why must the sensors be at low temperature?

**Exercise:**

**Problem:** Why are cloudy nights generally warmer than clear ones?

**Exercise:**

**Problem:**

Why are thermometers that are used in weather stations shielded from the sunshine? What does a thermometer measure if it is shielded from the sunshine and also if it is not?

**Exercise:**

**Problem:**

On average, would Earth be warmer or cooler without the atmosphere? Explain your answer.

## **Problems & Exercises**

**Exercise:**

**Problem:**

At what net rate does heat radiate from a  $275\text{-m}^2$  black roof on a night when the roof's temperature is  $30.0^\circ\text{C}$  and the surrounding temperature is  $15.0^\circ\text{C}$ ? The emissivity of the roof is 0.900.

---

**Solution:**

$-21.7\text{ kW}$

Note that the negative answer implies heat loss to the surroundings.

**Exercise:**

**Problem:**

(a) Cherry-red embers in a fireplace are at  $850^{\circ}\text{C}$  and have an exposed area of  $0.200\text{ m}^2$  and an emissivity of 0.980. The surrounding room has a temperature of  $18.0^{\circ}\text{C}$ . If 50% of the radiant energy enters the room, what is the net rate of radiant heat transfer in kilowatts? (b) Does your answer support the contention that most of the heat transfer into a room by a fireplace comes from infrared radiation?

**Exercise:****Problem:**

Radiation makes it impossible to stand close to a hot lava flow. Calculate the rate of heat transfer by radiation from  $1.00\text{ m}^2$  of  $1200^{\circ}\text{C}$  fresh lava into  $30.0^{\circ}\text{C}$  surroundings, assuming lava's emissivity is 1.00.

---

**Solution:**

−266 kW

**Exercise:****Problem:**

(a) Calculate the rate of heat transfer by radiation from a car radiator at  $110^{\circ}\text{C}$  into a  $50.0^{\circ}\text{C}$  environment, if the radiator has an emissivity of 0.750 and a  $1.20\text{-m}^2$  surface area. (b) Is this a significant fraction of the heat transfer by an automobile engine? To answer this, assume a horsepower of 200 hp (1.5 kW) and the efficiency of automobile engines as 25%.

**Exercise:**

**Problem:**

Find the net rate of heat transfer by radiation from a skier standing in the shade, given the following. She is completely clothed in white (head to foot, including a ski mask), the clothes have an emissivity of 0.200 and a surface temperature of  $10.0^{\circ}\text{C}$ , the surroundings are at  $-15.0^{\circ}\text{C}$ , and her surface area is  $1.60\text{ m}^2$ .

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**Solution:**

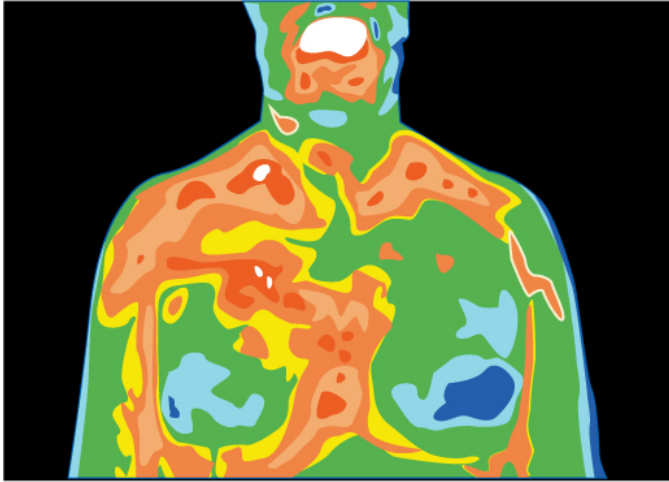
$-36.0\text{ W}$

**Exercise:****Problem:**

Suppose you walk into a sauna that has an ambient temperature of  $50.0^{\circ}\text{C}$ . (a) Calculate the rate of heat transfer to you by radiation given your skin temperature is  $37.0^{\circ}\text{C}$ , the emissivity of skin is 0.98, and the surface area of your body is  $1.50\text{ m}^2$ . (b) If all other forms of heat transfer are balanced (the net heat transfer is zero), at what rate will your body temperature increase if your mass is  $75.0\text{ kg}$ ?

**Exercise:****Problem:**

Thermography is a technique for measuring radiant heat and detecting variations in surface temperatures that may be medically, environmentally, or militarily meaningful. (a) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of  $34.0^{\circ}\text{C}$  compared with that at  $33.0^{\circ}\text{C}$ , such as on a person's skin? (b) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of  $34.0^{\circ}\text{C}$  compared with that at  $20.0^{\circ}\text{C}$ , such as for warm and cool automobile hoods?



Artist's rendition of a thermograph of a patient's upper body, showing the distribution of heat represented by different colors.

---

**Solution:**

(a) 1.31%

(b) 20.5%

**Exercise:**

**Problem:**

The Sun radiates like a perfect black body with an emissivity of exactly 1. (a) Calculate the surface temperature of the Sun, given that it is a sphere with a  $7.00 \times 10^8$ -m radius that radiates  $3.80 \times 10^{26}$  W into 3-K space. (b) How much power does the Sun radiate per square meter of its surface? (c) How much power in watts per square meter is that value at the distance of Earth,  $1.50 \times 10^{11}$  m away? (This number is called the solar constant.)

**Exercise:**

**Problem:**

A large body of lava from a volcano has stopped flowing and is slowly cooling. The interior of the lava is at  $1200^{\circ}\text{C}$ , its surface is at  $450^{\circ}\text{C}$ , and the surroundings are at  $27.0^{\circ}\text{C}$ . (a) Calculate the rate at which energy is transferred by radiation from  $1.00\text{ m}^2$  of surface lava into the surroundings, assuming the emissivity is 1.00. (b) Suppose heat conduction to the surface occurs at the same rate. What is the thickness of the lava between the  $450^{\circ}\text{C}$  surface and the  $1200^{\circ}\text{C}$  interior, assuming that the lava's conductivity is the same as that of brick?

---

**Solution:**

(a)  $-15.0\text{ kW}$

(b)  $4.2\text{ cm}$

**Exercise:****Problem:**

Calculate the temperature the entire sky would have to be in order to transfer energy by radiation at  $1000\text{ W/m}^2$ —about the rate at which the Sun radiates when it is directly overhead on a clear day. This value is the effective temperature of the sky, a kind of average that takes account of the fact that the Sun occupies only a small part of the sky but is much hotter than the rest. Assume that the body receiving the energy has a temperature of  $27.0^{\circ}\text{C}$ .

**Exercise:**

**Problem:**

(a) A shirtless rider under a circus tent feels the heat radiating from the sunlit portion of the tent. Calculate the temperature of the tent canvas based on the following information: The shirtless rider's skin temperature is  $34.0^{\circ}\text{C}$  and has an emissivity of 0.970. The exposed area of skin is  $0.400\text{ m}^2$ . He receives radiation at the rate of  $20.0\text{ W}$ —half what you would calculate if the entire region behind him was hot. The rest of the surroundings are at  $34.0^{\circ}\text{C}$ . (b) Discuss how this situation would change if the sunlit side of the tent was nearly pure white and if the rider was covered by a white tunic.

---

**Solution:**

(a)  $48.5^{\circ}\text{C}$

(b) A pure white object reflects more of the radiant energy that hits it, so a white tent would prevent more of the sunlight from heating up the inside of the tent, and the white tunic would prevent that heat which entered the tent from heating the rider. Therefore, with a white tent, the temperature would be lower than  $48.5^{\circ}\text{C}$ , and the rate of radiant heat transferred to the rider would be less than  $20.0\text{ W}$ .

**Exercise:****Problem: Integrated Concepts**

One  $30.0^{\circ}\text{C}$  day the relative humidity is 75.0%, and that evening the temperature drops to  $20.0^{\circ}\text{C}$ , well below the dew point. (a) How many grams of water condense from each cubic meter of air? (b) How much heat transfer occurs by this condensation? (c) What temperature increase could this cause in dry air?

**Exercise:****Problem: Integrated Concepts**

Large meteors sometimes strike the Earth, converting most of their kinetic energy into thermal energy. (a) What is the kinetic energy of a  $10^9$  kg meteor moving at 25.0 km/s? (b) If this meteor lands in a deep ocean and 80% of its kinetic energy goes into heating water, how many kilograms of water could it raise by  $5.0^\circ\text{C}$ ? (c) Discuss how the energy of the meteor is more likely to be deposited in the ocean and the likely effects of that energy.

---

**Solution:**

(a)  $3 \times 10^{17} \text{ J}$

(b)  $1 \times 10^{13} \text{ kg}$

(c) When a large meteor hits the ocean, it causes great tidal waves, dissipating large amount of its energy in the form of kinetic energy of the water.

**Exercise:**

**Problem: Integrated Concepts**

Frozen waste from airplane toilets has sometimes been accidentally ejected at high altitude. Ordinarily it breaks up and disperses over a large area, but sometimes it holds together and strikes the ground. Calculate the mass of  $0^\circ\text{C}$  ice that can be melted by the conversion of kinetic and gravitational potential energy when a 20.0 kg piece of frozen waste is released at 12.0 km altitude while moving at 250 m/s and strikes the ground at 100 m/s (since less than 20.0 kg melts, a significant mess results).

**Exercise:**

**Problem: Integrated Concepts**

(a) A large electrical power facility produces 1600 MW of “waste heat,” which is dissipated to the environment in cooling towers by warming air flowing through the towers by  $5.00^\circ\text{C}$ . What is the

necessary flow rate of air in  $\text{m}^3/\text{s}$ ? (b) Is your result consistent with the large cooling towers used by many large electrical power plants?

---

**Solution:**

(a)  $3.44 \times 10^5 \text{ m}^3/\text{s}$

(b) This is equivalent to 12 million cubic feet of air per second. That is tremendous. This is too large to be dissipated by heating the air by only  $5^\circ\text{C}$ . Many of these cooling towers use the circulation of cooler air over warmer water to increase the rate of evaporation. This would allow much smaller amounts of air necessary to remove such a large amount of heat because evaporation removes larger quantities of heat than was considered in part (a).

**Exercise:**

**Problem: Integrated Concepts**

(a) Suppose you start a workout on a Stairmaster, producing power at the same rate as climbing 116 stairs per minute. Assuming your mass is  $76.0 \text{ kg}$  and your efficiency is  $20.0\%$ , how long will it take for your body temperature to rise  $1.00^\circ\text{C}$  if all other forms of heat transfer in and out of your body are balanced? (b) Is this consistent with your experience in getting warm while exercising?

**Exercise:**

**Problem: Integrated Concepts**

A  $76.0\text{-kg}$  person suffering from hypothermia comes indoors and shivers vigorously. How long does it take the heat transfer to increase the person's body temperature by  $2.00^\circ\text{C}$  if all other forms of heat transfer are balanced?

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**Solution:**

20.9 min

**Exercise:****Problem: Integrated Concepts**

In certain large geographic regions, the underlying rock is hot. Wells can be drilled and water circulated through the rock for heat transfer for the generation of electricity. (a) Calculate the heat transfer that can be extracted by cooling  $1.00 \text{ km}^3$  of granite by  $100^\circ\text{C}$ . (b) How long will this take if heat is transferred at a rate of 300 MW, assuming no heat transfers back into the  $1.00 \text{ km}$  of rock by its surroundings?

**Exercise:****Problem: Integrated Concepts**

Heat transfers from your lungs and breathing passages by evaporating water. (a) Calculate the maximum number of grams of water that can be evaporated when you inhale  $1.50 \text{ L}$  of  $37^\circ\text{C}$  air with an original relative humidity of 40.0%. (Assume that body temperature is also  $37^\circ\text{C}$ .) (b) How many joules of energy are required to evaporate this amount? (c) What is the rate of heat transfer in watts from this method, if you breathe at a normal resting rate of 10.0 breaths per minute?

---

**Solution:**

(a)  $3.96 \times 10^{-2} \text{ g}$

(b)  $96.2 \text{ J}$

(c)  $16.0 \text{ W}$

**Exercise:****Problem: Integrated Concepts**

(a) What is the temperature increase of water falling  $55.0 \text{ m}$  over Niagara Falls? (b) What fraction must evaporate to keep the temperature constant?

**Exercise:****Problem: Integrated Concepts**

Hot air rises because it has expanded. It then displaces a greater volume of cold air, which increases the buoyant force on it. (a) Calculate the ratio of the buoyant force to the weight of  $50.0^{\circ}\text{C}$  air surrounded by  $20.0^{\circ}\text{C}$  air. (b) What energy is needed to cause  $1.00\text{ m}^3$  of air to go from  $20.0^{\circ}\text{C}$  to  $50.0^{\circ}\text{C}$ ? (c) What gravitational potential energy is gained by this volume of air if it rises  $1.00\text{ m}$ ? Will this cause a significant cooling of the air?

---

**Solution:**

(a) 1.102

(b)  $2.79 \times 10^4\text{ J}$

(c) 12.6 J. This will not cause a significant cooling of the air because it is much less than the energy found in part (b), which is the energy required to warm the air from  $20.0^{\circ}\text{C}$  to  $50.0^{\circ}\text{C}$ .

**Exercise:****Problem: Unreasonable Results**

(a) What is the temperature increase of an  $80.0\text{ kg}$  person who consumes  $2500\text{ kcal}$  of food in one day with  $95.0\%$  of the energy transferred as heat to the body? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

---

**Solution:**

(a)  $36^{\circ}\text{C}$

(b) Any temperature increase greater than about  $3^{\circ}\text{C}$  would be unreasonably large. In this case the final temperature of the person would rise to  $73^{\circ}\text{C}$  ( $163^{\circ}\text{F}$ ).

(c) The assumption of 95% heat retention is unreasonable.

**Exercise:**

**Problem: Unreasonable Results**

A slightly deranged Arctic inventor surrounded by ice thinks it would be much less mechanically complex to cool a car engine by melting ice on it than by having a water-cooled system with a radiator, water pump, antifreeze, and so on. (a) If 80.0% of the energy in 1.00 gal of gasoline is converted into “waste heat” in a car engine, how many kilograms of 0°C ice could it melt? (b) Is this a reasonable amount of ice to carry around to cool the engine for 1.00 gal of gasoline consumption? (c) What premises or assumptions are unreasonable?

**Exercise:**

**Problem: Unreasonable Results**

(a) Calculate the rate of heat transfer by conduction through a window with an area of  $1.00 \text{ m}^2$  that is 0.750 cm thick, if its inner surface is at 22.0°C and its outer surface is at 35.0°C. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

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**Solution:**

(a) 1.46 kW

(b) Very high power loss through a window. An electric heater of this power can keep an entire room warm.

(c) The surface temperatures of the window do not differ by as great an amount as assumed. The inner surface will be warmer, and the outer surface will be cooler.

**Exercise:**

**Problem: Unreasonable Results**

A meteorite 1.20 cm in diameter is so hot immediately after penetrating the atmosphere that it radiates 20.0 kW of power. (a) What is its temperature, if the surroundings are at 20.0°C and it has an emissivity of 0.800? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

**Exercise:**

**Problem: Construct Your Own Problem**

Consider a new model of commercial airplane having its brakes tested as a part of the initial flight permission procedure. The airplane is brought to takeoff speed and then stopped with the brakes alone. Construct a problem in which you calculate the temperature increase of the brakes during this process. You may assume most of the kinetic energy of the airplane is converted to thermal energy in the brakes and surrounding materials, and that little escapes. Note that the brakes are expected to become so hot in this procedure that they ignite and, in order to pass the test, the airplane must be able to withstand the fire for some time without a general conflagration.

**Exercise:**

**Problem: Construct Your Own Problem**

Consider a person outdoors on a cold night. Construct a problem in which you calculate the rate of heat transfer from the person by all three heat transfer methods. Make the initial circumstances such that at rest the person will have a net heat transfer and then decide how much physical activity of a chosen type is necessary to balance the rate of heat transfer. Among the things to consider are the size of the person, type of clothing, initial metabolic rate, sky conditions, amount of water evaporated, and volume of air breathed. Of course, there are many other factors to consider and your instructor may wish to guide you in the assumptions made as well as the detail of analysis and method of presenting your results.

## Glossary

emissivity

measure of how well an object radiates

greenhouse effect

warming of the Earth that is due to gases such as carbon dioxide and methane that absorb infrared radiation from the Earth's surface and reradiate it in all directions, thus sending a fraction of it back toward the surface of the Earth

net rate of heat transfer by radiation

is  $\frac{Q_{\text{net}}}{t} = \sigma e A (T_2^4 - T_1^4)$

radiation

energy transferred by electromagnetic waves directly as a result of a temperature difference

Stefan-Boltzmann law of radiation

$\frac{Q}{t} = \sigma e A T^4$  where  $\sigma$  is the Stefan-Boltzmann constant,  $A$  is the surface area of the object,  $T$  is the absolute temperature, and  $e$  is the emissivity

## Climate and the Effects of Global Climate Change

By the end of this section, you will be able to:

- Define global climate change
- Summarize the effects of the Industrial Revolution on global atmospheric carbon dioxide concentration
- Describe three natural factors affecting long-term global climate
- List two or more greenhouse gases and describe their role in the greenhouse effect

All biomes are universally affected by global conditions, such as climate, that ultimately shape each biome's environment. Scientists who study climate have noted a series of marked changes that have gradually become increasingly evident during the last sixty years. **Global climate change** is the term used to describe altered global weather patterns, including a worldwide increase in temperature, due largely to rising levels of atmospheric carbon dioxide.

## Climate and Weather

A common misconception about global climate change is that a specific weather event occurring in a particular region (for example, a very cool week in June in central Indiana) is evidence of global climate change. However, a cold week in June is a weather-related event and not a climate-related one. These misconceptions often arise because of confusion over the terms climate and weather.

**Climate** refers to the long-term, predictable atmospheric conditions of a specific area. The climate of a biome is characterized by having consistent temperature and annual rainfall ranges. Climate does not address the amount of rain that fell on one particular day in a biome or the colder-than-average temperatures that occurred on one day. In contrast, **weather** refers to the conditions of the atmosphere during a short period of time. Weather forecasts are usually made for 48-hour cycles. Long-range weather forecasts are available but can be unreliable.

To better understand the difference between climate and weather, imagine that you are planning an outdoor event in northern Wisconsin. You would be thinking about *climate* when you plan the event in the summer rather than the winter because you have long-term knowledge that any given Saturday in the months of May to August would be a better choice for an outdoor event in Wisconsin than any given Saturday in January. However, you cannot determine the specific day that the event should be held on because it is difficult to accurately predict the weather on a specific day. Climate can be considered “average” weather.

## **Global Climate Change**

Climate change can be understood by approaching three areas of study:

- current and past global climate change
- causes of past and present-day global climate change
- ancient and current results of climate change

It is helpful to keep these three different aspects of climate change clearly separated when consuming media reports about global climate change. It is common for reports and discussions about global climate change to confuse the data showing that Earth’s climate is changing with the factors that drive this climate change.

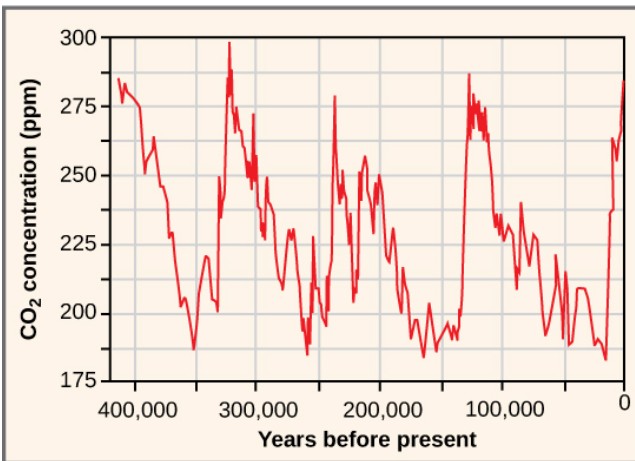
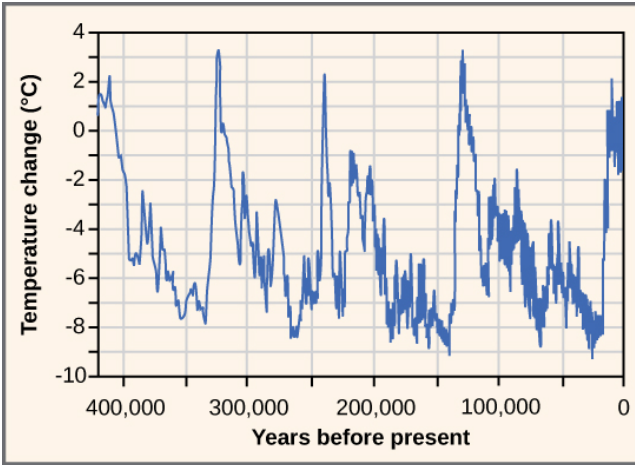
## **Evidence for Global Climate Change**

Since scientists cannot go back in time to directly measure climatic variables, such as average temperature and precipitation, they must instead indirectly measure temperature. To do this, scientists rely on historical evidence of Earth’s past climate.

Antarctic ice cores are a key example of such evidence. These ice cores are samples of polar ice obtained by means of drills that reach thousands of meters into ice sheets or high mountain glaciers. Viewing the ice cores is like traveling backwards through time; the deeper the sample, the earlier the time period. Trapped within the ice are bubbles of air and other biological

evidence that can reveal temperature and carbon dioxide data. Antarctic ice cores have been collected and analyzed to indirectly estimate the temperature of the Earth over the past 400,000 years ([\[link\]a](#)). The 0 °C on this graph refers to the long-term average. Temperatures that are greater than 0 °C exceed Earth's long-term average temperature. Conversely, temperatures that are less than 0 °C are less than Earth's average temperature. This figure shows that there have been periodic cycles of increasing and decreasing temperature.

Before the late 1800s, the Earth has been as much as 9 °C cooler and about 3 °C warmer. Note that the graph in [\[link\]b](#) shows that the atmospheric concentration of carbon dioxide has also risen and fallen in periodic cycles; note the relationship between carbon dioxide concentration and temperature. [\[link\]b](#) shows that carbon dioxide levels in the atmosphere have historically cycled between 180 and 300 parts per million (ppm) by volume.

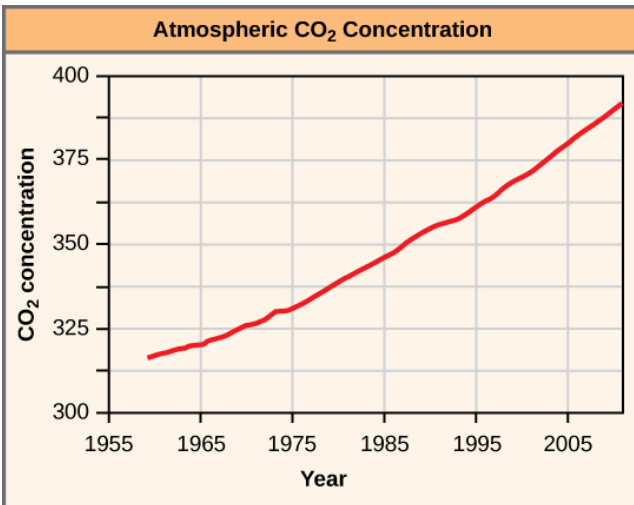


Ice at the Russian Vostok station in East Antarctica was laid down over the course 420,000 years and reached a depth of over 3,000 m. By measuring the amount of CO<sub>2</sub> trapped in the ice, scientists have determined past atmospheric CO<sub>2</sub> concentrations. Temperatures relative to modern day were determined from the amount of deuterium (an isotope of hydrogen) present.

[\[link\]](#) does not show the last 2,000 years with enough detail to compare the changes of Earth's temperature during the last 400,000 years with the temperature change that has occurred in the more recent past. Two significant temperature anomalies, or irregularities, have occurred in the last 2000 years. These are the Medieval Climate Anomaly (or the Medieval Warm Period) and the Little Ice Age. A third temperature anomaly aligns with the Industrial Era. The Medieval Climate Anomaly occurred between 900 and 1300 AD. During this time period, many climate scientists think that slightly warmer weather conditions prevailed in many parts of the world; the higher-than-average temperature changes varied between 0.10 °C and 0.20 °C above the norm. Although 0.10 °C does not seem large enough to produce any noticeable change, it did free seas of ice. Because of this warming, the Vikings were able to colonize Greenland.

The Little Ice Age was a cold period that occurred between 1550 AD and 1850 AD. During this time, a slight cooling of a little less than 1 °C was observed in North America, Europe, and possibly other areas of the Earth. This 1 °C change in global temperature is a seemingly small deviation in temperature (as was observed during the Medieval Climate Anomaly); however, it also resulted in noticeable changes. Historical accounts reveal a time of exceptionally harsh winters with much snow and frost.

The Industrial Revolution, which began around 1750, was characterized by changes in much of human society. Advances in agriculture increased the food supply, which improved the standard of living for people in Europe and the United States. New technologies were invented and provided jobs and cheaper goods. These new technologies were powered using fossil fuels, especially coal. The Industrial Revolution starting in the early nineteenth century ushered in the beginning of the Industrial Era. When a fossil fuel is burned, carbon dioxide is released. With the beginning of the Industrial Era, atmospheric carbon dioxide began to rise ([\[link\]](#)).



The atmospheric concentration of CO<sub>2</sub> has risen steadily since the beginning of industrialization.

## Current and Past Drivers of Global Climate Change

Since it is not possible to go back in time to directly observe and measure climate, scientists use indirect evidence to determine the drivers, or factors, that may be responsible for climate change. The indirect evidence includes data collected using ice cores, boreholes (a narrow shaft bored into the ground), tree rings, glacier lengths, pollen remains, and ocean sediments. The data shows a correlation between the timing of temperature changes and drivers of climate change: before the Industrial Era (pre-1780), there were three drivers of climate change that were not related to human activity or atmospheric gases. The first of these is the Milankovitch cycles. The **Milankovitch cycles** describe the effects of slight changes in the Earth's orbit on Earth's climate. The length of the Milankovitch cycles ranges between 19,000 and 100,000 years. In other words, one could expect to see some predictable changes in the Earth's climate associated with changes in the Earth's orbit at a minimum of every 19,000 years.

The variation in the sun's intensity is the second natural factor responsible for climate change. **Solar intensity** is the amount of solar power or energy the sun emits in a given amount of time. There is a direct relationship between solar intensity and temperature. As solar intensity increases (or decreases), the Earth's temperature correspondingly increases (or decreases). Changes in solar intensity have been proposed as one of several possible explanations for the Little Ice Age.

Finally, volcanic eruptions are a third natural driver of climate change. Volcanic eruptions can last a few days, but the solids and gases released during an eruption can influence the climate over a period of a few years, causing short-term climate changes. The gases and solids released by volcanic eruptions can include carbon dioxide, water vapor, sulfur dioxide, hydrogen sulfide, hydrogen, and carbon monoxide. Generally, volcanic eruptions cool the climate. This occurred in 1783 when volcanos in Iceland erupted and caused the release of large volumes of sulfuric oxide. This led to **haze-effect cooling**, a global phenomenon that occurs when dust, ash, or other suspended particles block out sunlight and trigger lower global temperatures as a result; haze-effect cooling usually extends for one or more years. In Europe and North America, haze-effect cooling produced some of the lowest average winter temperatures on record in 1783 and 1784.

Greenhouse gases are probably the most significant drivers of the climate. When heat energy from the sun strikes the Earth, gases known as **greenhouse gases** trap the heat in the atmosphere, as do the glass panes of a greenhouse keep heat from escaping. The greenhouse gases that affect Earth include carbon dioxide, methane, water vapor, nitrous oxide, and ozone. Approximately half of the radiation from the sun passes through these gases in the atmosphere and strikes the Earth. This radiation is converted into thermal radiation on the Earth's surface, and then a portion of that energy is re-radiated back into the atmosphere. Greenhouse gases, however, reflect much of the thermal energy back to the Earth's surface. The more greenhouse gases there are in the atmosphere, the more thermal energy is reflected back to the Earth's surface. Greenhouse gases absorb and emit radiation and are an important factor in the **greenhouse effect**: the warming

of Earth due to carbon dioxide and other greenhouse gases in the atmosphere.

Evidence supports the relationship between atmospheric concentrations of carbon dioxide and temperature: as carbon dioxide rises, global temperature rises. Since 1950, the concentration of atmospheric carbon dioxide has increased from about 280 ppm to 382 ppm in 2006. In 2011, the atmospheric carbon dioxide concentration was 392 ppm. However, the planet would not be inhabitable by current life forms if water vapor did not produce its drastic greenhouse warming effect.

Scientists look at patterns in data and try to explain differences or deviations from these patterns. The atmospheric carbon dioxide data reveal a historical pattern of carbon dioxide increasing and decreasing, cycling between a low of 180 ppm and a high of 300 ppm. Scientists have concluded that it took around 50,000 years for the atmospheric carbon dioxide level to increase from its low minimum concentration to its higher maximum concentration. However, starting recently, atmospheric carbon dioxide concentrations have increased beyond the historical maximum of 300 ppm. The current increases in atmospheric carbon dioxide have happened very quickly—in a matter of hundreds of years rather than thousands of years. What is the reason for this difference in the rate of change and the amount of increase in carbon dioxide? A key factor that must be recognized when comparing the historical data and the current data is the presence of modern human society; no other driver of climate change has yielded changes in atmospheric carbon dioxide levels at this rate or to this magnitude.

Human activity releases carbon dioxide and methane, two of the most important greenhouse gases, into the atmosphere in several ways. The primary mechanism that releases carbon dioxide is the burning of fossil fuels, such as gasoline, coal, and natural gas ([\[link\]](#)). Deforestation, cement manufacture, animal agriculture, the clearing of land, and the burning of forests are other human activities that release carbon dioxide. Methane ( $\text{CH}_4$ ) is produced when bacteria break down organic matter under anaerobic conditions. Anaerobic conditions can happen when organic matter is trapped underwater (such as in rice paddies) or in the intestines of

herbivores. Methane can also be released from natural gas fields and the decomposition that occurs in landfills. Another source of methane is the melting of clathrates. **Clathrates** are frozen chunks of ice and methane found at the bottom of the ocean. When water warms, these chunks of ice melt and methane is released. As the ocean's water temperature increases, the rate at which clathrates melt is increasing, releasing even more methane. This leads to increased levels of methane in the atmosphere, which further accelerates the rate of global warming. This is an example of the positive feedback loop that is leading to the rapid rate of increase of global temperatures.



The burning of fossil fuels in industry and by vehicles releases carbon dioxide and other greenhouse gases into the atmosphere. (credit: "Pöllö"/Wikimedia Commons)

## Documented Results of Climate Change: Past and Present

Scientists have geological evidence of the consequences of long-ago climate change. Modern-day phenomena such as retreating glaciers and melting polar ice cause a continual rise in sea level. Meanwhile, changes in climate can negatively affect organisms.

### Geological Climate Change

Global warming has been associated with at least one planet-wide extinction event during the geological past. The Permian extinction event occurred about 251 million years ago toward the end of the roughly 50-million-year-long geological time span known as the Permian period. This geologic time period was one of the three warmest periods in Earth's geologic history. Scientists estimate that approximately 70 percent of the terrestrial plant and animal species and 84 percent of marine species became extinct, vanishing forever near the end of the Permian period. Organisms that had adapted to wet and warm climatic conditions, such as annual rainfall of 300–400 cm (118–157 in) and 20 °C–30 °C (68 °F–86 °F) in the tropical wet forest, may not have been able to survive the Permian climate change.

#### Note:

Link to Learning



Watch this [NASA video](#) to discover the mixed effects of global warming on plant growth. While scientists found that warmer temperatures in the 1980s and 1990s caused an increase in plant productivity, this advantage has since been counteracted by more frequent droughts.

## Present Climate Change

A number of global events have occurred that may be attributed to climate change during our lifetimes. Glacier National Park in Montana is undergoing the retreat of many of its glaciers, a phenomenon known as glacier recession. In 1850, the area contained approximately 150 glaciers. By 2010, however, the park contained only about 24 glaciers greater than 25 acres in size. One of these glaciers is the Grinnell Glacier ([link](#)) at Mount Gould. Between 1966 and 2005, the size of Grinnell Glacier shrank by 40 percent. Similarly, the mass of the ice sheets in Greenland and the Antarctic is decreasing: Greenland lost 150–250 km<sup>3</sup> of ice per year between 2002 and 2006. In addition, the size and thickness of the Arctic sea ice is decreasing.



The effect of global warming can be seen in the continuing retreat of Grinnell Glacier. The mean annual temperature in the park has increased 1.33 °C since 1900. The loss of a glacier results in the loss of summer meltwaters, sharply reducing

seasonal water supplies and severely affecting local ecosystems. (credit: modification of work by USGS)

This loss of ice is leading to increases in the global sea level. On average, the sea is rising at a rate of 1.8 mm per year. However, between 1993 and 2010 the rate of sea level increase ranged between 2.9 and 3.4 mm per year. A variety of factors affect the volume of water in the ocean, including the temperature of the water (the density of water is related to its temperature) and the amount of water found in rivers, lakes, glaciers, polar ice caps, and sea ice. As glaciers and polar ice caps melt, there is a significant contribution of liquid water that was previously frozen.

In addition to some abiotic conditions changing in response to climate change, many organisms are also being affected by the changes in temperature. Temperature and precipitation play key roles in determining the geographic distribution and phenology of plants and animals. (Phenology is the study of the effects of climatic conditions on the timing of periodic lifecycle events, such as flowering in plants or migration in birds.) Researchers have shown that 385 plant species in Great Britain are flowering 4.5 days sooner than was recorded earlier during the previous 40 years. In addition, insect-pollinated species were more likely to flower earlier than wind-pollinated species. The impact of changes in flowering date would be mitigated if the insect pollinators emerged earlier. This mismatched timing of plants and pollinators could result in injurious ecosystem effects because, for continued survival, insect-pollinated plants must flower when their pollinators are present.

## **Section Summary**

The Earth has gone through periodic cycles of increases and decreases in temperature. During the past 2000 years, the Medieval Climate Anomaly was a warmer period, while the Little Ice Age was unusually cool. Both of these irregularities can be explained by natural causes of changes in climate, and, although the temperature changes were small, they had significant effects. Natural drivers of climate change include Milankovitch

cycles, changes in solar activity, and volcanic eruptions. None of these factors, however, leads to rapid increases in global temperature or sustained increases in carbon dioxide. The burning of fossil fuels is an important source of greenhouse gases, which plays a major role in the greenhouse effect. Long ago, global warming resulted in the Permian extinction: a large-scale extinction event that is documented in the fossil record. Currently, modern-day climate change is associated with the increased melting of glaciers and polar ice sheets, resulting in a gradual increase in sea level. Plants and animals can also be affected by global climate change when the timing of seasonal events, such as flowering or pollination, is affected by global warming.

## Review Questions

### Exercise:

**Problem:** Which of the following is an example of a weather event?

- a. The hurricane season lasts from June 1 through November 30.
- b. The amount of atmospheric CO<sub>2</sub> has steadily increased during the last century.
- c. A windstorm blew down trees in the Boundary Waters Canoe Area in Minnesota on July 4, 1999.
- d. Deserts are generally dry ecosystems having very little rainfall.

---

### Solution:

C

### Exercise:

### Problem:

Which of the following natural forces is responsible for the release of carbon dioxide and other atmospheric gases?

- a. the Milankovitch cycles

- b. volcanoes
- c. solar intensity
- d. burning of fossil fuels

---

**Solution:**

B

**Free Response**

**Exercise:**

**Problem:**

Compare and contrast how natural- and human-induced processes have influenced global climate change.

---

**Solution:**

Natural processes such as the Milankovitch cycles, variation in solar intensity, and volcanic eruptions can cause periodic, intermittent changes in global climate. Human activity, in the form of emissions from the burning of fossil fuels, has caused a progressive rise in the levels of atmospheric carbon dioxide.

**Exercise:**

**Problem:**

Predict possible consequences if carbon emissions from fossil fuels continue to rise.

---

**Solution:**

If carbon emissions continue to rise, the global temperature will continue to rise; thus, ocean waters will cause the rising of sea levels at the coastlines. Continued melting of glaciers and reduced spring and summer meltwaters may cause summertime water shortages. Changes

in seasonal temperatures may alter lifecycles and interrupt breeding patterns in many species of plants and animals.

## **Glossary**

clathrates

frozen chunks of ice and methane found at the bottom of the ocean

climate

long-term, predictable atmospheric conditions present in a specific area

global climate change

altered global weather patterns, including a worldwide increase in temperature, due largely to rising levels of atmospheric carbon dioxide

greenhouse effect

warming of Earth due to carbon dioxide and other greenhouse gases in the atmosphere

greenhouse gases

atmospheric gases such as carbon dioxide and methane that absorb and emit radiation, thus trapping heat in Earth's atmosphere

haze-effect cooling

effect of the gases and solids from a volcanic eruption on global climate

Milankovitch cycles

cyclic changes in the Earth's orbit that may affect climate

solar intensity

amount of solar power energy the sun emits in a given amount of time

weather

conditions of the atmosphere during a short period of time

## Case Study: Greenhouse Gases and Climate Change

In this module, two case studies provide examples of climate action plans – one for a city (Chicago) and one for an institution (the University of Illinois at Chicago).

### Introduction

If increased greenhouse gas emissions from human activity are causing climate change, then how do we reduce those emissions? Whether dictated by an international, national, or local regulation or a voluntary agreement, plans are needed to move to a low-carbon economy. In the absence of federal regulation, cities, states, government institutions, and colleges and universities, have all taken climate action initiatives. This case study provides two examples of climate action plans – one for a city (Chicago) and one for an institution (the University of Illinois at Chicago).

### Chicago's Climate Action Plan

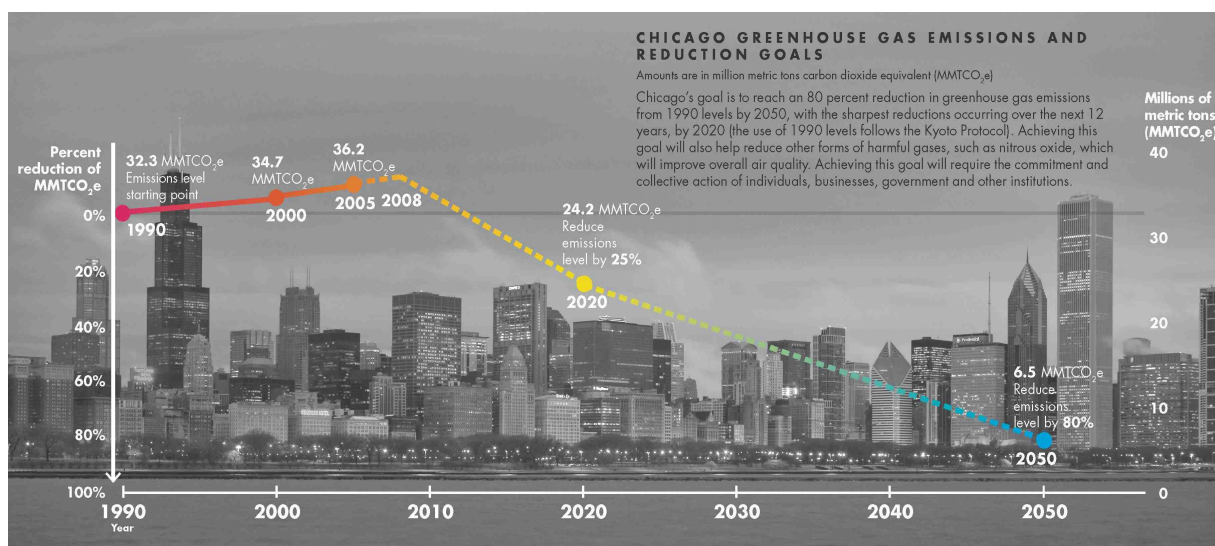
Urban areas produce a lot of waste. In fact, 75 percent of all greenhouse gas emissions are generated in urban areas. Therefore, it is important for cities to develop plans to address environmental issues. The [Chicago Climate Action Plan](#) (Chicago CAP) is one such example. The mid-term goal of this plan is a 25 percent reduction in greenhouse gas emissions by 2020 and final goal is 80 percent reduction below 1990 GHG levels by the year 2050.

The Chicago CAP outlines several benefits of a climate action plan. The first would obviously be the reduction of the effects of climate change. Under a higher emissions scenario as per the [Intergovernmental Panel on Climate Change](#) (IPCC), it is predicted that the number of 100 degree Fahrenheit days per year would increase to 31, under the lower emissions scenario it would only be eight. Established by the [United Nations Environment Programme](#) (UNEP), the IPCC is the leading international body that assesses climate change through the contributions of thousands of scientists.

Second, there is an economic benefit derived from increased efficiencies that reduce energy and water consumption. Third, local governments and

agencies have great influence over their city's greenhouse gas emissions and can enhance energy efficiency of buildings through codes and ordinances so they play a key role in climate action at all governmental levels. Finally, reducing our dependence on fossil fuels helps the United States achieve energy independence.

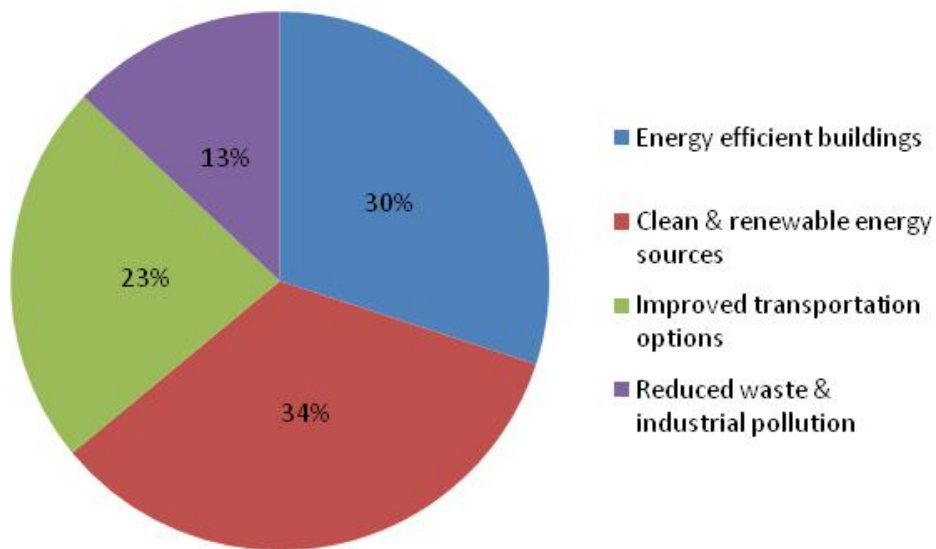
## Designing a Climate Action Plan



**Chicago Greenhouse Gas Emissions and Reduction Goals** Figure illustrates the emissions calculated for Chicago through 2005. *Source: [City of Chicago, Chicago Climate Action Plan](#)*

A good climate action plan includes reporting of greenhouse gas emissions, as far back as there is data, preferably to 1990. Figure [Chicago Greenhouse Gas Emissions and Reduction Goals](#) depicts the emissions calculated for Chicago through 2005. From that point there is an estimate (the dotted line) of a further increase before the reductions become evident and the goals portrayed can be obtained. The plan was released in September 2008 and provides a roadmap of five strategies with 35 actions to reduce greenhouse gas emissions (GHG) and adapt to climate change.

The strategies are shown in Table [Alignment of the Chicago and UIC Climate Action Plans](#). Figure [Sources of the Chicago CAP Emission Reductions by Strategy](#) identifies the proportion of emissions reductions from the various strategies.

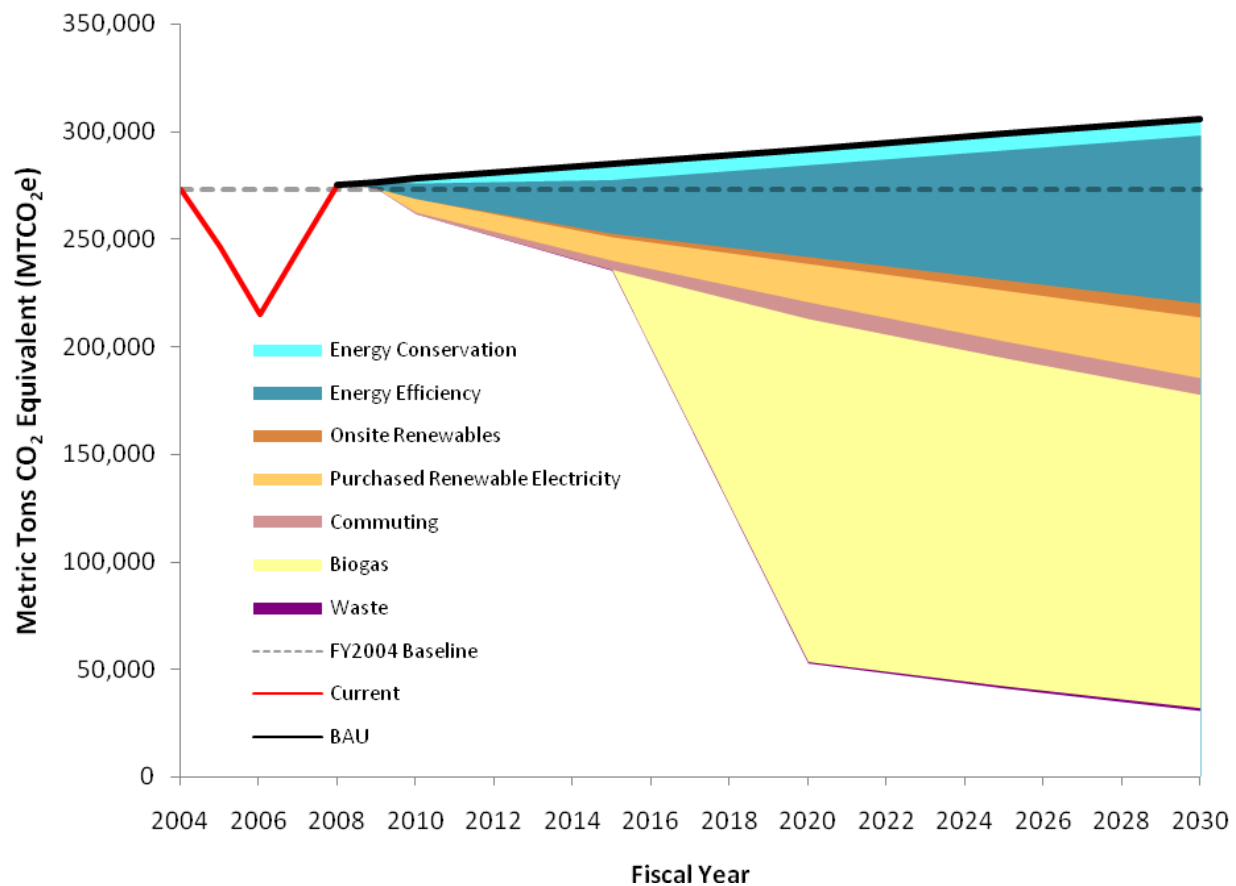


Graph shows the sources of the Chicago CAP emission reductions by strategy. Source: [C. Klein-Banai](#) using data from [City of Chicago, Chicago Climate Action Plan](#).

In 2010 CCAP put out a [progress report](#) wherein progress is measured by the many small steps that are being taken to implement the plan. It is not translated exactly to emissions reductions but reports on progress for each step such as the number of residential units that have been retrofitted for energy efficiency, the number of appliances traded in, the increase in the number of rides on public transit, and the amount of water conserved daily.

## University Climate Action Plan

Several factors caused a major Chicago university to develop a climate action plan. As part of the [American College and University Presidents' Climate Commitment](#) (ACUPCC), nearly 670 presidents have signed a commitment to inventory their greenhouse gases, publicly report it, and to develop a climate action plan. Part of the Chicago CAP is to engage businesses and organizations within the city in climate action planning. In order to be a better steward of the environment, the University of Illinois at Chicago (UIC) developed a [climate action plan](#). The goals are similar to Chicago's: a 40 percent GHG emissions reduction by 2030 and at least 80 percent by 2050, using a 2004 baseline. The strategies align with those of the city in which the campus resides (see Table [Alignment of the Chicago and UIC Climate Action Plans](#)). UIC's greenhouse gas reports are also made publically available on the [ACUPCC reporting site](#). Figure [UIC's Projected Emissions Reductions](#) displays UIC's calculated emissions inventory (in red) and then the predicted increases for growth if activities continue in a "business as usual (BAU)" approach. The triangular wedges below represent emissions reductions through a variety of strategies, similar to those of the [wedge approach](#) that Professors Sokolow and Pacala proposed. Those strategies are displayed in Table [Alignment of the Chicago and UIC Climate Action Plans](#), alongside Chicago's for comparative purposes.



**UIC's Projected Emissions Reductions** Projected emissions reductions from 2004 to 2030. Where BAU stands for Business as Usual, what would happen if no action were taken? Source: [UIC Climate Action Plan, figure 6](#).

The UIC CAP also has major strategy categories that are similar to Chicago's and within each strategy there are a number of recommended actions. Progress on this plan will be monitored both by reporting emissions at least every two years to the ACUPCC and by tracking individual actions and reporting to the campus community.

<b>CHICAGO CAP</b>	<b>UIC CAP</b>
<b>Energy Efficient Buildings</b>	<b>Energy Efficiency and Conservation</b>
Retrofit commercial and industrial buildings	Retrofit buildings
Retrofit residential buildings	Energy performance contracting
Trade in appliances	Monitoring and maintenance
Conserve water	Water conservation
Update City energy code	Establish green building standards
Establish new guidelines for renovations	
Cool with trees and green roofs	Green roofs/reflective roofs
Take easy steps	Energy conservation by campus community
<b>Clean &amp; Renewable Energy Sources</b>	<b>Clean and Renewable Energy</b>
Upgrade power plants	Modify power plants
Improve power plant efficiency	Purchase electricity from a renewable electricity provider
Build renewable electricity	Build renewable electricity

Increase distributed generation	
Promote household renewable power	Geothermal heating and cooling
<b>Improved Transportation Options</b>	<b>Improved Transportation Options</b>
Invest more in transit	
Expand transit incentives	Expand transit incentives
Promote transit-oriented development	
Make walking and biking easier	Make walking and biking easier
Car share and car pool	Car sharing/car pool program
Improve fleet efficiency	Continue to improve fleet efficiency
Achieve higher fuel efficiency standards	
Switch to cleaner fuels	
Support intercity rail	Reduce business travel (web conferencing)
Improve freight movement	Anti-Idling regulations/guidelines
<b>Reduced Waste &amp; Industrial Pollution</b>	<b>Recycling and Waste Management</b>

Reduce, reuse and recycle	Establishing recycling goals
Shift to alternative refrigerants	Composting
Capture stormwater on site	Sustainable food purchases & use of biodegradable packaging
	Collecting and converting vegetable oil
	Develop a user-friendly property management system
	Expand the waste minimization program
	Recycle construction debris
	Purchasing policies
<b>Preparation (Adaptation)</b>	<b>Improved Grounds Operations</b>
Manage heat	Capture stormwater on site
Protect air quality	Use native species
Manage stormwater	Reduce/eliminate irrigation
Implement green urban design	Integrated pest management
Preserve plants and trees	Tree care plan
Pursue innovative cooling	

Engage the public	<b>Education, Research and Public Engagement</b>
Engage businesses	<b>Employment Strategies</b>
Plan for the future	Telecommuting
	Flextime
	Childcare center
	Public Engagement

**Alignment of the Chicago and UIC Climate Action Plans** *Source: C. Klein-Banai using data from Chicago Climate Action Plan and UIC Climate Action Plan*

## Conclusion

There is no one approach that will effectively reduce greenhouse gas emissions. Climate action plans are helpful tools to represent strategies to reduce emissions. Governmental entities such as nations, states, and cities can develop plans, as can institutions and businesses. It is important that there be an alignment of plans when they intersect, such as a city and a university that resides within it.

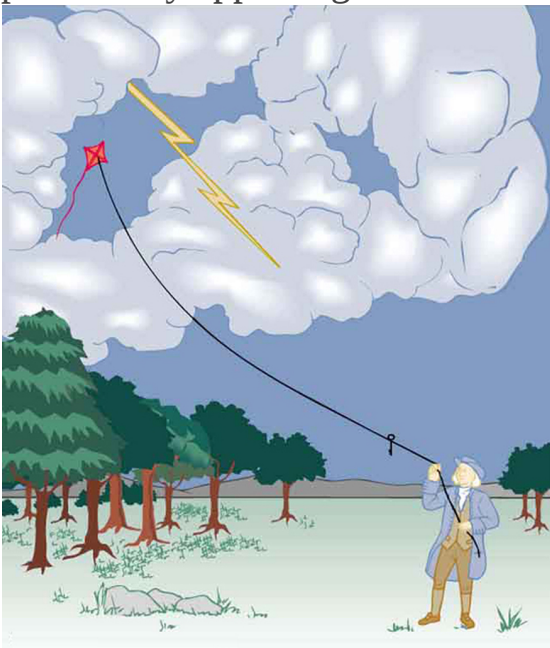
## Introduction to Electric Charge and Electric Field

class="introduction"

Static electricity from this plastic slide causes the child's hair to stand on end. The sliding motion stripped electrons away from the child's body, leaving an excess of positive charges, which repel each other along each strand of hair. (credit: Ken Bosma/Wikimedia Commons)



The image of American politician and scientist Benjamin Franklin (1706–1790) flying a kite in a thunderstorm is familiar to every schoolchild. (See [\[link\]](#).) In this experiment, Franklin demonstrated a connection between lightning and **static electricity**. Sparks were drawn from a key hung on a kite string during an electrical storm. These sparks were like those produced by static electricity, such as the spark that jumps from your finger to a metal doorknob after you walk across a wool carpet. What Franklin demonstrated in his dangerous experiment was a connection between phenomena on two different scales: one the grand power of an electrical storm, the other an effect of more human proportions. Connections like this one reveal the underlying unity of the laws of nature, an aspect we humans find particularly appealing.



When Benjamin Franklin demonstrated that lightning was related to static electricity, he made a connection that is now part of the evidence that all directly experienced forces except the gravitational force are manifestations of the electromagnetic force.

Much has been written about Franklin. His experiments were only part of the life of a man who was a scientist, inventor, revolutionary, statesman, and writer. Franklin's experiments were not performed in isolation, nor were they the only ones to reveal connections.

For example, the Italian scientist Luigi Galvani (1737–1798) performed a series of experiments in which static electricity was used to stimulate contractions of leg muscles of dead frogs, an effect already known in humans subjected to static discharges. But Galvani also found that if he joined two metal wires (say copper and zinc) end to end and touched the other ends to muscles, he produced the same effect in frogs as static discharge. Alessandro Volta (1745–1827), partly inspired by Galvani's work, experimented with various combinations of metals and developed the battery.

During the same era, other scientists made progress in discovering fundamental connections. The periodic table was developed as the systematic properties of the elements were discovered. This influenced the development and refinement of the concept of atoms as the basis of matter. Such submicroscopic descriptions of matter also help explain a great deal more.

Atomic and molecular interactions, such as the forces of friction, cohesion, and adhesion, are now known to be manifestations of the **electromagnetic force**. Static electricity is just one aspect of the electromagnetic force, which also includes moving electricity and magnetism.

All the macroscopic forces that we experience directly, such as the sensations of touch and the tension in a rope, are due to the electromagnetic force, one of the four fundamental forces in nature. The gravitational force, another fundamental force, is actually sensed through the electromagnetic interaction of molecules, such as between those in our feet and those on the top of a bathroom scale. (The other two fundamental forces, the strong nuclear force and the weak nuclear force, cannot be sensed on the human scale.)

This chapter begins the study of electromagnetic phenomena at a fundamental level. The next several chapters will cover static electricity, moving electricity, and magnetism—collectively known as electromagnetism. In this chapter, we begin with the study of electric phenomena due to charges that are at least temporarily stationary, called electrostatics, or static electricity.

## **Glossary**

static electricity

a buildup of electric charge on the surface of an object

electromagnetic force

one of the four fundamental forces of nature; the electromagnetic force consists of static electricity, moving electricity and magnetism

## Static Electricity and Charge: Conservation of Charge

- Define electric charge, and describe how the two types of charge interact.
- Describe three common situations that generate static electricity.
- State the law of conservation of charge.



Borneo amber was mined in Sabah, Malaysia, from shale-sandstone-mudstone veins.

When a piece of amber is rubbed with a piece of silk, the amber gains more electrons, giving it a net negative charge.

At the same time, the silk, having lost electrons, becomes positively charged. (credit: Sebakoamber, Wikimedia Commons)

What makes plastic wrap cling? Static electricity. Not only are applications of static electricity common these days, its existence has been known since ancient times. The first record of its effects dates to ancient Greeks who noted more than 500 years B.C. that polishing amber temporarily enabled it

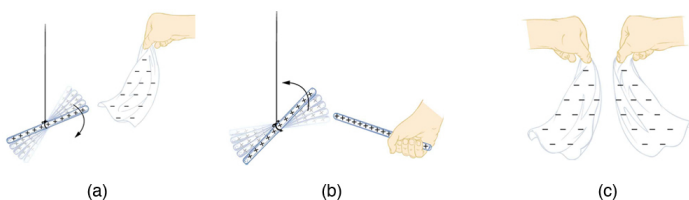
to attract bits of straw (see [\[link\]](#)). The very word *electric* derives from the Greek word for amber (*electron*).

Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Static cling generated in a clothes dryer and the attraction of straw to recently polished amber also result from rubbing. Similarly, lightning results from air movements under certain weather conditions. You can also rub a balloon on your hair, and the static electricity created can then make the balloon cling to a wall. We also have to be cautious of static electricity, especially in dry climates. When we pump gasoline, we are warned to discharge ourselves (after sliding across the seat) on a metal surface before grabbing the gas nozzle. Attendants in hospital operating rooms must wear booties with aluminum foil on the bottoms to avoid creating sparks which may ignite the oxygen being used.

Some of the most basic characteristics of static electricity include:

- The effects of static electricity are explained by a physical quantity not previously introduced, called electric charge.
- There are only two types of charge, one called positive and the other called negative.
- Like charges repel, whereas unlike charges attract.
- The force between charges decreases with distance.

How do we know there are two types of **electric charge**? When various materials are rubbed together in controlled ways, certain combinations of materials always produce one type of charge on one material and the opposite type on the other. By convention, we call one type of charge “positive”, and the other type “negative.” For example, when glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged. Since the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, since each rod has positive charge on it. Similarly, two silk cloths so rubbed will repel, since both cloths have negative charge. [\[link\]](#) shows how these simple materials can be used to explore the nature of the force between charges.



A glass rod becomes positively charged when rubbed with silk, while the silk becomes negatively charged.

(a) The glass rod is attracted to the silk because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel.

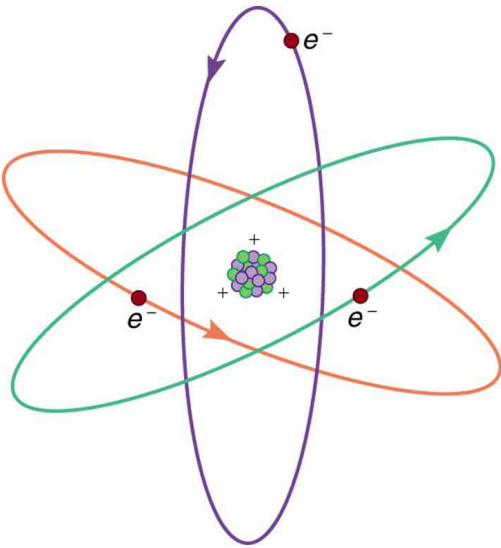
More sophisticated questions arise. Where do these charges come from? Can you create or destroy charge? Is there a smallest unit of charge? Exactly how does the force depend on the amount of charge and the distance between charges? Such questions obviously occurred to Benjamin Franklin and other early researchers, and they interest us even today.

## Charge Carried by Electrons and Protons

Franklin wrote in his letters and books that he could see the effects of electric charge but did not understand what caused the phenomenon. Today we have the advantage of knowing that normal matter is made of atoms, and that atoms contain positive and negative charges, usually in equal amounts.

[\[link\]](#) shows a simple model of an atom with negative **electrons** orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged **protons**. Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.) Other charge-carrying particles are observed in cosmic rays and nuclear decay, and are created in

particle accelerators. All but the electron and proton survive only a short time and are quite rare by comparison.



This simplified (and not to scale) view of an atom is called the planetary model of the atom.

Negative electrons orbit a much heavier positive nucleus, as the planets orbit the much heavier sun. There the similarity ends, because forces in the atom are electromagnetic, whereas those in the planetary system are gravitational.

Normal macroscopic amounts of matter contain immense numbers of atoms and molecules and, hence, even greater numbers of individual

negative and positive charges.

The charges of electrons and protons are identical in magnitude but opposite in sign. Furthermore, all charged objects in nature are integral multiples of this basic quantity of charge, meaning that all charges are made of combinations of a basic unit of charge. Usually, charges are formed by combinations of electrons and protons. The magnitude of this basic charge is

**Equation:**

$$|q_e| = 1.60 \times 10^{-19} \text{ C}.$$

The symbol  $q$  is commonly used for charge and the subscript  $e$  indicates the charge of a single electron (or proton).

The SI unit of charge is the coulomb (C). The number of protons needed to make a charge of 1.00 C is

**Equation:**

$$1.00 \text{ C} \times \frac{1 \text{ proton}}{1.60 \times 10^{-19} \text{ C}} = 6.25 \times 10^{18} \text{ protons}.$$

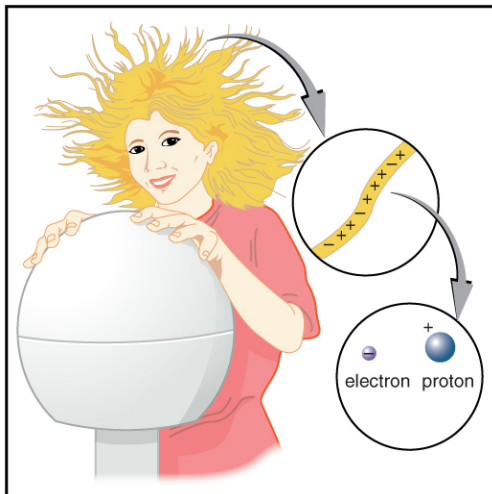
Similarly,  $6.25 \times 10^{18}$  electrons have a combined charge of  $-1.00$  coulomb. Just as there is a smallest bit of an element (an atom), there is a smallest bit of charge. There is no directly observed charge smaller than  $|q_e|$  (see [Things Great and Small: The Submicroscopic Origin of Charge](#)), and all observed charges are integral multiples of  $|q_e|$ .

**Note:**

Things Great and Small: The Submicroscopic Origin of Charge

With the exception of exotic, short-lived particles, all charge in nature is carried by electrons and protons. Electrons carry the charge we have named negative. Protons carry an equal-magnitude charge that we call positive. (See [\[link\]](#).) Electron and proton charges are considered fundamental building blocks, since all other charges are integral multiples of those carried by electrons and protons. Electrons and protons are also two of the three fundamental building blocks of ordinary matter. The neutron is the third and has zero total charge.

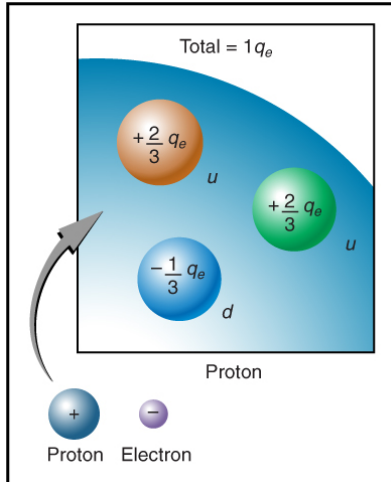
[\[link\]](#) shows a person touching a Van de Graaff generator and receiving excess positive charge. The expanded view of a hair shows the existence of both types of charges but an excess of positive. The repulsion of these positive like charges causes the strands of hair to repel other strands of hair and to stand up. The further blowup shows an artist's conception of an electron and a proton perhaps found in an atom in a strand of hair.



When this person touches  
a Van de Graaff  
generator, she receives an  
excess of positive charge,  
causing her hair to stand  
on end. The charges in

one hair are shown. An artist's conception of an electron and a proton illustrate the particles carrying the negative and positive charges. We cannot really see these particles with visible light because they are so small (the electron seems to be an infinitesimal point), but we know a great deal about their measurable properties, such as the charges they carry.

The electron seems to have no substructure; in contrast, when the substructure of protons is explored by scattering extremely energetic electrons from them, it appears that there are point-like particles inside the proton. These sub-particles, named quarks, have never been directly observed, but they are believed to carry fractional charges as seen in [\[link\]](#). Charges on electrons and protons and all other directly observable particles are unitary, but these quark substructures carry charges of either  $-\frac{1}{3}$  or  $+\frac{2}{3}$ . There are continuing attempts to observe fractional charge directly and to learn of the properties of quarks, which are perhaps the ultimate substructure of matter.



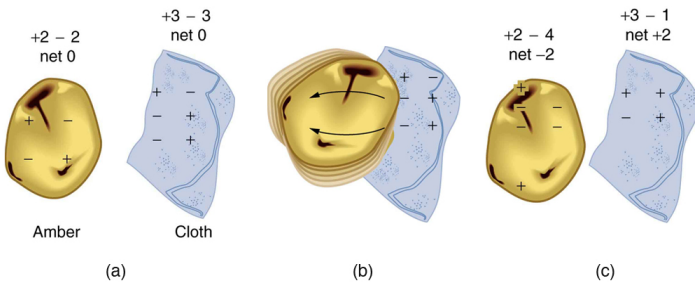
Artist's conception of fractional quark charges inside a proton. A group of three quark charges add up to the single positive charge on the proton:

$$-\frac{1}{3}q_e + \frac{2}{3}q_e + \frac{2}{3}q_e = +1q_e$$

.

## Separation of Charge in Atoms

Charges in atoms and molecules can be separated—for example, by rubbing materials together. Some atoms and molecules have a greater affinity for electrons than others and will become negatively charged by close contact in rubbing, leaving the other material positively charged. (See [link](#).) Positive charge can similarly be induced by rubbing. Methods other than rubbing can also separate charges. Batteries, for example, use combinations of substances that interact in such a way as to separate charges. Chemical interactions may transfer negative charge from one substance to the other, making one battery terminal negative and leaving the first one positive.



When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

No charge is actually created or destroyed when charges are separated as we have been discussing. Rather, existing charges are moved about. In fact, in all situations the total amount of charge is always constant. This universally obeyed law of nature is called the **law of conservation of charge**.

**Note:**

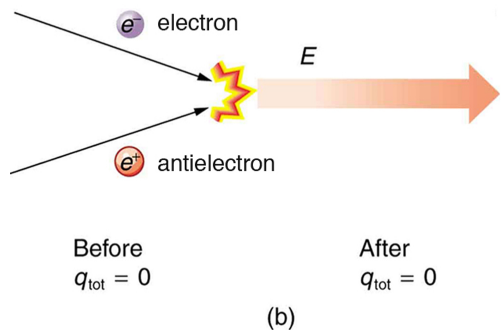
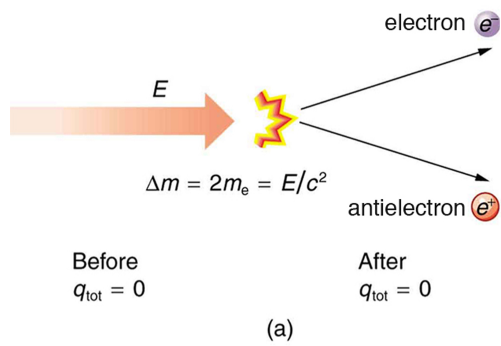
Law of Conservation of Charge

Total charge is constant in any process.

In more exotic situations, such as in particle accelerators, mass,  $\Delta m$ , can be created from energy in the amount  $\Delta m = \frac{E}{c^2}$ . Sometimes, the created mass is charged, such as when an electron is created. Whenever a charged particle is created, another having an opposite charge is always created along with it, so that the total charge created is zero. Usually, the two particles are “matter-antimatter” counterparts. For example, an antielectron would usually be created at the same time as an electron. The antielectron has a positive charge (it is called a positron), and so the total charge created is zero. (See [\[link\]](#).) All particles have antimatter counterparts with opposite signs. When matter and antimatter counterparts are brought together, they completely annihilate one another. By annihilate, we mean that the mass of the two particles is converted to energy  $E$ , again obeying the relationship  $\Delta m = \frac{E}{c^2}$ . Since the two particles have equal and opposite charge, the total charge is zero before and after the annihilation; thus, total charge is conserved.

**Note:****Making Connections: Conservation Laws**

Only a limited number of physical quantities are universally conserved. Charge is one—energy, momentum, and angular momentum are others. Because they are conserved, these physical quantities are used to explain more phenomena and form more connections than other, less basic quantities. We find that conserved quantities give us great insight into the rules followed by nature and hints to the organization of nature. Discoveries of conservation laws have led to further discoveries, such as the weak nuclear force and the quark substructure of protons and other particles.



(a) When enough energy is present, it can be converted into matter. Here the matter created is an electron–antielectron pair. ( $m_e$  is the electron’s mass.) The total charge before and after this event is zero. (b) When matter and antimatter collide, they annihilate each other; the total charge is conserved at zero before and after the annihilation.

The law of conservation of charge is absolute—it has never been observed to be violated. Charge, then, is a special physical quantity, joining a very

short list of other quantities in nature that are always conserved. Other conserved quantities include energy, momentum, and angular momentum.

**Note:**

**PhET Explorations: Balloons and Static Electricity**

Why does a balloon stick to your sweater? Rub a balloon on a sweater, then let go of the balloon and it flies over and sticks to the sweater. View the charges in the sweater, balloons, and the wall.

[https://phet.colorado.edu/sims/html/balloons-and-static-electricity/latest/balloons-and-static-electricity\\_en.html](https://phet.colorado.edu/sims/html/balloons-and-static-electricity/latest/balloons-and-static-electricity_en.html)

## Section Summary

- There are only two types of charge, which we call positive and negative.
- Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, while the vast majority of negative charge is carried by electrons.
- The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge  $|q_e|$  is

**Equation:**

$$|q_e| = 1.60 \times 10^{-19} \text{ C}.$$

- Whenever charge is created or destroyed, equal amounts of positive and negative are involved.
- Most often, existing charges are separated from neutral objects to obtain some net charge.

- Both positive and negative charges exist in neutral objects and can be separated by rubbing one object with another. For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge ensures that whenever a charge is created, an equal charge of the opposite sign is created at the same time.

## Conceptual Questions

### Exercise:

#### Problem:

There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

### Exercise:

#### Problem:

Why do most objects tend to contain nearly equal numbers of positive and negative charges?

## Problems & Exercises

### Exercise:

#### Problem:

Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of  $-2.00 \text{ nC}$  (b) How many electrons must be removed from a neutral object to leave a net charge of  $0.500 \mu\text{C}$ ?

---

#### Solution:

(a)  $1.25 \times 10^{10}$

(b)  $3.13 \times 10^{12}$

**Exercise:**

**Problem:**

If  $1.80 \times 10^{20}$  electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

**Exercise:**

**Problem:**

To start a car engine, the car battery moves  $3.75 \times 10^{21}$  electrons through the starter motor. How many coulombs of charge were moved?

---

**Solution:**

-600 C

**Exercise:**

**Problem:**

A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge  $|q_e|$  is this?

## Glossary

electric charge

a physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electromagnetic force

law of conservation of charge

states that whenever a charge is created, an equal amount of charge with the opposite sign is created simultaneously

electron

a particle orbiting the nucleus of an atom and carrying the smallest unit of negative charge

proton

a particle in the nucleus of an atom and carrying a positive charge equal in magnitude and opposite in sign to the amount of negative charge carried by an electron

## Conductors and Insulators

- Define conductor and insulator, explain the difference, and give examples of each.
- Describe three methods for charging an object.
- Explain what happens to an electric force as you move farther from the source.
- Define polarization.

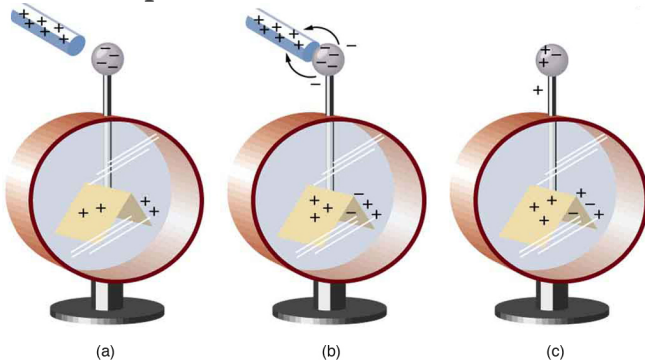


This power adapter uses metal wires and connectors to conduct electricity from the wall socket to a laptop computer. The conducting wires allow electrons to move freely through the cables, which are shielded by rubber and plastic. These materials act as insulators that don't allow electric charge to escape outward. (credit: Evan-Amos, Wikimedia Commons)

Some substances, such as metals and salty water, allow charges to move through them with relative ease. Some of the electrons in metals and similar conductors are not bound to individual atoms or sites in the material. These **free electrons** can move through the material much as air moves through loose sand. Any substance that has free electrons and allows charge to move

relatively freely through it is called a **conductor**. The moving electrons may collide with fixed atoms and molecules, losing some energy, but they can move in a conductor. Superconductors allow the movement of charge without any loss of energy. Salty water and other similar conducting materials contain free ions that can move through them. An ion is an atom or molecule having a positive or negative (nonzero) total charge. In other words, the total number of electrons is not equal to the total number of protons.

Other substances, such as glass, do not allow charges to move through them. These are called **insulators**. Electrons and ions in insulators are bound in the structure and cannot move easily—as much as  $10^{23}$  times more slowly than in conductors. Pure water and dry table salt are insulators, for example, whereas molten salt and salty water are conductors.



An electroscope is a favorite instrument in physics demonstrations and student laboratories. It is typically made with gold foil leaves hung from a (conducting) metal stem and is insulated from the room air in a glass-walled container. (a) A positively charged glass rod is brought near the tip of the electroscope, attracting electrons to the top and leaving a net positive charge on the leaves. Like charges in the light flexible gold leaves

repel, separating them. (b) When the rod is touched against the ball, electrons are attracted and transferred, reducing the net charge on the glass rod but leaving the electroscope positively charged. (c) The excess charges are evenly distributed in the stem and leaves of the electroscope once the glass rod is removed.

## Charging by Contact

[\[link\]](#) shows an electroscope being charged by touching it with a positively charged glass rod. Because the glass rod is an insulator, it must actually touch the electroscope to transfer charge to or from it. (Note that the extra positive charges reside on the surface of the glass rod as a result of rubbing it with silk before starting the experiment.) Since only electrons move in metals, we see that they are attracted to the top of the electroscope. There, some are transferred to the positive rod by touch, leaving the electroscope with a net positive charge.

**Electrostatic repulsion** in the leaves of the charged electroscope separates them. The electrostatic force has a horizontal component that results in the leaves moving apart as well as a vertical component that is balanced by the gravitational force. Similarly, the electroscope can be negatively charged by contact with a negatively charged object.

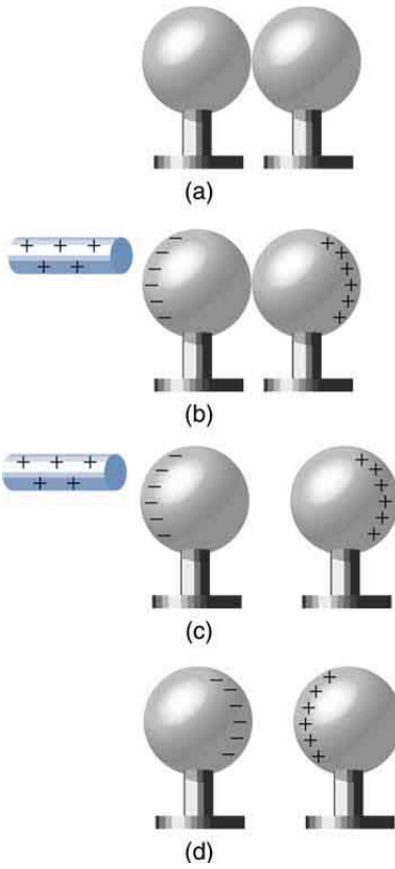
## Charging by Induction

It is not necessary to transfer excess charge directly to an object in order to charge it. [\[link\]](#) shows a method of **induction** wherein a charge is created in a nearby object, without direct contact. Here we see two neutral metal spheres in contact with one another but insulated from the rest of the world.

A positively charged rod is brought near one of them, attracting negative charge to that side, leaving the other sphere positively charged.

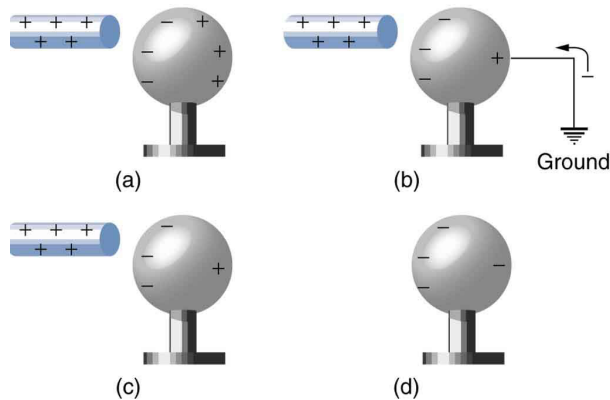
This is an example of induced **polarization** of neutral objects. Polarization is the separation of charges in an object that remains neutral. If the spheres are now separated (before the rod is pulled away), each sphere will have a net charge. Note that the object closest to the charged rod receives an opposite charge when charged by induction. Note also that no charge is removed from the charged rod, so that this process can be repeated without depleting the supply of excess charge.

Another method of charging by induction is shown in [\[link\]](#). The neutral metal sphere is polarized when a charged rod is brought near it. The sphere is then grounded, meaning that a conducting wire is run from the sphere to the ground. Since the earth is large and most ground is a good conductor, it can supply or accept excess charge easily. In this case, electrons are attracted to the sphere through a wire called the ground wire, because it supplies a conducting path to the ground. The ground connection is broken before the charged rod is removed, leaving the sphere with an excess charge opposite to that of the rod. Again, an opposite charge is achieved when charging by induction and the charged rod loses none of its excess charge.



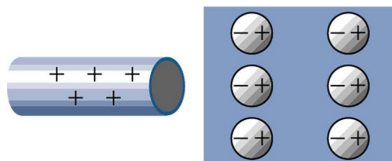
Charging by induction. (a) Two uncharged or neutral metal spheres are in contact with each other but insulated from the rest of the world. (b) A positively charged glass rod is brought near the sphere on the left, attracting negative charge and leaving the other sphere positively charged. (c) The

spheres are separated before the rod is removed, thus separating negative and positive charge. (d) The spheres retain net charges after the inducing rod is removed—without ever having been touched by a charged object.

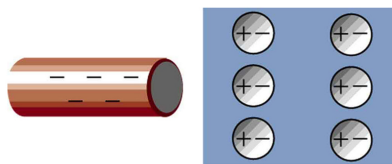


Charging by induction, using a ground connection. (a) A positively charged rod is brought near a neutral metal sphere, polarizing it. (b) The sphere is grounded, allowing electrons to be attracted from the earth's ample supply. (c) The ground connection is broken. (d) The positive rod is

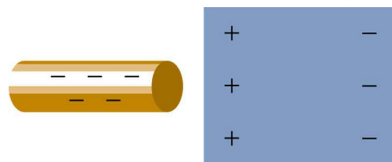
removed, leaving the sphere  
with an induced negative  
charge.



(a)



(b)



(c)

Both positive and  
negative objects  
attract a neutral  
object by polarizing  
its molecules. (a) A  
positive object  
brought near a  
neutral insulator  
polarizes its  
molecules. There is  
a slight shift in the  
distribution of the  
electrons orbiting  
the molecule, with

unlike charges being brought nearer and like charges moved away. Since the electrostatic force decreases with distance, there is a net attraction. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction.

Neutral objects can be attracted to any charged object. The pieces of straw attracted to polished amber are neutral, for example. If you run a plastic comb through your hair, the charged comb can pick up neutral pieces of paper. [\[link\]](#) shows how the polarization of atoms and molecules in neutral objects results in their attraction to a charged object.

When a charged rod is brought near a neutral substance, an insulator in this case, the distribution of charge in atoms and molecules is shifted slightly. Opposite charge is attracted nearer the external charged rod, while like charge is repelled. Since the electrostatic force decreases with distance, the repulsion of like charges is weaker than the attraction of unlike charges, and so there is a net attraction. Thus a positively charged glass rod attracts neutral pieces of paper, as will a negatively charged rubber rod. Some

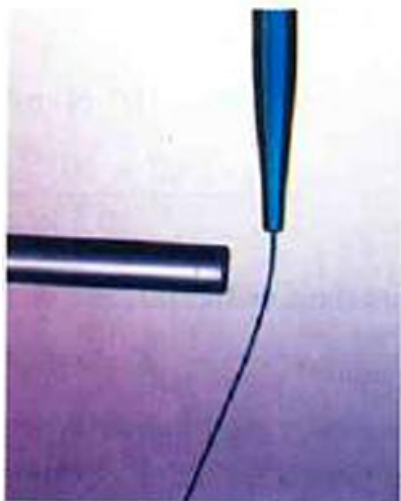
molecules, like water, are polar molecules. Polar molecules have a natural or inherent separation of charge, although they are neutral overall. Polar molecules are particularly affected by other charged objects and show greater polarization effects than molecules with naturally uniform charge distributions.

**Exercise:**

**Check Your Understanding**

**Problem:**

Can you explain the attraction of water to the charged rod in the figure below?



---

**Solution:**

**Answer**

Water molecules are polarized, giving them slightly positive and slightly negative sides. This makes water even more susceptible to a charged rod's attraction. As the water flows downward, due to the force of gravity, the charged conductor exerts a net attraction to the opposite charges in the stream of water, pulling it closer.

**Note:**

PhET Explorations: John Travoltage

Make sparks fly with John Travoltage. Wiggle Johnnie's foot and he picks up charges from the carpet. Bring his hand close to the door knob and get rid of the excess charge.

[https://phet.colorado.edu/sims/html/john-travoltage/latest/john-travoltage\\_en.html](https://phet.colorado.edu/sims/html/john-travoltage/latest/john-travoltage_en.html)

## Section Summary

- Polarization is the separation of positive and negative charges in a neutral object.
- A conductor is a substance that allows charge to flow freely through its atomic structure.
- An insulator holds charge within its atomic structure.
- Objects with like charges repel each other, while those with unlike charges attract each other.
- A conducting object is said to be grounded if it is connected to the Earth through a conductor. Grounding allows transfer of charge to and from the earth's large reservoir.
- Objects can be charged by contact with another charged object and obtain the same sign charge.
- If an object is temporarily grounded, it can be charged by induction, and obtains the opposite sign charge.
- Polarized objects have their positive and negative charges concentrated in different areas, giving them a non-symmetrical charge.
- Polar molecules have an inherent separation of charge.

## Conceptual Questions

### Exercise:

#### Problem:

An eccentric inventor attempts to levitate by first placing a large negative charge on himself and then putting a large positive charge on the ceiling of his workshop. Instead, while attempting to place a large negative charge on himself, his clothes fly off. Explain.

**Exercise:****Problem:**

If you have charged an electroscope by contact with a positively charged object, describe how you could use it to determine the charge of other objects. Specifically, what would the leaves of the electroscope do if other charged objects were brought near its knob?

**Exercise:****Problem:**

When a glass rod is rubbed with silk, it becomes positive and the silk becomes negative—yet both attract dust. Does the dust have a third type of charge that is attracted to both positive and negative? Explain.

**Exercise:****Problem:**

Why does a car always attract dust right after it is polished? (Note that car wax and car tires are insulators.)

**Exercise:****Problem:**

Describe how a positively charged object can be used to give another object a negative charge. What is the name of this process?

**Exercise:****Problem:**

What is grounding? What effect does it have on a charged conductor? On a charged insulator?

**Problems & Exercises****Exercise:**

**Problem:**

Suppose a speck of dust in an electrostatic precipitator has  $1.0000 \times 10^{12}$  protons in it and has a net charge of  $-5.00 \text{ nC}$  (a very large charge for a small speck). How many electrons does it have?

---

**Solution:**

$$1.03 \times 10^{12}$$

**Exercise:****Problem:**

An amoeba has  $1.00 \times 10^{16}$  protons and a net charge of  $0.300 \text{ pC}$ . (a) How many fewer electrons are there than protons? (b) If you paired them up, what fraction of the protons would have no electrons?

**Exercise:****Problem:**

A  $50.0 \text{ g}$  ball of copper has a net charge of  $2.00 \mu\text{C}$ . What fraction of the copper's electrons has been removed? (Each copper atom has 29 protons, and copper has an atomic mass of 63.5.)

---

**Solution:**

$$9.09 \times 10^{-13}$$

**Exercise:****Problem:**

What net charge would you place on a  $100 \text{ g}$  piece of sulfur if you put an extra electron on  $1 \text{ in } 10^{12}$  of its atoms? (Sulfur has an atomic mass of 32.1.)

**Exercise:**

**Problem:**

How many coulombs of positive charge are there in 4.00 kg of plutonium, given its atomic mass is 244 and that each plutonium atom has 94 protons?

---

**Solution:**

$$1.48 \times 10^8 \text{ C}$$

**Glossary**

free electron

an electron that is free to move away from its atomic orbit

conductor

a material that allows electrons to move separately from their atomic orbits

insulator

a material that holds electrons securely within their atomic orbits

grounded

when a conductor is connected to the Earth, allowing charge to freely flow to and from Earth's unlimited reservoir

induction

the process by which an electrically charged object brought near a neutral object creates a charge in that object

polarization

slight shifting of positive and negative charges to opposite sides of an atom or molecule

electrostatic repulsion

the phenomenon of two objects with like charges repelling each other

## Coulomb's Law

- State Coulomb's law in terms of how the electrostatic force changes with the distance between two objects.
- Calculate the electrostatic force between two charged point forces, such as electrons or protons.
- Compare the electrostatic force to the gravitational attraction for a proton and an electron; for a human and the Earth.



This NASA image of Arp 87 shows the result of a strong gravitational attraction between two galaxies. In contrast, at the subatomic level, the electrostatic attraction between two objects, such as an electron and a proton, is far greater than their mutual attraction due to gravity. (credit: NASA/HST)

Through the work of scientists in the late 18th century, the main features of the **electrostatic force**—the existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called **Coulomb's law** after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.

### Note:

Coulomb's Law

### Equation:

$$F = k \frac{|q_1 q_2|}{r^2}.$$

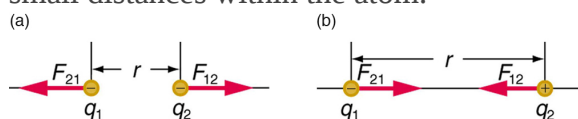
Coulomb's law calculates the magnitude of the force  $F$  between two point charges,  $q_1$  and  $q_2$ , separated by a distance  $r$ . In SI units, the constant  $k$  is equal to

**Equation:**

$$k = 8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \approx 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}.$$

The electrostatic force is a vector quantity and is expressed in units of newtons. The force is understood to be along the line joining the two charges. (See [\[link\]](#).)

Although the formula for Coulomb's law is simple, it was no mean task to prove it. The experiments Coulomb did, with the primitive equipment then available, were difficult. Modern experiments have verified Coulomb's law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared ( $F \propto 1/r^2$ ) to an accuracy of 1 part in  $10^{16}$ . No exceptions have ever been found, even at the small distances within the atom.



The magnitude of the electrostatic force  $F$  between point charges  $q_1$  and  $q_2$  separated by a distance  $r$  is given by Coulomb's law. Note that

Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on  $q_1$  is equal in magnitude and opposite in direction to the force it exerts on  $q_2$ .

(a) Like charges. (b) Unlike charges.

### Example:

#### How Strong is the Coulomb Force Relative to the Gravitational Force?

Compare the electrostatic force between an electron and proton separated by  $0.530 \times 10^{-10} \text{ m}$  with the gravitational force between them. This distance is their average separation in a hydrogen atom.

#### Strategy

To compare the two forces, we first compute the electrostatic force using Coulomb's law,  $F = k \frac{|q_1 q_2|}{r^2}$ . We then calculate the gravitational force using Newton's universal law of

gravitation. Finally, we take a ratio to see how the forces compare in magnitude.

**Solution**

Entering the given and known information about the charges and separation of the electron and proton into the expression of Coulomb's law yields

**Equation:**

$$F = k \frac{|q_1 q_2|}{r^2}$$

**Equation:**

$$= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{(1.60 \times 10^{-19} \text{ C})(1.60 \times 10^{-19} \text{ C})}{(0.530 \times 10^{-10} \text{ m})^2}$$

Thus the Coulomb force is

**Equation:**

$$F = 8.19 \times 10^{-8} \text{ N}.$$

The charges are opposite in sign, so this is an attractive force. This is a very large force for an electron—it would cause an acceleration of  $8.99 \times 10^{22} \text{ m/s}^2$  (verification is left as an end-of-section problem). The gravitational force is given by Newton's law of gravitation as:

**Equation:**

$$F_G = G \frac{mM}{r^2},$$

where  $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ . Here  $m$  and  $M$  represent the electron and proton masses, which can be found in the appendices. Entering values for the knowns yields

**Equation:**

$$F_G = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \times \frac{(9.11 \times 10^{-31} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}{(0.530 \times 10^{-10} \text{ m})^2} = 3.61 \times 10^{-47} \text{ N}$$

This is also an attractive force, although it is traditionally shown as positive since gravitational force is always attractive. The ratio of the magnitude of the electrostatic force to gravitational force in this case is, thus,

**Equation:**

$$\frac{F}{F_G} = 2.27 \times 10^{39}.$$

**Discussion**

This is a remarkably large ratio! Note that this will be the ratio of electrostatic force to gravitational force for an electron and a proton at any distance (taking the ratio before entering numerical values shows that the distance cancels). This ratio gives some indication

of just how much larger the Coulomb force is than the gravitational force between two of the most common particles in nature.

As the example implies, gravitational force is completely negligible on a small scale, where the interactions of individual charged particles are important. On a large scale, such as between the Earth and a person, the reverse is true. Most objects are nearly electrically neutral, and so attractive and repulsive **Coulomb forces** nearly cancel. Gravitational force on a large scale dominates interactions between large objects because it is always attractive, while Coulomb forces tend to cancel.

## Section Summary

- Frenchman Charles Coulomb was the first to publish the mathematical equation that describes the electrostatic force between two objects.
- Coulomb's law gives the magnitude of the force between point charges. It is **Equation:**

$$F = k \frac{|q_1 q_2|}{r^2},$$

where  $q_1$  and  $q_2$  are two point charges separated by a distance  $r$ , and  $k \approx 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$

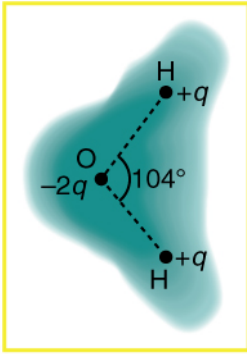
- This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.
- The Coulomb force is extraordinarily strong compared with the gravitational force, another basic force—but unlike gravitational force it can cancel, since it can be either attractive or repulsive.
- The electrostatic force between two subatomic particles is far greater than the gravitational force between the same two particles.

## Conceptual Questions

### Exercise:

#### Problem:

[\[link\]](#) shows the charge distribution in a water molecule, which is called a polar molecule because it has an inherent separation of charge. Given water's polar character, explain what effect humidity has on removing excess charge from objects.



Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a *polar molecule*. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.

**Exercise:****Problem:**

Using [\[link\]](#), explain, in terms of Coulomb's law, why a polar molecule (such as in [\[link\]](#)) is attracted by both positive and negative charges.

**Exercise:****Problem:**

Given the polar character of water molecules, explain how ions in the air form nucleation centers for rain droplets.

**Problems & Exercises****Exercise:****Problem:**

What is the repulsive force between two pith balls that are 8.00 cm apart and have equal charges of  $-30.0\text{ nC}$ ?

**Exercise:****Problem:**

(a) How strong is the attractive force between a glass rod with a  $0.700\text{ }\mu\text{C}$  charge and a silk cloth with a  $-0.600\text{ }\mu\text{C}$  charge, which are 12.0 cm apart, using the approximation that they act like point charges? (b) Discuss how the answer to this problem might be affected if the charges are distributed over some area and do not act like point charges.

---

**Solution:**

(a) 0.263 N

(b) If the charges are distributed over some area, there will be a concentration of charge along the side closest to the oppositely charged object. This effect will increase the net force.

**Exercise:****Problem:**

Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three?

**Exercise:****Problem:**

Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?

---

**Solution:**

The separation decreased by a factor of 5.

**Exercise:****Problem:**

How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them?

**Exercise:****Problem:**

If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them? You will see that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.

**Exercise:****Problem:**

A test charge of  $+2 \mu\text{C}$  is placed halfway between a charge of  $+6 \mu\text{C}$  and another of  $+4 \mu\text{C}$  separated by 10 cm. (a) What is the magnitude of the force on the test charge? (b) What is the direction of this force (away from or toward the  $+6 \mu\text{C}$  charge)?

**Exercise:****Problem:**

Bare free charges do not remain stationary when close together. To illustrate this, calculate the acceleration of two isolated protons separated by 2.00 nm (a typical distance between gas atoms). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

---

**Solution:**

$$\begin{aligned} F &= k \frac{|q_1 q_2|}{r^2} = ma \Rightarrow a = \frac{kq^2}{mr^2} \\ &= \frac{(9.00 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ m})^2}{(1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^{-9} \text{ m})^2} \\ &= 3.45 \times 10^{16} \text{ m/s}^2 \end{aligned}$$

**Exercise:****Problem:**

(a) By what factor must you change the distance between two point charges to change the force between them by a factor of 10? (b) Explain how the distance can either increase or decrease by this factor and still cause a factor of 10 change in the force.

---

**Solution:**

(a) 3.2

(b) If the distance increases by 3.2, then the force will decrease by a factor of 10 ; if the distance decreases by 3.2, then the force will increase by a factor of 10. Either way, the force changes by a factor of 10.

**Exercise:**

**Problem:**

Suppose you have a total charge  $q_{\text{tot}}$  that you can split in any manner. Once split, the separation distance is fixed. How do you split the charge to achieve the greatest force?

**Exercise:**

**Problem:**

(a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of a 10.0 mg piece of tape held 1.00 cm above another. (b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.

---

**Solution:**

(a)  $1.04 \times 10^{-9} \text{ C}$

(b) This charge is approximately 1 nC, which is consistent with the magnitude of charge typical for static electricity

**Exercise:**

**Problem:**

(a) Find the ratio of the electrostatic to gravitational force between two electrons. (b) What is this ratio for two protons? (c) Why is the ratio different for electrons and protons?

**Exercise:**

**Problem:**

At what distance is the electrostatic force between two protons equal to the weight of one proton?

**Exercise:**

**Problem:**

A certain five cent coin contains 5.00 g of nickel. What fraction of the nickel atoms' electrons, removed and placed 1.00 m above it, would support the weight of this coin? The atomic mass of nickel is 58.7, and each nickel atom contains 28 electrons and 28 protons.

---

**Solution:**

$$1.02 \times 10^{-11}$$

**Exercise:****Problem:**

(a) Two point charges totaling  $8.00 \mu\text{C}$  exert a repulsive force of 0.150 N on one another when separated by 0.500 m. What is the charge on each? (b) What is the charge on each if the force is attractive?

**Exercise:****Problem:**

Point charges of  $5.00 \mu\text{C}$  and  $-3.00 \mu\text{C}$  are placed 0.250 m apart. (a) Where can a third charge be placed so that the net force on it is zero? (b) What if both charges are positive?

---

**Solution:**

- a. 0.859 m beyond negative charge on line connecting two charges
- b. 0.109 m from lesser charge on line connecting two charges

**Exercise:****Problem:**

Two point charges  $q_1$  and  $q_2$  are 3.00 m apart, and their total charge is  $20 \mu\text{C}$ . (a) If the force of repulsion between them is 0.075N, what are magnitudes of the two charges? (b) If one charge attracts the other with a force of 0.525N, what are the magnitudes of the two charges? Note that you may need to solve a quadratic equation to reach your answer.

**Glossary****Coulomb's law**

the mathematical equation calculating the electrostatic force vector between two charged particles

**Coulomb force**

another term for the electrostatic force

electrostatic force

the amount and direction of attraction or repulsion between two charged bodies

## Introduction to Magnetism

class="introduction"

The  
magnificent  
spectacle  
of the  
Aurora  
Borealis, or  
northern  
lights,  
glows in  
the  
northern  
sky above  
Bear Lake  
near  
Eielson Air  
Force Base,  
Alaska.  
Shaped by  
the Earth's  
magnetic  
field, this  
light is  
produced  
by  
radiation  
spewed  
from solar  
storms.  
(credit:  
Senior  
Airman  
Joshua  
Strang, via  
Flickr)



One evening, an Alaskan sticks a note to his refrigerator with a small magnet. Through the kitchen window, the Aurora Borealis glows in the night sky. This grand spectacle is shaped by the same force that holds the note to the refrigerator.

People have been aware of magnets and magnetism for thousands of years. The earliest records date to well before the time of Christ, particularly in a region of Asia Minor called Magnesia (the name of this region is the source of words like *magnetic*). Magnetic rocks found in Magnesia, which is now part of western Turkey, stimulated interest during ancient times. A practical application for magnets was found later, when they were employed as navigational compasses. The use of magnets in compasses resulted not only in improved long-distance sailing, but also in the names of “north” and “south” being given to the two types of magnetic poles.

Today magnetism plays many important roles in our lives. Physicists’ understanding of magnetism has enabled the development of technologies that affect our everyday lives. The iPod in your purse or backpack, for example, wouldn’t have been possible without the applications of magnetism and electricity on a small scale.

The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology. The 2007 Nobel Prize in Physics went to Albert Fert from France and Peter Grunberg from Germany for this discovery of *giant magnetoresistance* and its applications to computer memory.

All electric motors, with uses as diverse as powering refrigerators, starting cars, and moving elevators, contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Hundreds of millions of dollars are spent annually on magnetic containment of fusion as a future energy source. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is used to explain atomic energy levels, cosmic rays, and charged particles trapped in the Van Allen belts. Once again, we will find all these disparate phenomena are linked by a small number of underlying physical principles.

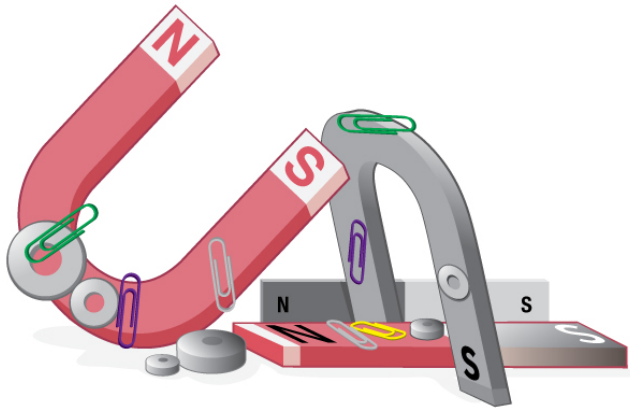


Engineering of  
technology like iPods  
would not be possible  
without a deep  
understanding  
magnetism. (credit: Jesse!  
S?, Flickr)



## Magnets

- Describe the difference between the north and south poles of a magnet.
- Describe how magnetic poles interact with each other.



Magnets come in various shapes, sizes, and strengths. All have both a north pole and a south pole. There is never an isolated pole (a monopole).

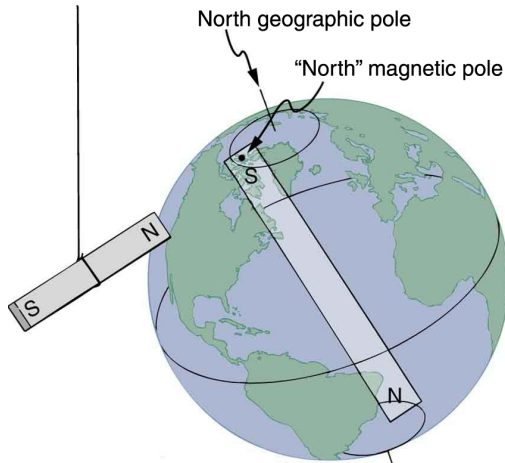
All magnets attract iron, such as that in a refrigerator door. However, magnets may attract or repel other magnets. Experimentation shows that all magnets have two poles. If freely suspended, one pole will point toward the north. The two poles are thus named the **north magnetic pole** and the **south magnetic pole** (or more properly, north-seeking and south-seeking poles, for the attractions in those directions).

### Note:

#### Universal Characteristics of Magnets and Magnetic Poles

It is a universal characteristic of all magnets that *like poles repel and unlike poles attract*. (Note the similarity with electrostatics: unlike charges attract and like charges repel.)

Further experimentation shows that it is *impossible to separate north and south poles* in the manner that + and – charges can be separated.



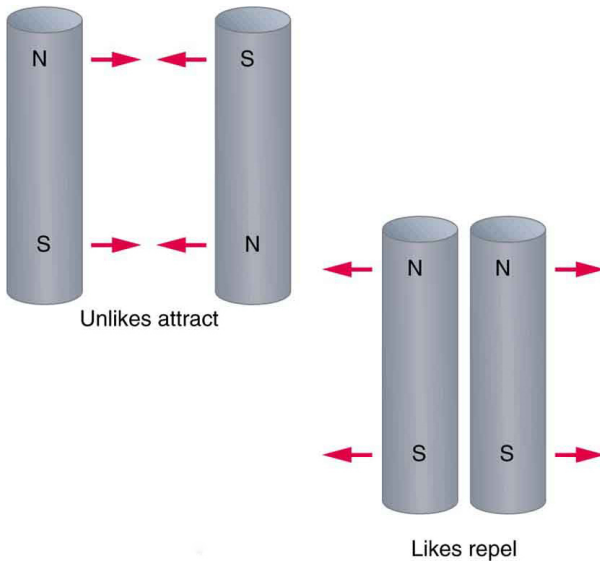
One end of a bar magnet is suspended from a thread that points toward north. The magnet's two poles are labeled N and S for north-seeking and south-seeking poles, respectively.

**Note:**

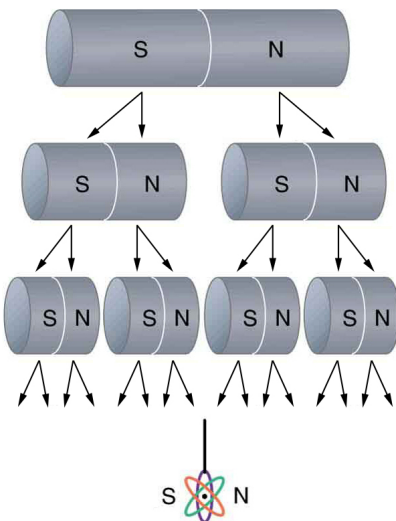
**Misconception Alert: Earth's Geographic North Pole Hides an S**

The Earth acts like a very large bar magnet with its south-seeking pole near the geographic North Pole. That is why the north pole of your compass is attracted toward the geographic north pole of the Earth—because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole! Confusion arises because the geographic term “North Pole” has come to be used (incorrectly) for the magnetic pole that is near the

North Pole. Thus, “North magnetic pole” is actually a misnomer—it should be called the South magnetic pole.



Unlike poles attract, whereas  
like poles repel.



North and south  
poles always occur  
in pairs. Attempts

to separate them  
result in more pairs  
of poles. If we  
continue to split the  
magnet, we will  
eventually get  
down to an iron  
atom with a north  
pole and a south  
pole—these, too,  
cannot be  
separated.

The fact that magnetic poles always occur in pairs of north and south is true from the very large scale—for example, sunspots always occur in pairs that are north and south magnetic poles—all the way down to the very small scale. Magnetic atoms have both a north pole and a south pole, as do many types of subatomic particles, such as electrons, protons, and neutrons.

**Note:**

**Making Connections: Take-Home Experiment—Refrigerator Magnets**

We know that like magnetic poles repel and unlike poles attract. See if you can show this for two refrigerator magnets. Will the magnets stick if you turn them over? Why do they stick to the door anyway? What can you say about the magnetic properties of the door next to the magnet? Do refrigerator magnets stick to metal or plastic spoons? Do they stick to all types of metal?

## Section Summary

- Magnetism is a subject that includes the properties of magnets, the effect of the magnetic force on moving charges and currents, and the

- creation of magnetic fields by currents.
- There are two types of magnetic poles, called the north magnetic pole and south magnetic pole.
  - North magnetic poles are those that are attracted toward the Earth's geographic north pole.
  - Like poles repel and unlike poles attract.
  - Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

## Conceptual Questions

### Exercise:

#### Problem:

Volcanic and other such activity at the mid-Atlantic ridge extrudes material to fill the gap between separating tectonic plates associated with continental drift. The magnetization of rocks is found to reverse in a coordinated manner with distance from the ridge. What does this imply about the Earth's magnetic field and how could the knowledge of the spreading rate be used to give its historical record?

## Glossary

north magnetic pole

the end or the side of a magnet that is attracted toward Earth's geographic north pole

south magnetic pole

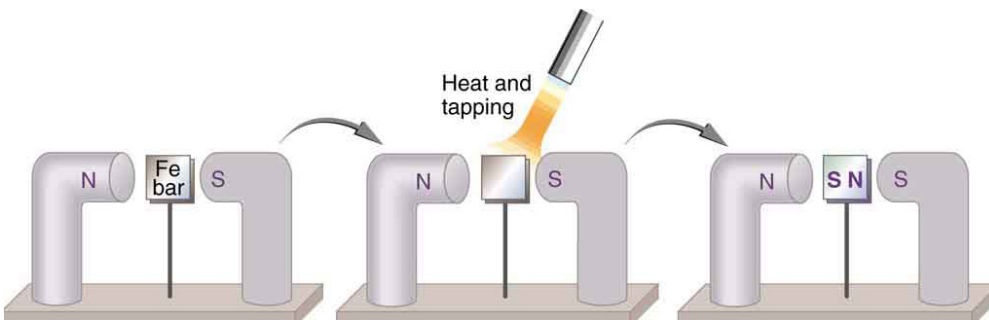
the end or the side of a magnet that is attracted toward Earth's geographic south pole

## Ferromagnets and Electromagnets

- Define ferromagnet.
- Describe the role of magnetic domains in magnetization.
- Explain the significance of the Curie temperature.
- Describe the relationship between electricity and magnetism.

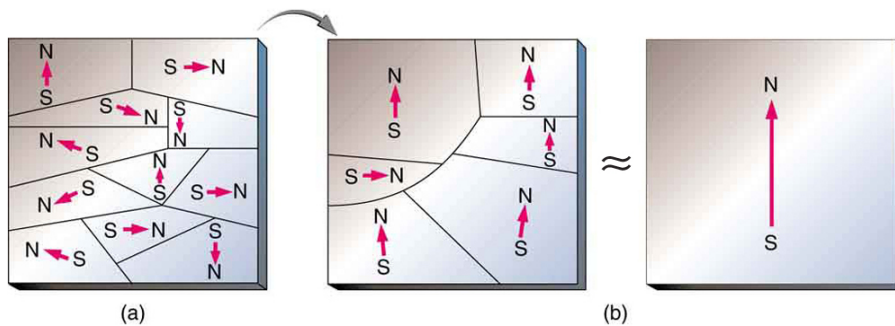
### Ferromagnets

Only certain materials, such as iron, cobalt, nickel, and gadolinium, exhibit strong magnetic effects. Such materials are called **ferromagnetic**, after the Latin word for iron, *ferrum*. A group of materials made from the alloys of the rare earth elements are also used as strong and permanent magnets; a popular one is neodymium. Other materials exhibit weak magnetic effects, which are detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets (the way iron is attracted to magnets), they can also be **magnetized** themselves—that is, they can be induced to be magnetic or made into permanent magnets.



An unmagnetized piece of iron is placed between two magnets, heated, and then cooled, or simply tapped when cold. The iron becomes a permanent magnet with the poles aligned as shown: its south pole is adjacent to the north pole of the original magnet, and its north pole is adjacent to the south pole of the original magnet. Note that there are attractive forces between the magnets.

When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest, as in [\[link\]](#). (This results in the attraction of the previously unmagnetized material to the magnet.) What happens on a microscopic scale is illustrated in [\[link\]](#). The regions within the material called **domains** act like small bar magnets. Within domains, the poles of individual atoms are aligned. Each atom acts like a tiny bar magnet. Domains are small and randomly oriented in an unmagnetized ferromagnetic object. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves as shown in [\[link\]](#)(b). This induced magnetization can be made permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets.



(a) An unmagnetized piece of iron (or other ferromagnetic material) has randomly oriented domains. (b) When magnetized by an external field, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within domains; each atom acts like a tiny bar magnet.

Conversely, a permanent magnet can be demagnetized by hard blows or by heating it in the absence of another magnet. Increased thermal motion at higher temperature can disrupt and randomize the orientation and the size of

the domains. There is a well-defined temperature for ferromagnetic materials, which is called the **Curie temperature**, above which they cannot be magnetized. The Curie temperature for iron is 1043 K (770°C), which is well above room temperature. There are several elements and alloys that have Curie temperatures much lower than room temperature and are ferromagnetic only below those temperatures.

## Electromagnets

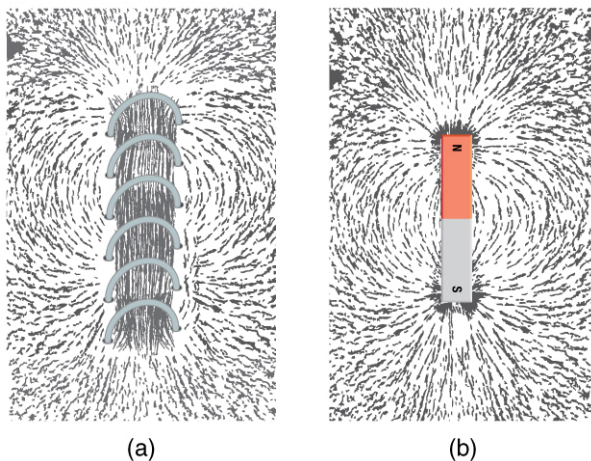
Early in the 19th century, it was discovered that electrical currents cause magnetic effects. The first significant observation was by the Danish scientist Hans Christian Oersted (1777–1851), who found that a compass needle was deflected by a current-carrying wire. This was the first significant evidence that the movement of charges had any connection with magnets. **Electromagnetism** is the use of electric current to make magnets. These temporarily induced magnets are called **electromagnets**. Electromagnets are employed for everything from a wrecking yard crane that lifts scrapped cars to controlling the beam of a 90-km-circumference particle accelerator to the magnets in medical imaging machines (See [\[link\]](#)).



Instrument for magnetic resonance imaging (MRI). The device uses a superconducting

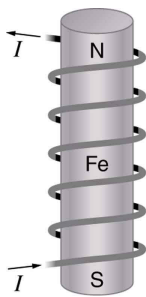
cylindrical coil for the main magnetic field. The patient goes into this “tunnel” on the gurney. (credit: Bill McChesney, Flickr)

[\[link\]](#) shows that the response of iron filings to a current-carrying coil and to a permanent bar magnet. The patterns are similar. In fact, electromagnets and ferromagnets have the same basic characteristics—for example, they have north and south poles that cannot be separated and for which like poles repel and unlike poles attract.



Iron filings near (a) a current-carrying coil and (b) a magnet act like tiny compass needles, showing the shape of their fields. Their response to a current-carrying coil and a permanent magnet is seen to be very similar, especially near the ends of the coil and the magnet.

Combining a ferromagnet with an electromagnet can produce particularly strong magnetic effects. (See [\[link\]](#).) Whenever strong magnetic effects are needed, such as lifting scrap metal, or in particle accelerators, electromagnets are enhanced by ferromagnetic materials. Limits to how strong the magnets can be made are imposed by coil resistance (it will overheat and melt at sufficiently high current), and so superconducting magnets may be employed. These are still limited, because superconducting properties are destroyed by too great a magnetic field.

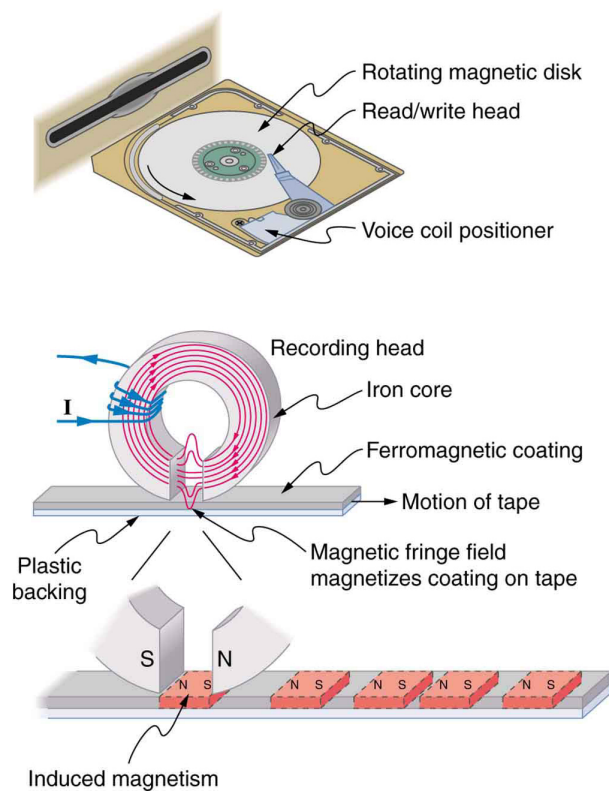


An  
electromagnet  
with a  
ferromagnetic  
core can  
produce very  
strong  
magnetic  
effects.

Alignment of  
domains in the  
core produces  
a magnet, the  
poles of which  
are aligned  
with the  
electromagnet

.

[\[link\]](#) shows a few uses of combinations of electromagnets and ferromagnets. Ferromagnetic materials can act as memory devices, because the orientation of the magnetic fields of small domains can be reversed or erased. Magnetic information storage on videotapes and computer hard drives are among the most common applications. This property is vital in our digital world.



An electromagnet induces regions of permanent magnetism on a floppy disk coated with a ferromagnetic material. The information stored here is digital (a region is either magnetic or not); in other applications, it can be analog

(with a varying strength), such  
as on audiotapes.

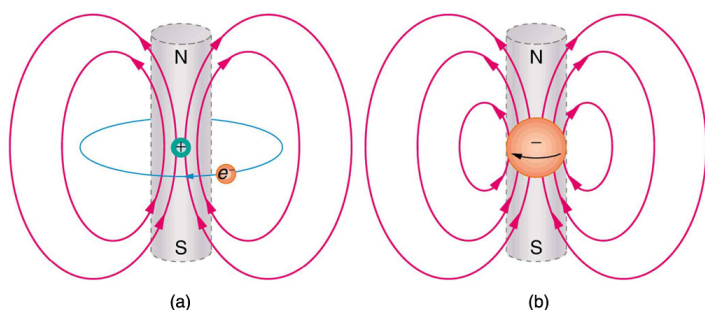
## Current: The Source of All Magnetism

An electromagnet creates magnetism with an electric current. In later sections we explore this more quantitatively, finding the strength and direction of magnetic fields created by various currents. But what about ferromagnets? [\[link\]](#) shows models of how electric currents create magnetism at the submicroscopic level. (Note that we cannot directly observe the paths of individual electrons about atoms, and so a model or visual image, consistent with all direct observations, is made. We can directly observe the electron's orbital angular momentum, its spin momentum, and subsequent magnetic moments, all of which are explained with electric-current-creating subatomic magnetism.) Currents, including those associated with other submicroscopic particles like protons, allow us to explain ferromagnetism and all other magnetic effects. Ferromagnetism, for example, results from an internal cooperative alignment of electron spins, possible in some materials but not in others.

Crucial to the statement that electric current is the source of all magnetism is the fact that it is impossible to separate north and south magnetic poles. (This is far different from the case of positive and negative charges, which are easily separated.) A current loop always produces a magnetic dipole—that is, a magnetic field that acts like a north pole and south pole pair. Since isolated north and south magnetic poles, called **magnetic monopoles**, are not observed, currents are used to explain all magnetic effects. If magnetic monopoles did exist, then we would have to modify this underlying connection that all magnetism is due to electrical current. There is no known reason that magnetic monopoles should not exist—they are simply never observed—and so searches at the subnuclear level continue. If they *do not* exist, we would like to find out why not. If they *do* exist, we would like to see evidence of them.

**Note:****Electric Currents and Magnetism**

Electric current is the source of all magnetism.



(a) In the planetary model of the atom, an electron orbits a nucleus, forming a closed-current loop and producing a magnetic field with a north pole and a south pole. (b) Electrons have spin and can be crudely pictured as rotating charge, forming a current that produces a magnetic field with a north pole and a south pole. Neither the planetary model nor the image of a spinning electron is completely consistent with modern physics. However, they do provide a useful way of understanding phenomena.

**Note:****PhET Explorations: Magnets and Electromagnets**

Explore the interactions between a compass and bar magnet. Discover how you can use a battery and wire to make a magnet! Can you make it a stronger magnet? Can you make the magnetic field reverse?

## Section Summary

- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.
- All magnetism is created by electric current.
- Ferromagnetic materials, such as iron, are those that exhibit strong magnetic effects.
- The atoms in ferromagnetic materials act like small magnets (due to currents within the atoms) and can be aligned, usually in millimeter-sized regions called domains.
- Domains can grow and align on a larger scale, producing permanent magnets. Such a material is magnetized, or induced to be magnetic.
- Above a material's Curie temperature, thermal agitation destroys the alignment of atoms, and ferromagnetism disappears.
- Electromagnets employ electric currents to make magnetic fields, often aided by induced fields in ferromagnetic materials.

## Glossary

ferromagnetic

materials, such as iron, cobalt, nickel, and gadolinium, that exhibit strong magnetic effects

magnetized

to be turned into a magnet; to be induced to be magnetic

domains

regions within a material that behave like small bar magnets

Curie temperature

the temperature above which a ferromagnetic material cannot be magnetized

electromagnetism

the use of electrical currents to induce magnetism

electromagnet

an object that is temporarily magnetic when an electrical current is passed through it

magnetic monopoles

an isolated magnetic pole; a south pole without a north pole, or vice versa (no magnetic monopole has ever been observed)

## More Applications of Magnetism

- Describe some applications of magnetism.

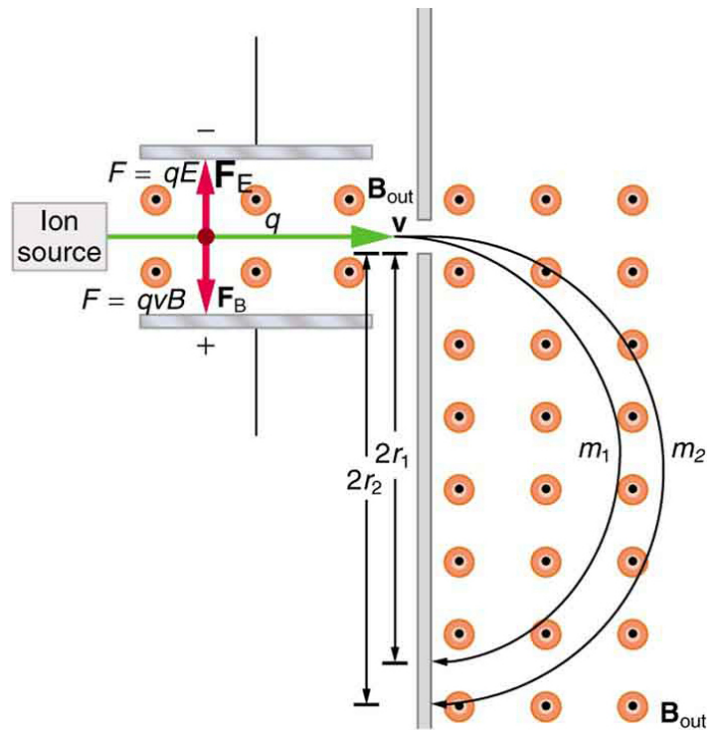
### Mass Spectrometry

The curved paths followed by charged particles in magnetic fields can be put to use. A charged particle moving perpendicular to a magnetic field travels in a circular path having a radius  $r$ .

**Equation:**

$$r = \frac{mv}{qB}$$

It was noted that this relationship could be used to measure the mass of charged particles such as ions. A mass spectrometer is a device that measures such masses. Most mass spectrometers use magnetic fields for this purpose, although some of them have extremely sophisticated designs. Since there are five variables in the relationship, there are many possibilities. However, if  $v$ ,  $q$ , and  $B$  can be fixed, then the radius of the path  $r$  is simply proportional to the mass  $m$  of the charged particle. Let us examine one such mass spectrometer that has a relatively simple design. (See [\[link\]](#).) The process begins with an ion source, a device like an electron gun. The ion source gives ions their charge, accelerates them to some velocity  $v$ , and directs a beam of them into the next stage of the spectrometer. This next region is a *velocity selector* that only allows particles with a particular value of  $v$  to get through.



This mass spectrometer uses a velocity selector to fix  $v$  so that the radius of the path is proportional to mass.

The velocity selector has both an electric field and a magnetic field, perpendicular to one another, producing forces in opposite directions on the ions. Only those ions for which the forces balance travel in a straight line into the next region. If the forces balance, then the electric force  $F = qE$  equals the magnetic force  $F = qvB$ , so that  $qE = qvB$ . Noting that  $q$

**Equation:**

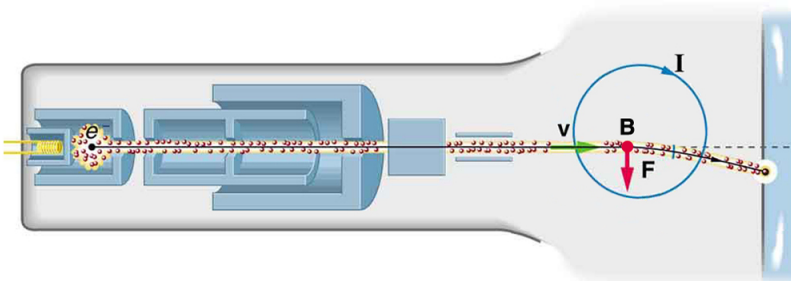
$$v = \frac{E}{B}$$

is the velocity particles must have to make it through the velocity selector, and further, that  $v$  can be selected by varying  $E$  and  $B$ . In the final region, there is only a uniform magnetic field, and so the charged particles move in circular arcs with radii proportional to particle mass. The paths also depend on charge  $q$ , but since  $q$  is in multiples of electron charges, it is easy to determine and to discriminate between ions in different charge states.

Mass spectrometry today is used extensively in chemistry and biology laboratories to identify chemical and biological substances according to their mass-to-charge ratios. In medicine, mass spectrometers are used to measure the concentration of isotopes used as tracers. Usually, biological molecules such as proteins are very large, so they are broken down into smaller fragments before analyzing. Recently, large virus particles have been analyzed as a whole on mass spectrometers. Sometimes a gas chromatograph or high-performance liquid chromatograph provides an initial separation of the large molecules, which are then input into the mass spectrometer.

## Cathode Ray Tubes—CRTs—and the Like

What do non-flat-screen TVs, old computer monitors, x-ray machines, and the 2-mile-long Stanford Linear Accelerator have in common? All of them accelerate electrons, making them different versions of the electron gun. Many of these devices use magnetic fields to steer the accelerated electrons. [\[link\]](#) shows the construction of the type of cathode ray tube (CRT) found in some TVs, oscilloscopes, and old computer monitors. Two pairs of coils are used to steer the electrons, one vertically and the other horizontally, to their desired destination.



The cathode ray tube (CRT) is so named because rays of electrons originate at the cathode in the electron gun. Magnetic coils are used to steer the beam in many CRTs. In this case, the beam is moved down. Another pair of horizontal coils would steer the beam horizontally.

## Magnetic Resonance Imaging

**Magnetic resonance imaging (MRI)** is one of the most useful and rapidly growing medical imaging tools. It non-invasively produces two-dimensional and three-dimensional images of the body that provide important medical information with none of the hazards of x-rays. MRI is based on an effect called **nuclear magnetic resonance (NMR)** in which an externally applied magnetic field interacts with the nuclei of certain atoms, particularly those of hydrogen (protons). These nuclei possess their own small magnetic fields, similar to those of electrons and the current loops discussed earlier in this chapter.

When placed in an external magnetic field, such nuclei experience a torque that pushes or aligns the nuclei into one of two new energy states—depending on the orientation of its spin (analogous to the N pole and S pole in a bar magnet). Transitions from the lower to higher energy state can be achieved by using an external radio frequency signal to “flip” the orientation of the small magnets. (This is actually a quantum mechanical process. The direction of the nuclear magnetic field is quantized as is energy in the radio waves. We will return to these topics in later chapters.) The specific frequency of the radio waves that are absorbed and reemitted depends sensitively on the type of nucleus, the chemical environment, and the external magnetic field strength. Therefore, this is a *resonance* phenomenon in which *nuclei* in a *magnetic* field act like resonators (analogous to those discussed in the treatment of sound in [Oscillatory](#)

[Motion and Waves](#)) that absorb and reemit only certain frequencies. Hence, the phenomenon is named *nuclear magnetic resonance (NMR)*.

NMR has been used for more than 50 years as an analytical tool. It was formulated in 1946 by F. Bloch and E. Purcell, with the 1952 Nobel Prize in Physics going to them for their work. Over the past two decades, NMR has been developed to produce detailed images in a process now called magnetic resonance imaging (MRI), a name coined to avoid the use of the word “nuclear” and the concomitant implication that nuclear radiation is involved. (It is not.) The 2003 Nobel Prize in Medicine went to P. Lauterbur and P. Mansfield for their work with MRI applications.

The largest part of the MRI unit is a superconducting magnet that creates a magnetic field, typically between 1 and 2 T in strength, over a relatively large volume. MRI images can be both highly detailed and informative about structures and organ functions. It is helpful that normal and non-normal tissues respond differently for slight changes in the magnetic field. In most medical images, the protons that are hydrogen nuclei are imaged. (About 2/3 of the atoms in the body are hydrogen.) Their location and density give a variety of medically useful information, such as organ function, the condition of tissue (as in the brain), and the shape of structures, such as vertebral disks and knee-joint surfaces. MRI can also be used to follow the movement of certain ions across membranes, yielding information on active transport, osmosis, dialysis, and other phenomena. With excellent spatial resolution, MRI can provide information about tumors, strokes, shoulder injuries, infections, etc.

An image requires position information as well as the density of a nuclear type (usually protons). By varying the magnetic field slightly over the volume to be imaged, the resonant frequency of the protons is made to vary with position. Broadcast radio frequencies are swept over an appropriate range and nuclei absorb and reemit them only if the nuclei are in a magnetic field with the correct strength. The imaging receiver gathers information through the body almost point by point, building up a tissue map. The reception of reemitted radio waves as a function of frequency thus gives position information. These “slices” or cross sections through the body are only several mm thick. The intensity of the reemitted radio waves is

proportional to the concentration of the nuclear type being flipped, as well as information on the chemical environment in that area of the body.

Various techniques are available for enhancing contrast in images and for obtaining more information. Scans called T1, T2, or proton density scans rely on different relaxation mechanisms of nuclei. Relaxation refers to the time it takes for the protons to return to equilibrium after the external field is turned off. This time depends upon tissue type and status (such as inflammation).

While MRI images are superior to x rays for certain types of tissue and have none of the hazards of x rays, they do not completely supplant x-ray images. MRI is less effective than x rays for detecting breaks in bone, for example, and in imaging breast tissue, so the two diagnostic tools complement each other. MRI images are also expensive compared to simple x-ray images and tend to be used most often where they supply information not readily obtained from x rays. Another disadvantage of MRI is that the patient is totally enclosed with detectors close to the body for about 30 minutes or more, leading to claustrophobia. It is also difficult for the obese patient to be in the magnet tunnel. New “open-MRI” machines are now available in which the magnet does not completely surround the patient.

Over the last decade, the development of much faster scans, called “functional MRI” (fMRI), has allowed us to map the functioning of various regions in the brain responsible for thought and motor control. This technique measures the change in blood flow for activities (thought, experiences, action) in the brain. The nerve cells increase their consumption of oxygen when active. Blood hemoglobin releases oxygen to active nerve cells and has somewhat different magnetic properties when oxygenated than when deoxygenated. With MRI, we can measure this and detect a blood oxygen-dependent signal. Most of the brain scans today use fMRI.

## **Other Medical Uses of Magnetic Fields**

Currents in nerve cells and the heart create magnetic fields like any other currents. These can be measured but with some difficulty since their strengths are about  $10^{-6}$  to  $10^{-8}$  less than the Earth’s magnetic field. Recording of the heart’s magnetic field as it beats is called a

**magnetocardiogram (MCG)**, while measurements of the brain's magnetic field is called a **magnetoencephalogram (MEG)**. Both give information that differs from that obtained by measuring the electric fields of these organs (ECGs and EEGs), but they are not yet of sufficient importance to make these difficult measurements common.

In both of these techniques, the sensors do not touch the body. MCG can be used in fetal studies, and is probably more sensitive than echocardiography. MCG also looks at the heart's electrical activity whose voltage output is too small to be recorded by surface electrodes as in EKG. It has the potential of being a rapid scan for early diagnosis of cardiac ischemia (obstruction of blood flow to the heart) or problems with the fetus.

MEG can be used to identify abnormal electrical discharges in the brain that produce weak magnetic signals. Therefore, it looks at brain activity, not just brain structure. It has been used for studies of Alzheimer's disease and epilepsy. Advances in instrumentation to measure very small magnetic fields have allowed these two techniques to be used more in recent years. What is used is a sensor called a SQUID, for superconducting quantum interference device. This operates at liquid helium temperatures and can measure magnetic fields thousands of times smaller than the Earth's.

Finally, there is a burgeoning market for magnetic cures in which magnets are applied in a variety of ways to the body, from magnetic bracelets to magnetic mattresses. The best that can be said for such practices is that they are apparently harmless, unless the magnets get close to the patient's computer or magnetic storage disks. Claims are made for a broad spectrum of benefits from cleansing the blood to giving the patient more energy, but clinical studies have not verified these claims, nor is there an identifiable mechanism by which such benefits might occur.

**Note:**

PhET Explorations: Magnet and Compass

Ever wonder how a compass worked to point you to the Arctic? Explore the interactions between a compass and bar magnet, and then add the Earth and find the surprising answer! Vary the magnet's strength, and see how

things change both inside and outside. Use the field meter to measure how the magnetic field changes.

<https://archive.cnx.org/specials/5ca3e2cc-ae74-11e5-b6d3-f3c228f04b5c/magnet-and-compass/#sim-bar-magnet>

## Section Summary

- Crossed (perpendicular) electric and magnetic fields act as a velocity filter, giving equal and opposite forces on any charge with velocity perpendicular to the fields and of magnitude

**Equation:**

$$v = \frac{E}{B}.$$

## Conceptual Questions

**Exercise:**

**Problem:**

Measurements of the weak and fluctuating magnetic fields associated with brain activity are called magnetoencephalograms (MEGs). Do the brain's magnetic fields imply coordinated or uncoordinated nerve impulses? Explain.

**Exercise:**

**Problem:**

Discuss the possibility that a Hall voltage would be generated on the moving heart of a patient during MRI imaging. Also discuss the same effect on the wires of a pacemaker. (The fact that patients with pacemakers are not given MRIs is significant.)

**Exercise:**

**Problem:**

A patient in an MRI unit turns his head quickly to one side and experiences momentary dizziness and a strange taste in his mouth. Discuss the possible causes.

**Exercise:****Problem:**

You are told that in a certain region there is either a uniform electric or magnetic field. What measurement or observation could you make to determine the type? (Ignore the Earth's magnetic field.)

**Exercise:****Problem:**

An example of magnetohydrodynamics (MHD) comes from the flow of a river (salty water). This fluid interacts with the Earth's magnetic field to produce a potential difference between the two river banks. How would you go about calculating the potential difference?

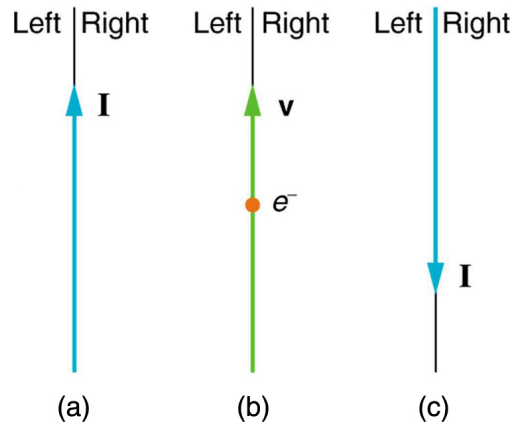
**Exercise:****Problem:**

Draw gravitational field lines between 2 masses, electric field lines between a positive and a negative charge, electric field lines between 2 positive charges and magnetic field lines around a magnet. Qualitatively describe the differences between the fields and the entities responsible for the field lines.

**Problems & Exercises****Exercise:**

**Problem:**

Indicate whether the magnetic field created in each of the three situations shown in [\[link\]](#) is into or out of the page on the left and right of the current.

**Solution:**

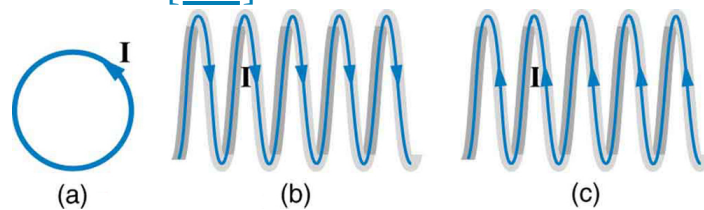
(a) right-into page, left-out of page

(b) right-out of page, left-into page

(c) right-out of page, left-into page

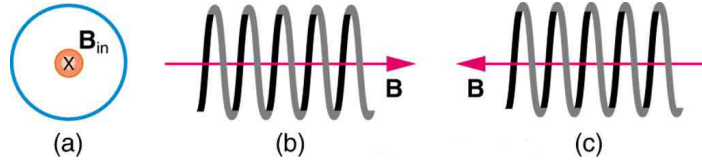
**Exercise:****Problem:**

What are the directions of the fields in the center of the loop and coils shown in [\[link\]](#)?

**Exercise:**

**Problem:**

What are the directions of the currents in the loop and coils shown in [\[link\]](#)?

**Solution:**

- (a) clockwise
- (b) clockwise as seen from the left
- (c) clockwise as seen from the right

**Exercise:****Problem:**

To see why an MRI utilizes iron to increase the magnetic field created by a coil, calculate the current needed in a 400-loop-per-meter circular coil 0.660 m in radius to create a 1.20-T field (typical of an MRI instrument) at its center with no iron present. The magnetic field of a proton is approximately like that of a circular current loop  $0.650 \times 10^{-15}$  m in radius carrying  $1.05 \times 10^4$  A. What is the field at the center of such a loop?

**Solution:**

$$1.01 \times 10^{13} \text{ T}$$

**Exercise:****Problem:**

Inside a motor, 30.0 A passes through a 250-turn circular loop that is 10.0 cm in radius. What is the magnetic field strength created at its center?

**Exercise:****Problem:**

Nonnuclear submarines use batteries for power when submerged. (a) Find the magnetic field 50.0 cm from a straight wire carrying 1200 A from the batteries to the drive mechanism of a submarine. (b) What is the field if the wires to and from the drive mechanism are side by side? (c) Discuss the effects this could have for a compass on the submarine that is not shielded.

---

**Solution:**

(a)  $4.80 \times 10^{-4} \text{ T}$

(b) Zero

(c) If the wires are not paired, the field is about 10 times stronger than Earth's magnetic field and so could severely disrupt the use of a compass.

**Exercise:****Problem:**

How strong is the magnetic field inside a solenoid with 10,000 turns per meter that carries 20.0 A?

**Exercise:****Problem:**

What current is needed in the solenoid described in [\[link\]](#) to produce a magnetic field  $10^4$  times the Earth's magnetic field of  $5.00 \times 10^{-5} \text{ T}$ ?

---

**Solution:**

39.8 A

**Exercise:**

**Problem:**

How far from the starter cable of a car, carrying 150 A, must you be to experience a field less than the Earth's ( $5.00 \times 10^{-5} \text{ T}$ )? Assume a long straight wire carries the current. (In practice, the body of your car shields the dashboard compass.)

**Exercise:****Problem:**

Measurements affect the system being measured, such as the current loop in [\[link\]](#). (a) Estimate the field the loop creates by calculating the field at the center of a circular loop 20.0 cm in diameter carrying 5.00 A. (b) What is the smallest field strength this loop can be used to measure, if its field must alter the measured field by less than 0.0100%?

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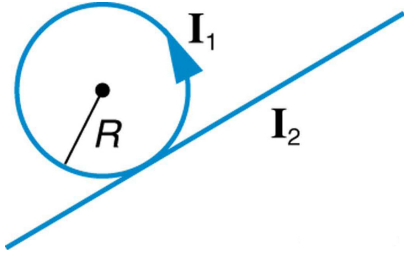
**Solution:**

(a)  $3.14 \times 10^{-5} \text{ T}$

(b) 0.314 T

**Exercise:****Problem:**

[\[link\]](#) shows a long straight wire just touching a loop carrying a current  $I_1$ . Both lie in the same plane. (a) What direction must the current  $I_2$  in the straight wire have to create a field at the center of the loop in the direction opposite to that created by the loop? (b) What is the ratio of  $I_1/I_2$  that gives zero field strength at the center of the loop? (c) What is the direction of the field directly above the loop under this circumstance?



**Exercise:**

**Problem:**

Find the magnitude and direction of the magnetic field at the point equidistant from the wires in [\[link\]\(a\)](#), using the rules of vector addition to sum the contributions from each wire.

**Solution:**

$$7.55 \times 10^{-5} \text{ T}, 23.4^\circ$$

**Exercise:**

**Problem:**

Find the magnitude and direction of the magnetic field at the point equidistant from the wires in [\[link\]\(b\)](#), using the rules of vector addition to sum the contributions from each wire.

**Exercise:**

**Problem:**

What current is needed in the top wire in [\[link\]\(a\)](#) to produce a field of zero at the point equidistant from the wires, if the currents in the bottom two wires are both 10.0 A into the page?

**Solution:**

$$10.0 \text{ A}$$

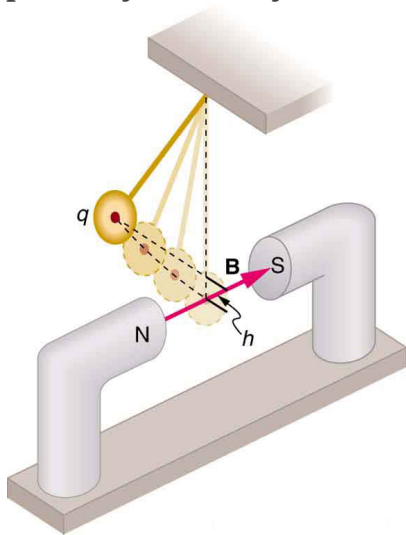
**Exercise:**

**Problem:**

Calculate the size of the magnetic field 20 m below a high voltage power line. The line carries 450 MW at a voltage of 300,000 V.

**Exercise:****Problem: Integrated Concepts**

(a) A pendulum is set up so that its bob (a thin copper disk) swings between the poles of a permanent magnet as shown in [\[link\]](#). What is the magnitude and direction of the magnetic force on the bob at the lowest point in its path, if it has a positive  $0.250\ \mu\text{C}$  charge and is released from a height of 30.0 cm above its lowest point? The magnetic field strength is 1.50 T. (b) What is the acceleration of the bob at the bottom of its swing if its mass is 30.0 grams and it is hung from a flexible string? Be certain to include a free-body diagram as part of your analysis.

**Solution:**

(a)  $9.09 \times 10^{-7}\ \text{N}$  upward

(b)  $3.03 \times 10^{-5}\ \text{m/s}^2$

**Exercise:**

**Problem: Integrated Concepts**

(a) What voltage will accelerate electrons to a speed of  $6.00 \times 10^{-7} \text{ m/s}$ ? (b) Find the radius of curvature of the path of a *proton* accelerated through this potential in a 0.500-T field and compare this with the radius of curvature of an electron accelerated through the same potential.

**Exercise:****Problem: Integrated Concepts**

Find the radius of curvature of the path of a 25.0-MeV proton moving perpendicularly to the 1.20-T field of a cyclotron.

---

**Solution:**

60.2 cm

**Exercise:****Problem: Integrated Concepts**

To construct a nonmechanical water meter, a 0.500-T magnetic field is placed across the supply water pipe to a home and the Hall voltage is recorded. (a) Find the flow rate in liters per second through a 3.00-cm-diameter pipe if the Hall voltage is 60.0 mV. (b) What would the Hall voltage be for the same flow rate through a 10.0-cm-diameter pipe with the same field applied?

**Exercise:****Problem: Integrated Concepts**

(a) Using the values given for an MHD drive in [\[link\]](#), and assuming the force is uniformly applied to the fluid, calculate the pressure created in  $\text{N/m}^2$ . (b) Is this a significant fraction of an atmosphere?

---

**Solution:**

(a)  $1.02 \times 10^3 \text{ N/m}^2$

(b) Not a significant fraction of an atmosphere

**Exercise:****Problem: Integrated Concepts**

(a) Calculate the maximum torque on a 50-turn, 1.50 cm radius circular current loop carrying 50  $\mu\text{A}$  in a 0.500-T field. (b) If this coil is to be used in a galvanometer that reads 50  $\mu\text{A}$  full scale, what force constant spring must be used, if it is attached 1.00 cm from the axis of rotation and is stretched by the  $60^\circ$  arc moved?

**Exercise:****Problem: Integrated Concepts**

A current balance used to define the ampere is designed so that the current through it is constant, as is the distance between wires. Even so, if the wires change length with temperature, the force between them will change. What percent change in force per degree will occur if the wires are copper?

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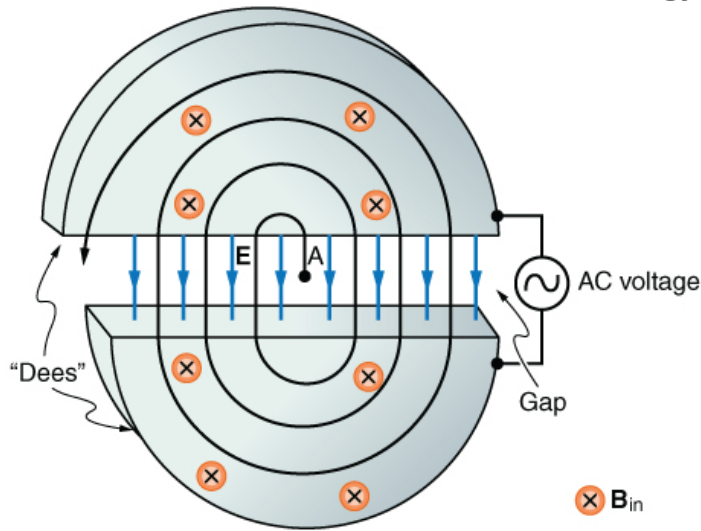
**Solution:**

$$17.0 \times 10^{-4} \% / ^\circ\text{C}$$

**Exercise:****Problem: Integrated Concepts**

(a) Show that the period of the circular orbit of a charged particle moving perpendicularly to a uniform magnetic field is  $T = 2\pi m / (qB)$ . (b) What is the frequency  $f$ ? (c) What is the angular

velocity  $\omega$ ? Note that these results are independent of the velocity and radius of the orbit and, hence, of the energy of the particle. ([link](#).)



Cyclotrons accelerate charged particles orbiting in a magnetic field by placing an AC voltage on the metal Dees, between which the particles move, so that energy is added twice each orbit. The frequency is constant, since it is independent of the particle energy—the radius of the orbit simply increases with energy until the particles approach the edge and are extracted for various experiments and applications.

### Exercise:

#### Problem: Integrated Concepts

A cyclotron accelerates charged particles as shown in [link](#). Using the results of the previous problem, calculate the frequency of the accelerating voltage needed for a proton in a 1.20-T field.

---

**Solution:**

18.3 MHz

**Exercise:****Problem: Integrated Concepts**

(a) A 0.140-kg baseball, pitched at 40.0 m/s horizontally and perpendicular to the Earth's horizontal  $5.00 \times 10^{-5}$  T field, has a 100-nC charge on it. What distance is it deflected from its path by the magnetic force, after traveling 30.0 m horizontally? (b) Would you suggest this as a secret technique for a pitcher to throw curve balls?

**Exercise:****Problem: Integrated Concepts**

(a) What is the direction of the force on a wire carrying a current due east in a location where the Earth's field is due north? Both are parallel to the ground. (b) Calculate the force per meter if the wire carries 20.0 A and the field strength is  $3.00 \times 10^{-5}$  T. (c) What diameter copper wire would have its weight supported by this force? (d) Calculate the resistance per meter and the voltage per meter needed.

---

**Solution:**

(a) Straight up

(b)  $6.00 \times 10^{-4}$  N/m

(c) 94.1  $\mu$ m

(d) 2.47  $\Omega$ /m, 49.4 V/m

**Exercise:****Problem: Integrated Concepts**

One long straight wire is to be held directly above another by repulsion between their currents. The lower wire carries 100 A and the wire 7.50 cm above it is 10-gauge (2.588 mm diameter) copper wire. (a) What current must flow in the upper wire, neglecting the Earth's field? (b) What is the smallest current if the Earth's  $3.00 \times 10^{-5}$  T field is parallel to the ground and is not neglected? (c) Is the supported wire in a stable or unstable equilibrium if displaced vertically? If displaced horizontally?

**Exercise:**

**Problem: Unreasonable Results**

(a) Find the charge on a baseball, thrown at 35.0 m/s perpendicular to the Earth's  $5.00 \times 10^{-5}$  T field, that experiences a 1.00-N magnetic force. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

---

**Solution:**

(a) 571 C

(b) Impossible to have such a large separated charge on such a small object.

(c) The 1.00-N force is much too great to be realistic in the Earth's field.

**Exercise:**

**Problem: Unreasonable Results**

A charged particle having mass  $6.64 \times 10^{-27}$  kg (that of a helium atom) moving at  $8.70 \times 10^5$  m/s perpendicular to a 1.50-T magnetic field travels in a circular path of radius 16.0 mm. (a) What is the charge of the particle? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

**Exercise:**

**Problem: Unreasonable Results**

An inventor wants to generate 120-V power by moving a 1.00-m-long wire perpendicular to the Earth's  $5.00 \times 10^{-5}$  T field. (a) Find the speed with which the wire must move. (b) What is unreasonable about this result? (c) Which assumption is responsible?

---

**Solution:**

(a)  $2.40 \times 10^6$  m/s

(b) The speed is too high to be practical  $\leq 1\%$  speed of light

(c) The assumption that you could reasonably generate such a voltage with a single wire in the Earth's field is unreasonable

**Exercise:****Problem: Unreasonable Results**

Frustrated by the small Hall voltage obtained in blood flow measurements, a medical physicist decides to increase the applied magnetic field strength to get a 0.500-V output for blood moving at 30.0 cm/s in a 1.50-cm-diameter vessel. (a) What magnetic field strength is needed? (b) What is unreasonable about this result? (c) Which premise is responsible?

**Exercise:****Problem: Unreasonable Results**

A surveyor 100 m from a long straight 200-kV DC power line suspects that its magnetic field may equal that of the Earth and affect compass readings. (a) Calculate the current in the wire needed to create a  $5.00 \times 10^{-5}$  T field at this distance. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

---

**Solution:**

(a) 25.0 kA

(b) This current is unreasonably high. It implies a total power delivery in the line of  $50.0 \times 10^9 \text{ W}$ , which is much too high for standard transmission lines.

(c) 100 meters is a long distance to obtain the required field strength. Also coaxial cables are used for transmission lines so that there is virtually no field for DC power lines, because of cancellation from opposing currents. The surveyor's concerns are not a problem for his magnetic field measurements.

**Exercise:****Problem: Construct Your Own Problem**

Consider a mass separator that applies a magnetic field perpendicular to the velocity of ions and separates the ions based on the radius of curvature of their paths in the field. Construct a problem in which you calculate the magnetic field strength needed to separate two ions that differ in mass, but not charge, and have the same initial velocity. Among the things to consider are the types of ions, the velocities they can be given before entering the magnetic field, and a reasonable value for the radius of curvature of the paths they follow. In addition, calculate the separation distance between the ions at the point where they are detected.

**Exercise:****Problem: Construct Your Own Problem**

Consider using the torque on a current-carrying coil in a magnetic field to detect relatively small magnetic fields (less than the field of the Earth, for example). Construct a problem in which you calculate the maximum torque on a current-carrying loop in a magnetic field. Among the things to be considered are the size of the coil, the number

of loops it has, the current you pass through the coil, and the size of the field you wish to detect. Discuss whether the torque produced is large enough to be effectively measured. Your instructor may also wish for you to consider the effects, if any, of the field produced by the coil on the surroundings that could affect detection of the small field.

## **Glossary**

magnetic resonance imaging (MRI)

a medical imaging technique that uses magnetic fields create detailed images of internal tissues and organs

nuclear magnetic resonance (NMR)

a phenomenon in which an externally applied magnetic field interacts with the nuclei of certain atoms

magnetocardiogram (MCG)

a recording of the heart's magnetic field as it beats

magnetoencephalogram (MEG)

a measurement of the brain's magnetic field

## Useful Information

This appendix is broken into several tables.

- [\[link\]](#), Important Constants
- [\[link\]](#), Submicroscopic Masses
- [\[link\]](#), Solar System Data
- [\[link\]](#), Metric Prefixes for Powers of Ten and Their Symbols
- [\[link\]](#), The Greek Alphabet
- [\[link\]](#), SI units
- [\[link\]](#), Selected British Units
- [\[link\]](#), Other Units
- [\[link\]](#), Useful Formulae

Symbol	Meaning	Best Value	Approximate Value
$c$	Speed of light in vacuum	$2.99792458 \times 10^8 \text{ m/s}$	$3.00 \times 10^8 \text{ m/s}$
$G$	Gravitational constant	$6.67408(31) \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$	$6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$
$N_A$	Avogadro's number	$6.02214129(27) \times 10^{23}$	$6.02 \times 10^{23}$
$k$	Boltzmann's constant	$1.3806488(13) \times 10^{-23} \text{ J/K}$	$1.38 \times 10^{-23} \text{ J/K}$
$R$	Gas constant	$8.3144621(75) \text{ J/mol} \cdot \text{K}$	$8.31 \text{ J/mol} \cdot \text{K} = 1.99 \text{ cal/mol} \cdot \text{K} =$
$\sigma$	Stefan-Boltzmann constant	$5.670373(21) \times 10^{-8} \text{ W/m}^2 \cdot \text{K}$	$5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}$
$k$	Coulomb force constant	$8.987551788... \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$	$8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
$q_e$	Charge on electron	$-1.602176565(35) \times 10^{-19} \text{ C}$	$-1.60 \times 10^{-19} \text{ C}$
$\epsilon_0$	Permittivity of free space	$8.854187817... \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$	$8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$
$\mu_0$	Permeability of free space	$4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$	$1.26 \times 10^{-6} \text{ T} \cdot \text{m/A}$
$h$	Planck's constant	$6.62606957(29) \times 10^{-34} \text{ J} \cdot \text{s}$	$6.63 \times 10^{-34} \text{ J} \cdot \text{s}$

Important Constants<sup>[footnote]</sup>

Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, [www.physics.nist.gov/cuu](http://www.physics.nist.gov/cuu) (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Symbol	Meaning	Best Value	Approximate Value
$m_e$	Electron mass	$9.10938291(40) \times 10^{-31}\text{kg}$	$9.11 \times 10^{-31}\text{kg}$
$m_p$	Proton mass	$1.672621777(74) \times 10^{-27}\text{kg}$	$1.6726 \times 10^{-27}\text{kg}$
$m_n$	Neutron mass	$1.674927351(74) \times 10^{-27}\text{kg}$	$1.6749 \times 10^{-27}\text{kg}$
u	Atomic mass unit	$1.660538921(73) \times 10^{-27}\text{kg}$	$1.6605 \times 10^{-27}\text{kg}$

#### Submicroscopic Masses<sup>[footnote]</sup>

Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, [www.physics.nist.gov/cuu](http://www.physics.nist.gov/cuu) (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

<b>Sun</b>	mass	$1.99 \times 10^{30}\text{kg}$
	average radius	$6.96 \times 10^8\text{m}$
	Earth-sun distance (average)	$1.496 \times 10^{11}\text{m}$
<b>Earth</b>	mass	$5.9736 \times 10^{24}\text{kg}$
	average radius	$6.376 \times 10^6\text{m}$
	orbital period	$3.16 \times 10^7\text{s}$



Epsilon	Ε	ε	Lambda	Λ	λ	Rho	Ρ	ρ	Psi	Ψ	ψ
Zeta	Ζ	ζ	Mu	Μ	μ	Sigma	Σ	σ	Omega	Ω	ω

The Greek Alphabet

	Entity	Abbreviation	Name
Fundamental units	Length	m	meter
	Mass	kg	kilogram
	Time	s	second
	Current	A	ampere
Supplementary unit	Angle	rad	radian
Derived units	Force	$N = kg \cdot m/s^2$	newton
	Energy	$J = kg \cdot m^2/s^2$	joule
	Power	$W = J/s$	watt
	Pressure	$Pa = N/m^2$	pascal
	Frequency	$Hz = 1/s$	hertz
	Electronic potential	$V = J/C$	volt
	Capacitance	$F = C/V$	farad
	Charge	$C = s \cdot A$	coulomb
	Resistance	$\Omega = V/A$	ohm

	Entity	Abbreviation	Name
	Magnetic field	$T = N/(A \cdot m)$	tesla
	Nuclear decay rate	$Bq = 1/s$	becquerel

#### SI Units

Length	1 inch (in.) = 2.54 cm (exactly)
	1 foot (ft) = 0.3048 m
	1 mile (mi) = 1.609 km
Force	1 pound (lb) = 4.448 N
Energy	1 British thermal unit (Btu) = $1.055 \times 10^3$ J
Power	1 horsepower (hp) = 746 W
Pressure	$1 \text{ lb/in}^2 = 6.895 \times 10^3 \text{ Pa}$

#### Selected British Units

Length	1 light year (ly) = $9.46 \times 10^{15}$ m
	1 astronomical unit (au) = $1.50 \times 10^{11}$ m
	1 nautical mile = 1.852 km
	1 angstrom( $\text{\AA}$ ) = $10^{-10}$ m
Area	1 acre (ac) = $4.05 \times 10^3 \text{ m}^2$
	1 square foot ( $\text{ft}^2$ ) = $9.29 \times 10^{-2} \text{ m}^2$
	1 barn ( $b$ ) = $10^{-28} \text{ m}^2$
Volume	1 liter ( $L$ ) = $10^{-3} \text{ m}^3$

	1 U.S. gallon (gal) = $3.785 \times 10^{-3} \text{ m}^3$
Mass	1 solar mass = $1.99 \times 10^{30} \text{ kg}$
	1 metric ton = $10^3 \text{ kg}$
	1 atomic mass unit ( $u$ ) = $1.6605 \times 10^{-27} \text{ kg}$
Time	1 year ( $y$ ) = $3.16 \times 10^7 \text{ s}$
	1 day ( $d$ ) = 86,400 s
Speed	1 mile per hour (mph) = 1.609 km/h
	1 nautical mile per hour (naut) = 1.852 km/h
Angle	1 degree ( $^\circ$ ) = $1.745 \times 10^{-2} \text{ rad}$
	1 minute of arc ( $'$ ) = 1/60 degree
	1 second of arc ( $''$ ) = 1/60 minute of arc
	1 grad = $1.571 \times 10^{-2} \text{ rad}$
Energy	1 kiloton TNT (kT) = $4.2 \times 10^{12} \text{ J}$
	1 kilowatt hour (kW · h) = $3.60 \times 10^6 \text{ J}$
	1 food calorie (kcal) = 4186 J
	1 calorie (cal) = 4.186 J
	1 electron volt (eV) = $1.60 \times 10^{-19} \text{ J}$
Pressure	1 atmosphere (atm) = $1.013 \times 10^5 \text{ Pa}$
	1 millimeter of mercury (mm Hg) = 133.3 Pa
	1 torricelli (torr) = 1 mm Hg = 133.3 Pa
Nuclear decay rate	1 curie (Ci) = $3.70 \times 10^{10} \text{ Bq}$

#### Other Units

Circumference of a circle with radius $r$ or diameter $d$	$C = 2\pi r = \pi d$
Area of a circle with radius $r$ or diameter $d$	$A = \pi r^2 = \pi d^2/4$
Area of a sphere with radius $r$	$A = 4\pi r^2$

Volume of a sphere with radius  $r$

$$V = (4/3)(\pi r^3)$$

Useful Formulae

## Glossary of Key Symbols and Notation

In this glossary, key symbols and notation are briefly defined.

Symbol	Definition
any symbol	average (indicated by a bar over a symbol— e.g., $\bar{v}$ is average velocity)
$^{\circ}\text{C}$	Celsius degree
$^{\circ}\text{F}$	Fahrenheit degree
//	parallel
$\perp$	perpendicular
$\propto$	proportional to
$\pm$	plus or minus

Symbol	Definition
0	zero as a subscript denotes an initial value
$\alpha$	alpha rays
$\alpha$	angular acceleration
$\alpha$	temperature coefficient(s) of resistivity
$\beta$	beta rays
$\beta$	sound level
$\beta$	volume coefficient of expansion
$\beta^{-}$	electron emitted in nuclear beta decay
$\beta^{+}$	positron decay
$\gamma$	gamma rays

Symbol	Definition
$\gamma$	surface tension
$\gamma = 1/\sqrt{1 - v^2/c^2}$	a constant used in relativity
$\Delta$	change in whatever quantity follows
$\delta$	uncertainty in whatever quantity follows
$\Delta E$	change in energy between the initial and final orbits of an electron in an atom
$\Delta E$	uncertainty in energy
$\Delta m$	difference in mass between initial and final products
$\Delta N$	number of decays that occur
$\Delta p$	change in momentum

Symbol	Definition
$\Delta p$	uncertainty in momentum
$\Delta PE_g$	change in gravitational potential energy
$\Delta\theta$	rotation angle
$\Delta s$	distance traveled along a circular path
$\Delta t$	uncertainty in time
$\Delta t_0$	proper time as measured by an observer at rest relative to the process
$\Delta V$	potential difference
$\Delta x$	uncertainty in position
$\epsilon_0$	permittivity of free space
$\eta$	viscosity

Symbol	Definition
$\theta$	angle between the force vector and the displacement vector
$\theta$	angle between two lines
$\theta$	contact angle
$\theta$	direction of the resultant
$\theta_b$	Brewster's angle
$\theta_c$	critical angle
$\kappa$	dielectric constant
$\lambda$	decay constant of a nuclide
$\lambda$	wavelength
$\lambda_n$	wavelength in a medium

Symbol	Definition
$\mu_0$	permeability of free space
$\mu_k$	coefficient of kinetic friction
$\mu_s$	coefficient of static friction
$\nu_e$	electron neutrino
$\pi^+$	positive pion
$\pi^-$	negative pion
$\pi^0$	neutral pion
$\rho$	density
$\rho_c$	critical density, the density needed to just halt universal expansion
$\rho_{fl}$	fluid density

Symbol	Definition
$\rho_{\text{obj}}$	average density of an object
$\rho/\rho_{\text{w}}$	specific gravity
$\tau$	characteristic time constant for a resistance and inductance (RL) or resistance and capacitance (RC) circuit
$\tau$	characteristic time for a resistor and capacitor (RC) circuit
$\tau$	torque
$\Upsilon$	upsilon meson
$\Phi$	magnetic flux
$\phi$	phase angle
$\Omega$	ohm (unit)
$\omega$	angular velocity

Symbol	Definition
A	ampere (current unit)
$A$	area
$A$	cross-sectional area
$A$	total number of nucleons
$a$	acceleration
$a_B$	Bohr radius
$a_c$	centripetal acceleration
$a_t$	tangential acceleration
AC	alternating current
AM	amplitude modulation

Symbol	Definition
atm	atmosphere
$B$	baryon number
$B$	blue quark color
$B$	antiblue (yellow) antiquark color
$b$	quark flavor bottom or beauty
$B$	bulk modulus
$B$	magnetic field strength
$B_{\text{int}}$	electron's intrinsic magnetic field
$B_{\text{orb}}$	orbital magnetic field
BE	binding energy of a nucleus—it is the energy required to completely disassemble it into separate protons and neutrons

Symbol	Definition
$BE/A$	binding energy per nucleon
Bq	becquerel—one decay per second
$C$	capacitance (amount of charge stored per volt)
$C$	coulomb (a fundamental SI unit of charge)
$C_p$	total capacitance in parallel
$C_s$	total capacitance in series
CG	center of gravity
CM	center of mass
$c$	quark flavor charm
$c$	specific heat

Symbol	Definition
$c$	speed of light
Cal	kilocalorie
cal	calorie
$COP_{hp}$	heat pump's coefficient of performance
$COP_{ref}$	coefficient of performance for refrigerators and air conditioners
$\cos \theta$	cosine
$\cot \theta$	cotangent
$\csc \theta$	cosecant
$D$	diffusion constant
$d$	displacement

Symbol	Definition
$d$	quark flavor down
dB	decibel
$d_i$	distance of an image from the center of a lens
$d_o$	distance of an object from the center of a lens
DC	direct current
$E$	electric field strength
$\varepsilon$	emf (voltage) or Hall electromotive force
emf	electromotive force
$E$	energy of a single photon
$E$	nuclear reaction energy

Symbol	Definition
$E$	relativistic total energy
$E$	total energy
$E_0$	ground state energy for hydrogen
$E_0$	rest energy
EC	electron capture
$E_{\text{cap}}$	energy stored in a capacitor
Eff	efficiency—the useful work output divided by the energy input
Eff <sub>C</sub>	Carnot efficiency
$E_{\text{in}}$	energy consumed (food digested in humans)
$E_{\text{ind}}$	energy stored in an inductor

Symbol	Definition
$E_{\text{out}}$	energy output
$e$	emissivity of an object
$e^+$	antielectron or positron
eV	electron volt
F	farad (unit of capacitance, a coulomb per volt)
F	focal point of a lens
<b>F</b>	force
$F$	magnitude of a force
$F$	restoring force
$F_{\text{B}}$	buoyant force

Symbol	Definition
$F_c$	centripetal force
$F_i$	force input
$\mathbf{F}_{\text{net}}$	net force
$F_o$	force output
FM	frequency modulation
$f$	focal length
$f$	frequency
$f_0$	resonant frequency of a resistance, inductance, and capacitance (RLC) series circuit
$f_0$	threshold frequency for a particular material (photoelectric effect)

Symbol	Definition
$f_1$	fundamental
$f_2$	first overtone
$f_3$	second overtone
$f_B$	beat frequency
$f_k$	magnitude of kinetic friction
$f_s$	magnitude of static friction
$G$	gravitational constant
$G$	green quark color
$\bar{G}$	antigreen (magenta) antiquark color

Symbol	Definition
$g$	acceleration due to gravity
$g$	gluons (carrier particles for strong nuclear force)
$h$	change in vertical position
$h$	height above some reference point
$h$	maximum height of a projectile
$h$	Planck's constant
$hf$	photon energy
$h_i$	height of the image
$h_o$	height of the object
$I$	electric current

Symbol	Definition
$I$	intensity
$I$	intensity of a transmitted wave
$I$	moment of inertia (also called rotational inertia)
$I_0$	intensity of a polarized wave before passing through a filter
$I_{\text{ave}}$	average intensity for a continuous sinusoidal electromagnetic wave
$I_{\text{rms}}$	average current
J	joule
$J/\Psi$	Joules/psi meson
K	kelvin
$k$	Boltzmann constant

Symbol	Definition
$k$	force constant of a spring
$K_{\alpha}$	x rays created when an electron falls into an $n = 1$ shell vacancy from the $n = 3$ shell
$K_{\beta}$	x rays created when an electron falls into an $n = 2$ shell vacancy from the $n = 3$ shell
kcal	kilocalorie
KE	translational kinetic energy
KE + PE	mechanical energy
$\text{KE}_e$	kinetic energy of an ejected electron
$\text{KE}_{\text{rel}}$	relativistic kinetic energy
$\text{KE}_{\text{rot}}$	rotational kinetic energy
KE	thermal energy

Symbol	Definition
kg	kilogram (a fundamental SI unit of mass)
$L$	angular momentum
L	liter
$L$	magnitude of angular momentum
$L$	self-inductance
$\ell$	angular momentum quantum number
$L_{\alpha}$	x rays created when an electron falls into an $n = 2$ shell from the $n = 3$ shell
$L_e$	electron total family number
$L_{\mu}$	muon family total number
$L_{\tau}$	tau family total number

Symbol	Definition
$L_f$	heat of fusion
$L_f$ and $L_v$	latent heat coefficients
$L_{orb}$	orbital angular momentum
$L_s$	heat of sublimation
$L_v$	heat of vaporization
$L_z$	z - component of the angular momentum
$M$	angular magnification
$M$	mutual inductance
m	indicates metastable state
$m$	magnification

Symbol	Definition
$m$	mass
$m$	mass of an object as measured by a person at rest relative to the object
m	meter (a fundamental SI unit of length)
$m$	order of interference
$m$	overall magnification (product of the individual magnifications)
$m(^AX)$	atomic mass of a nuclide
MA	mechanical advantage
$m_e$	magnification of the eyepiece
$m_e$	mass of the electron
$m_\ell$	angular momentum projection quantum number

Symbol	Definition
$m_n$	mass of a neutron
$m_o$	magnification of the objective lens
mol	mole
$m_p$	mass of a proton
$m_s$	spin projection quantum number
$N$	magnitude of the normal force
N	newton
<b>N</b>	normal force
$N$	number of neutrons
$n$	index of refraction

Symbol	Definition
$n$	number of free charges per unit volume
$N_A$	Avogadro's number
$N_r$	Reynolds number
$N \cdot m$	newton-meter (work-energy unit)
$N \cdot m$	newtons times meters (SI unit of torque)
OE	other energy
$P$	power
$P$	power of a lens
$P$	pressure
<b>p</b>	momentum

Symbol	Definition
$p$	momentum magnitude
$p$	relativistic momentum
$\mathbf{p}_{\text{tot}}$	total momentum
$\mathbf{p}'_{\text{tot}}$	total momentum some time later
$P_{\text{abs}}$	absolute pressure
$P_{\text{atm}}$	atmospheric pressure
$P_{\text{atm}}$	standard atmospheric pressure
PE	potential energy
PE <sub>el</sub>	elastic potential energy
PE <sub>elec</sub>	electric potential energy

Symbol	Definition
$PE_s$	potential energy of a spring
$P_g$	gauge pressure
$P_{in}$	power consumption or input
$P_{out}$	useful power output going into useful work or a desired, form of energy
$Q$	latent heat
$Q$	net heat transferred into a system
$Q$	flow rate—volume per unit time flowing past a point
$+Q$	positive charge
$-Q$	negative charge

Symbol	Definition
$q$	electron charge
$q_p$	charge of a proton
$q$	test charge
QF	quality factor
$R$	activity, the rate of decay
$R$	radius of curvature of a spherical mirror
$R$	red quark color
$R$	antired (cyan) quark color
$R$	resistance
R	resultant or total displacement

Symbol	Definition
$R$	Rydberg constant
$R$	universal gas constant
$r$	distance from pivot point to the point where a force is applied
$r$	internal resistance
$r_{\perp}$	perpendicular lever arm
$r$	radius of a nucleus
$r$	radius of curvature
$r$	resistivity
r or rad	radiation dose unit
rem	roentgen equivalent man

Symbol	Definition
rad	radian
RBE	relative biological effectiveness
RC	resistor and capacitor circuit
rms	root mean square
$r_n$	radius of the $n$ th H-atom orbit
$R_p$	total resistance of a parallel connection
$R_s$	total resistance of a series connection
$R_s$	Schwarzschild radius
$S$	entropy
<b>S</b>	intrinsic spin (intrinsic angular momentum)

Symbol	Definition
$S$	magnitude of the intrinsic (internal) spin angular momentum
$S$	shear modulus
$S$	strangeness quantum number
$s$	quark flavor strange
s	second (fundamental SI unit of time)
$s$	spin quantum number
<b>s</b>	total displacement
$\sec \theta$	secant
$\sin \theta$	sine
$s_z$	z-component of spin angular momentum

Symbol	Definition
$T$	period—time to complete one oscillation
$T$	temperature
$T_c$	critical temperature—temperature below which a material becomes a superconductor
$T$	tension
T	tesla (magnetic field strength $B$ )
$t$	quark flavor top or truth
$t$	time
$t_{1/2}$	half-life—the time in which half of the original nuclei decay
$\tan \theta$	tangent
$U$	internal energy

Symbol	Definition
$u$	quark flavor up
u	unified atomic mass unit
<b>u</b>	velocity of an object relative to an observer
<b>u'</b>	velocity relative to another observer
$V$	electric potential
$V$	terminal voltage
V	volt (unit)
$V$	volume
<b>v</b>	relative velocity between two observers
$v$	speed of light in a material

Symbol	Definition
$\mathbf{v}$	velocity
$\mathbf{v}$	average fluid velocity
$V_B - V_A$	change in potential
$\mathbf{v}_d$	drift velocity
$V_p$	transformer input voltage
$V_{\text{rms}}$	rms voltage
$V_s$	transformer output voltage
$\mathbf{v}_{\text{tot}}$	total velocity
$v_w$	propagation speed of sound or other wave
$\mathbf{v}_w$	wave velocity

Symbol	Definition
$W$	work
$W$	net work done by a system
$W$	watt
$w$	weight
$w_{\text{fl}}$	weight of the fluid displaced by an object
$W_{\text{c}}$	total work done by all conservative forces
$W_{\text{nc}}$	total work done by all nonconservative forces
$W_{\text{out}}$	useful work output
$X$	amplitude
$X$	symbol for an element

Symbol	Definition
${}_A^Z X_N$	notation for a particular nuclide
$x$	deformation or displacement from equilibrium
$x$	displacement of a spring from its undeformed position
$x$	horizontal axis
$X_C$	capacitive reactance
$X_L$	inductive reactance
$x_{\text{rms}}$	root mean square diffusion distance
$y$	vertical axis
$Y$	elastic modulus or Young's modulus
$Z$	atomic number (number of protons in a nucleus)

Symbol	Definition
$Z$	impedance